Dominances en programmation linéaire :
ordonnancement autour d'une date d'échéance commune

soutenance de thèse d'**Anne-Elisabeth FALQ**

encadrée par Pierre Fouilhoux et Safia Kedad-Sidhoum

2 Novembre 2020, LIP6, Paris

slides et tapuscrit disponibles sur
http://perso.eleves.ens-rennes.fr/~afalq494/recherche-these.html

Comment est née cette thèse? (only slide in french)

2016/2017 cours de M2 à l'UPMC, donné par Pierre et Safia

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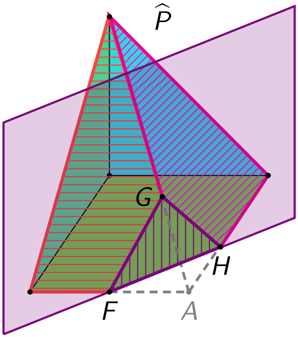
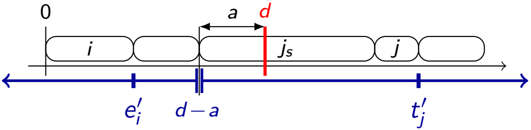
PLNE et polyèdres



Ordonnancement juste-à-temps



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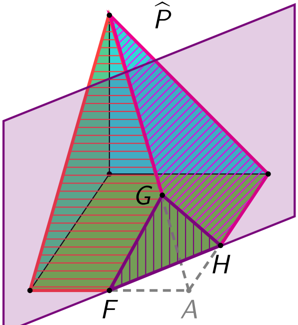

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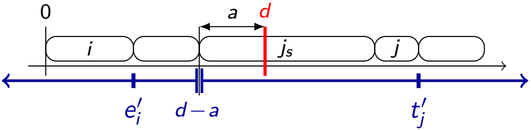

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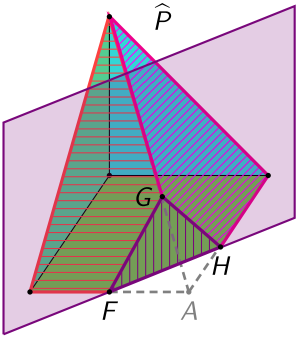

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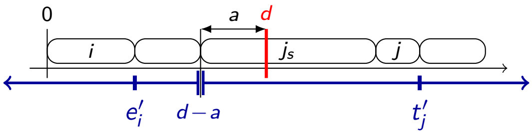
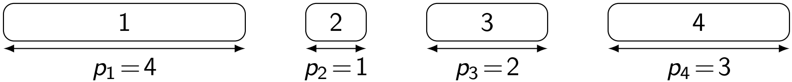


Outline

1. Introduction
   - Scheduling around a common due date
   - Known results about UCDDP and CDDP
   - How to encode schedules?
2. Focus 1: A formulation for UCDDP using natural variables
3. Interlude : a first attempt to eliminate dominated solutions
4. Focus 2: dominance inequality
5. Conclusion and perspectives

Scheduling around a common due-date on a single machine

An instance = • a set of tasks : $J = \{1, 2, 3, 4\}$



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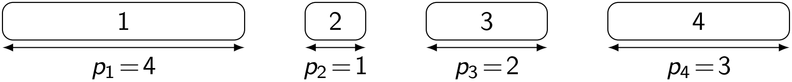

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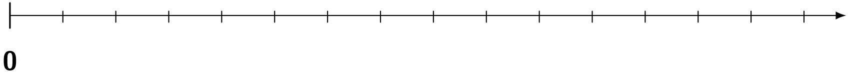

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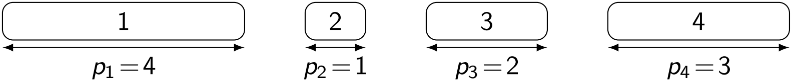

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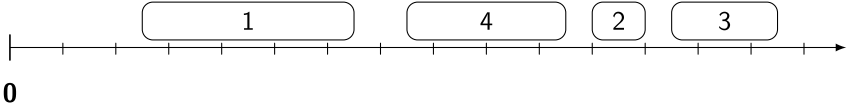

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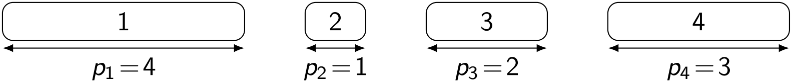

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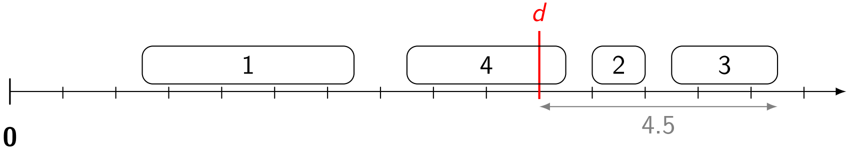

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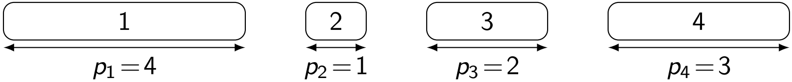

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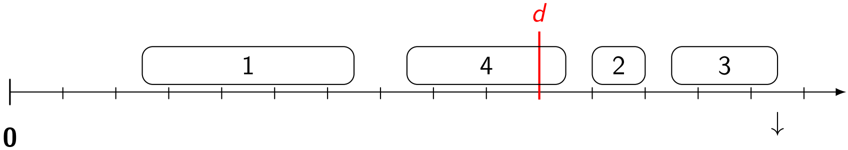

$4,5 \times 2 = 9$

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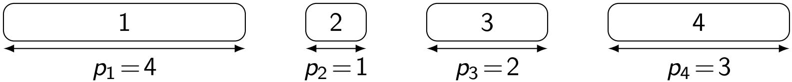

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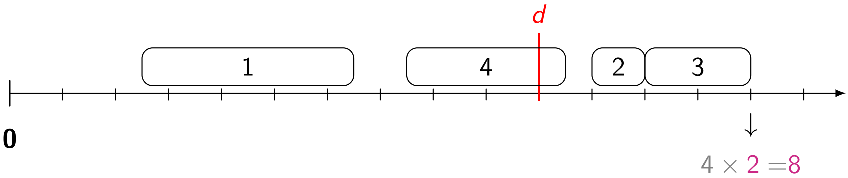

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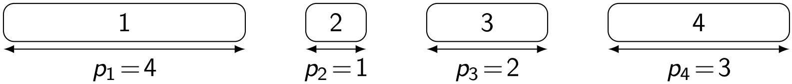

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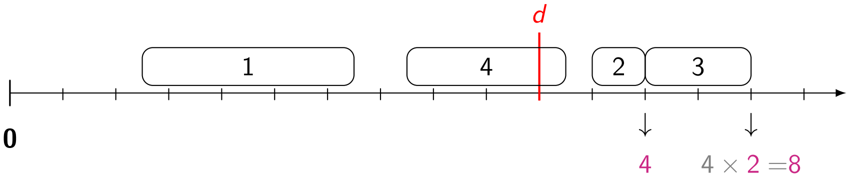

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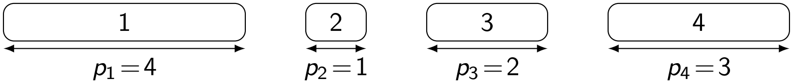

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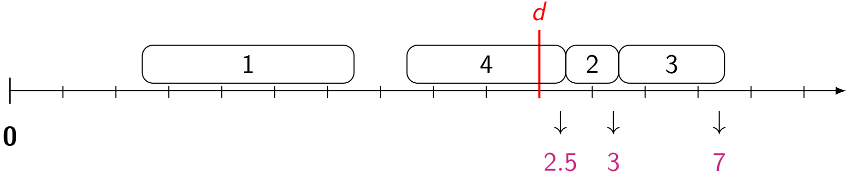

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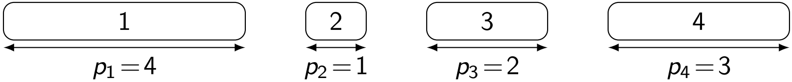

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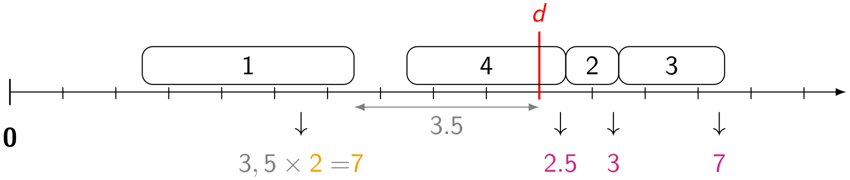

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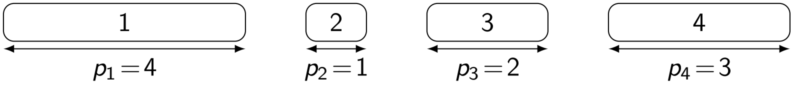

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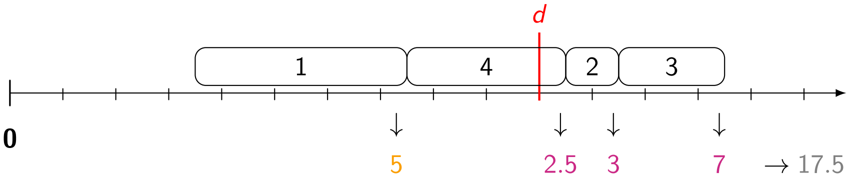

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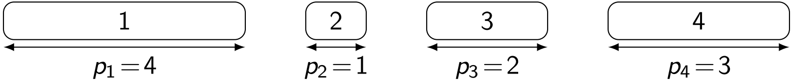

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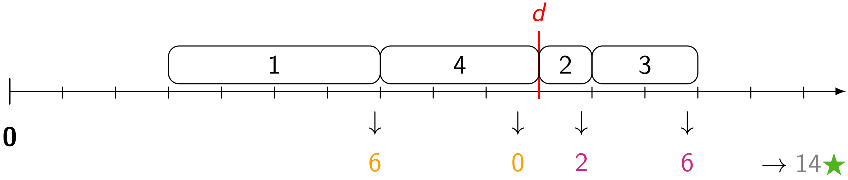

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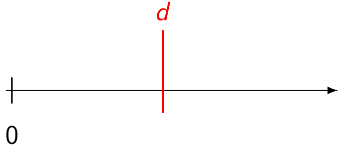

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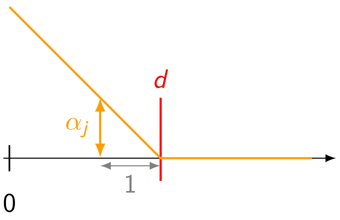

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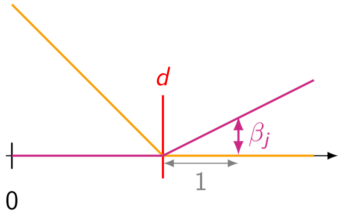

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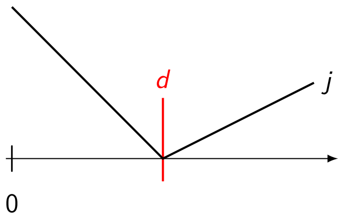

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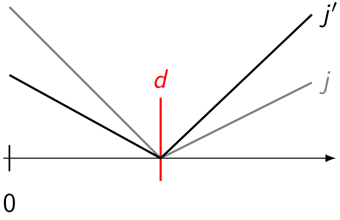

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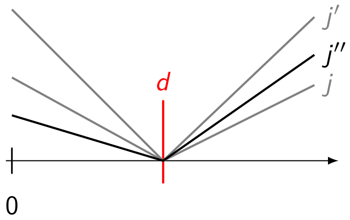

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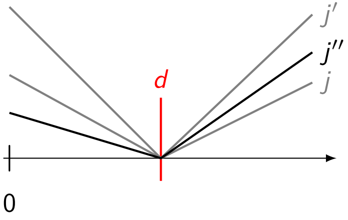

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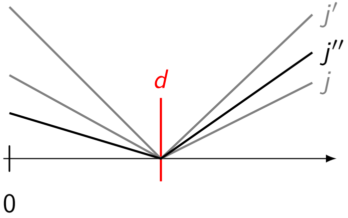

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The objective = $\min \sum_{j \in J} \alpha_j E_j + \beta_j T_j$ = minimize the sum of earliness and tardiness penalties

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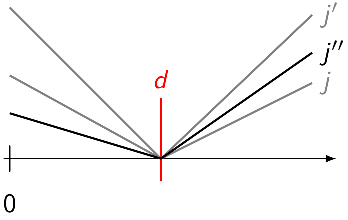

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In both cases,

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$\rightarrow$ other solutions can be discarded

Dominance properties and complexity

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dominance properties	

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Kanet, 1981, Naval Research Logistics Quaterly

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Hall and Posner, 1991, Operations research

Dominance properties and complexity

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dominance properties	• without idle time  • one on time task  (+ "V-shaped")
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Hoogeveen and van de Velde, 1991, European Journal of Operational research
Sourd, 2009, Informs Journal on Computing

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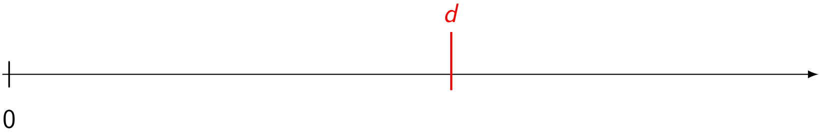
	unrestrictive case $d \geqslant \sum_j p_j$	general case
dominance properties	• without idle time • one on time task (+ "V-shaped")	• without idle time • one on time task **or** beginning at 0 ~~V-shaped~~
complexity	$\forall j \in J,\ \alpha_j = \beta_j = \omega \rightarrow$ P $\forall j \in J,\ \alpha_j = \beta_j \rightarrow$ weakly NP-hard arbitrary $\alpha_j, \beta_j \rightarrow$ NP-hard	$\forall j \in J,\ \alpha_j = \beta_j = \omega \rightarrow$ NP-hard $\forall j \in J,\ \alpha_j = \beta_j \rightarrow$ weakly NP-hard arbitrary $\alpha_j, \beta_j \rightarrow$ Branch-and-Bound can solve up to 1000-task instances

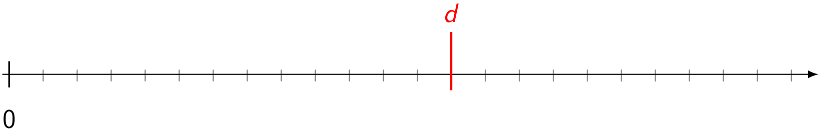
Kanet, 1981, Naval Research Logistics Quaterly
Hall and Posner, 1991, Operations research
Hoogeveen and van de Velde, 1991, European Journal of Operational research
Sourd, 2009, Informs Journal on Computing

Time-indexed variables



First let us discretize the time horizon .

Time-indexed variables



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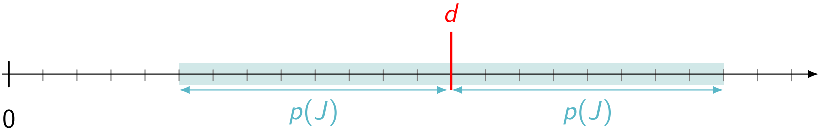

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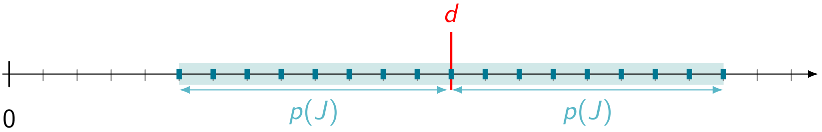

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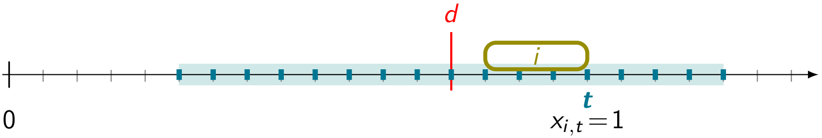
Time-indexed variables



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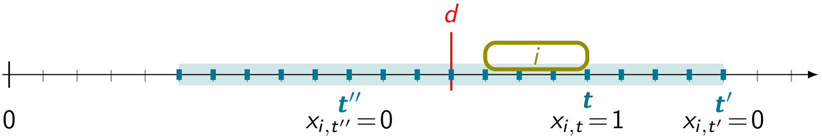

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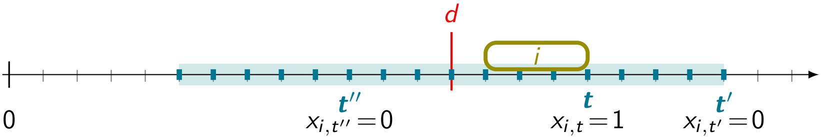

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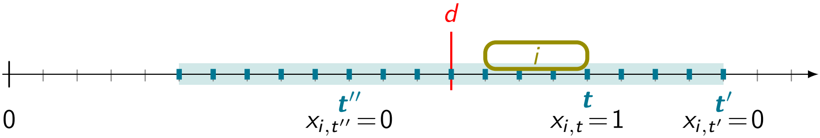

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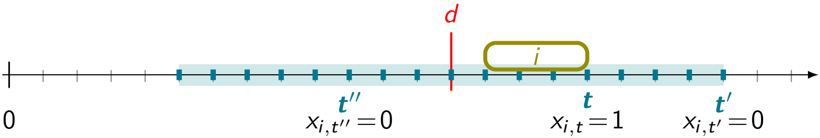
Constraints:
- $\forall i \in J, \sum_{t\in\mathcal{T}} x_{i,t} = 1$ task i is placed
- $\forall t \in \mathcal{T}, \sum_{i\in J}\sum_{\substack{s\in\mathcal{T}\\ s\in[t,t+p_i[}} x_{i,s} \leqslant 1$ at most 1 task is in progress at t
- $\forall i \in J, \forall t \in \mathcal{T}, x_{i,t} \in \mathbb{Z}$ integrity constraint

Time-indexed variables



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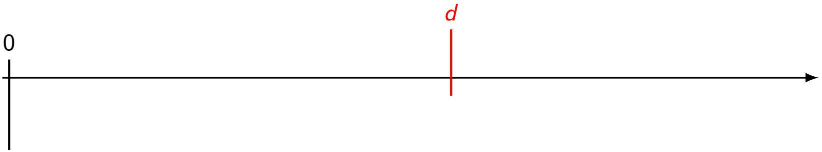
- \+ easy to formulate as a **MIP**
- \+ good relaxation value
- − $2n\,p(J)$ binary variables = a pseudo polynomial number
- − $n + n\,p(J)$ inequalities = a pseudo polynomial number

Completion time variables



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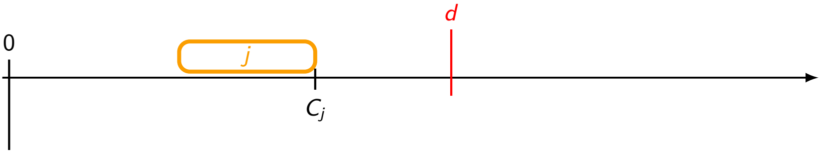

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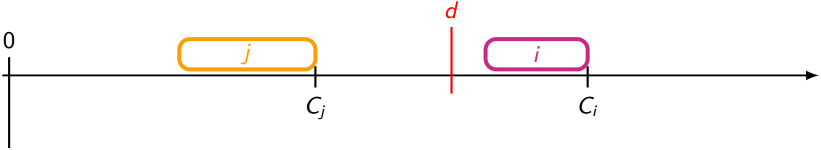

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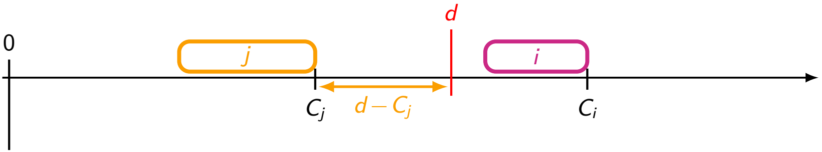

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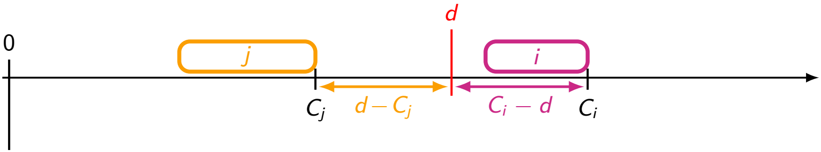

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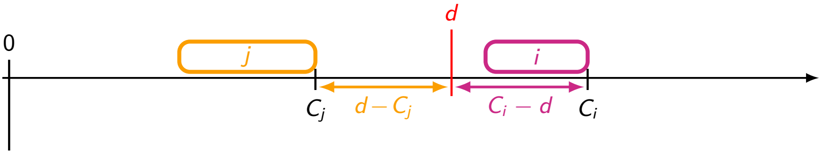

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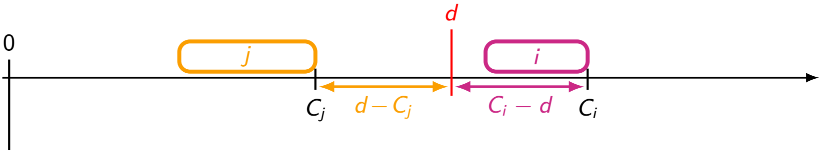

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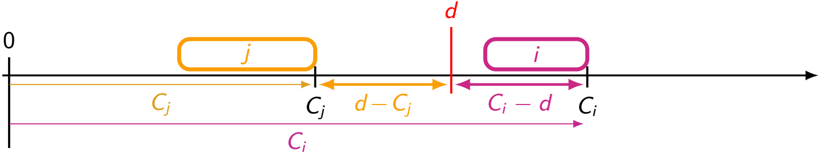

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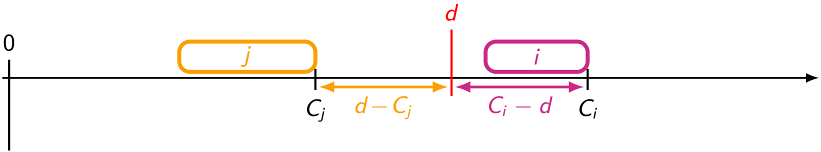

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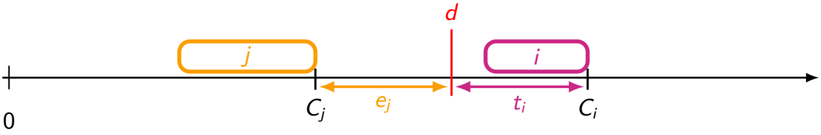
Earliness–Tardiness variables



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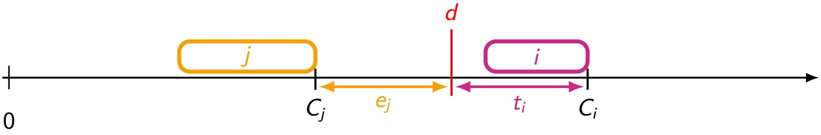

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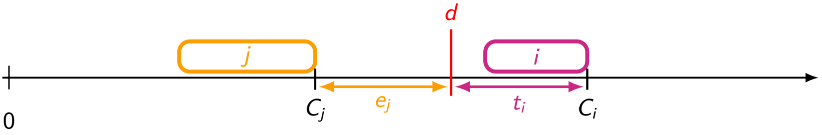

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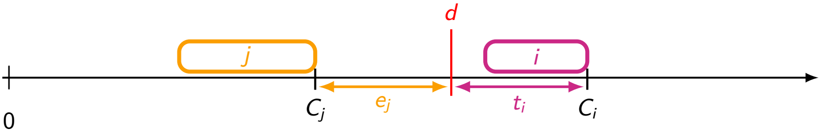

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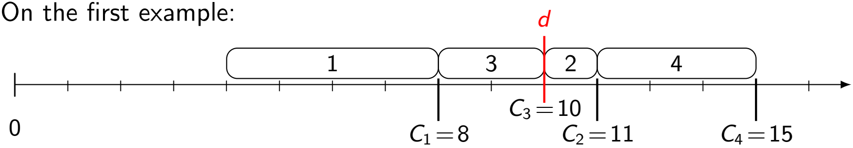

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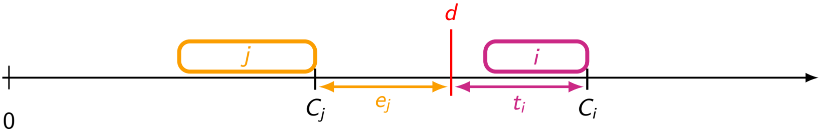

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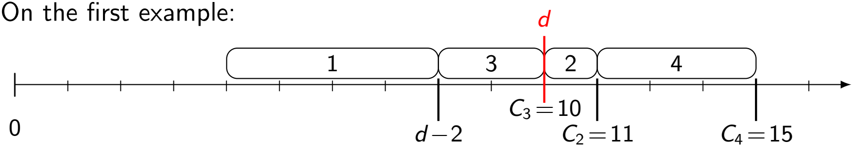

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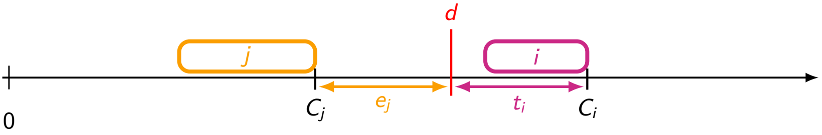

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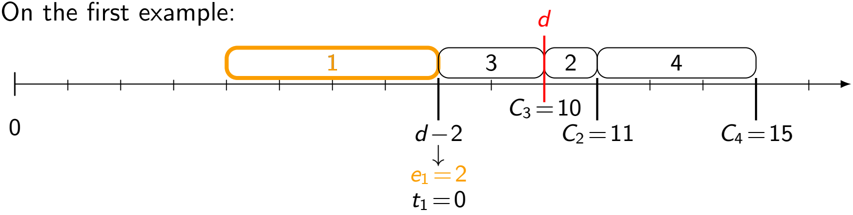

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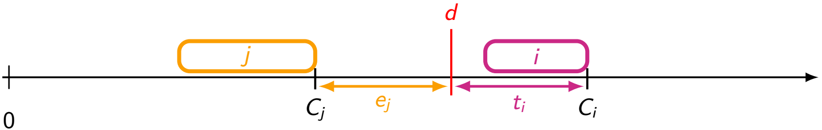

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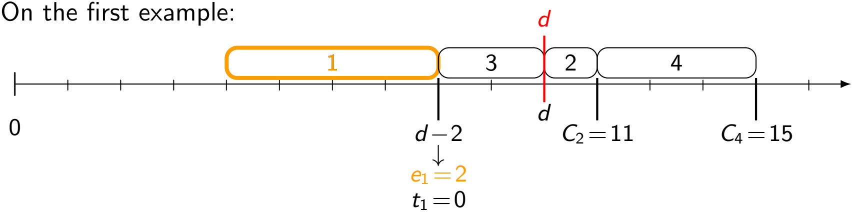

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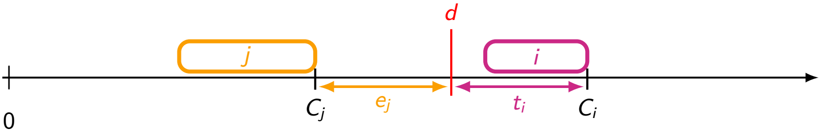

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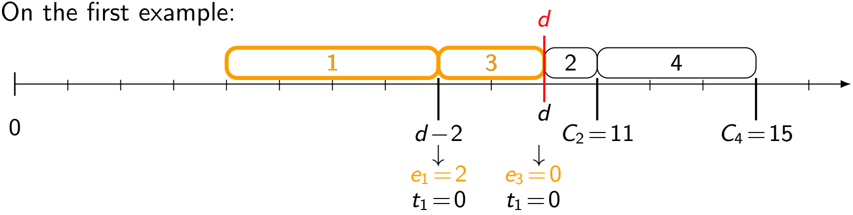

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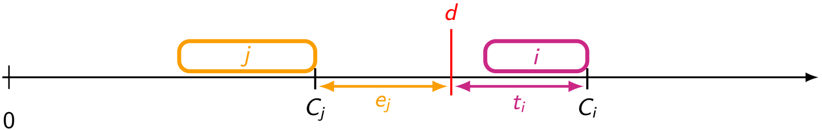

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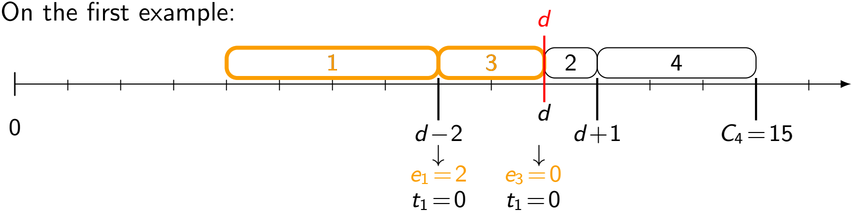

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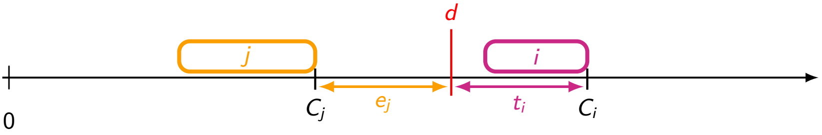

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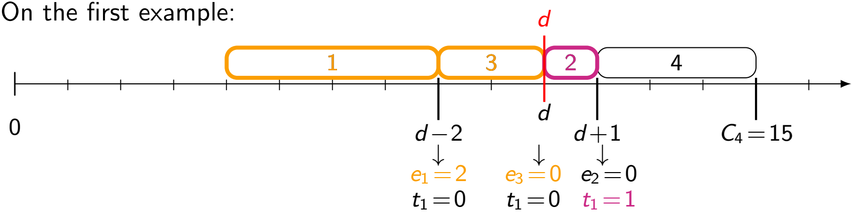

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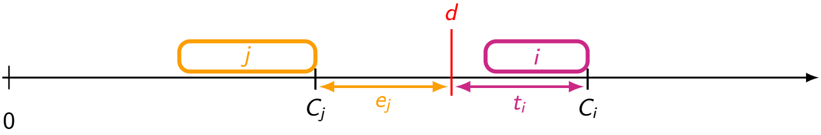

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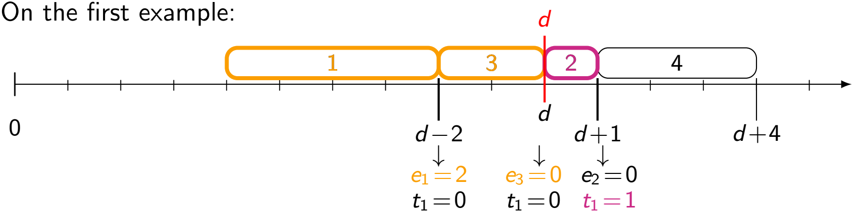

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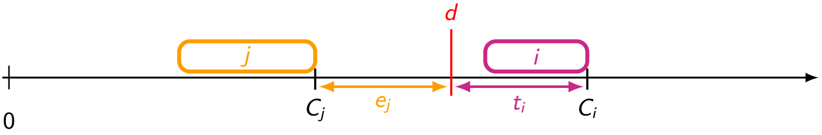

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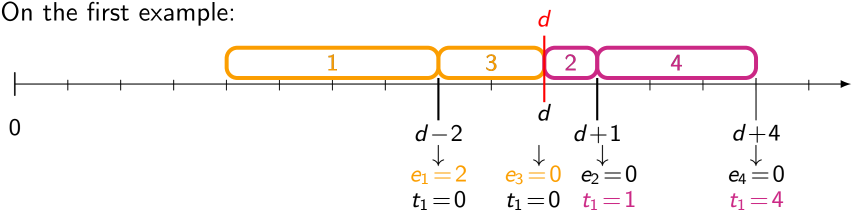

Thesis guideline and presentation outline

Questions of the thesis:

- How can we use linear programming to formulate scheduling problems for an exact solving?

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parts of Chapter 1 and 2

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parts of Chapter 3 and 4

Focus 2: Dominance inequalities to reinforce such a formulation
part of Chapter 6

Outline

1. Introduction
2. Focus 1: A formulation for UCDDP using natural variables
   Describing the solution set for (e, t) variables
   How to extend this formulation
   How to manage this kind of formulations in practice
3. Interlude : a first attempt to eliminate dominated solutions
4. Focus 2: dominance inequality
5. Conclusion and perspectives

What is the goal?

We already have: $\text{UCDDP} \iff \min_{(e,t)\in\mathscr{S}} g_{\alpha,\beta}(e,t)$

where : $\rightarrow$ $g_{\alpha,\beta}$ is linear $\left(g_{\alpha,\beta} = (e,t) \mapsto \sum_{j\in J} \alpha_j e_j + \beta_j t_j\right)$

$\rightarrow$ $\mathscr{S}$ is the set of (e, t) vectors that describe a schedule

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We want to describe $\mathscr{S}$ with:

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  if $\mathcal{P} \neq \mathcal{NP}$, it cannot be sufficient (LP-solving $\in \mathcal{P}$ and UCDDP $\in \mathcal{NP}$)

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where : $\rightarrow$ $g_{\alpha,\beta}$ is linear $\left(g_{\alpha,\beta} = (e,t) \mapsto \sum_{j\in J} \alpha_j e_j + \beta_j t_j\right)$

$\rightarrow$ $\mathscr{S}$ is the set of (e,t) vectors that describe a schedule

We want to describe $\mathscr{S}$ with:

- linear inequalities that define a polyhedron P
  if $\mathcal{P} \neq \mathcal{NP}$, it cannot be sufficient (LP-solving $\in \mathcal{P}$ and UCDDP $\in \mathcal{NP}$)
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An instance =

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- the processing times of these tasks $(p_j)_{j \in J}$
- an unrestrictive common due-date $d \geqslant \sum p_j$
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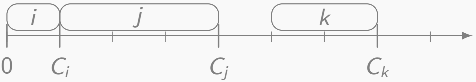
Non-overlapping inequalities for $1 \mid - \mid \min \sum \omega_j C_j$

Queyranne's non-overlapping inequalities



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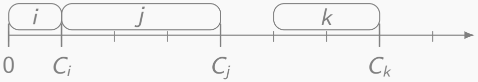

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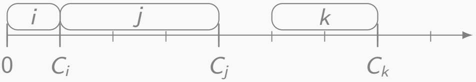

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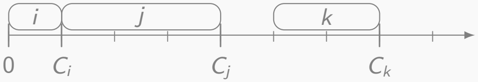

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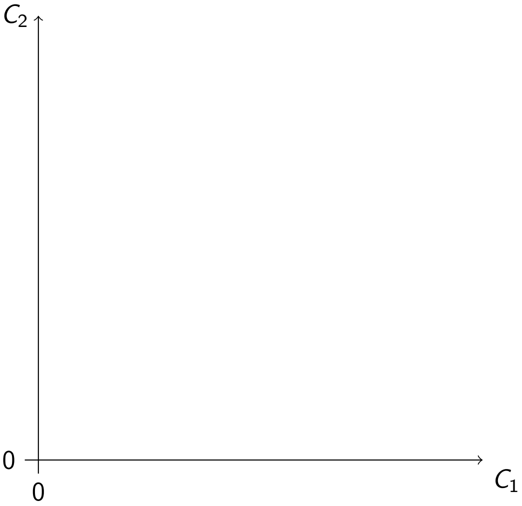
Queyranne, 1993, Mathematical Programming

The set of vectors (C_1, C_2) encoding a 2-task schedule



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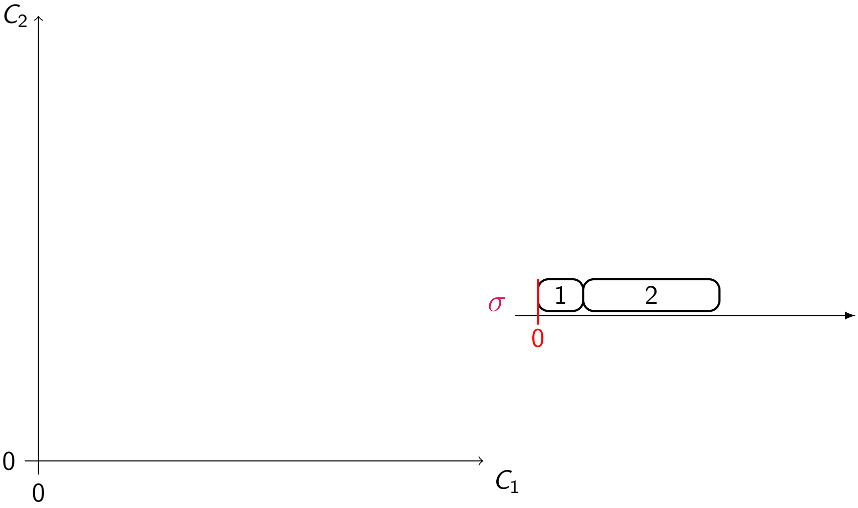

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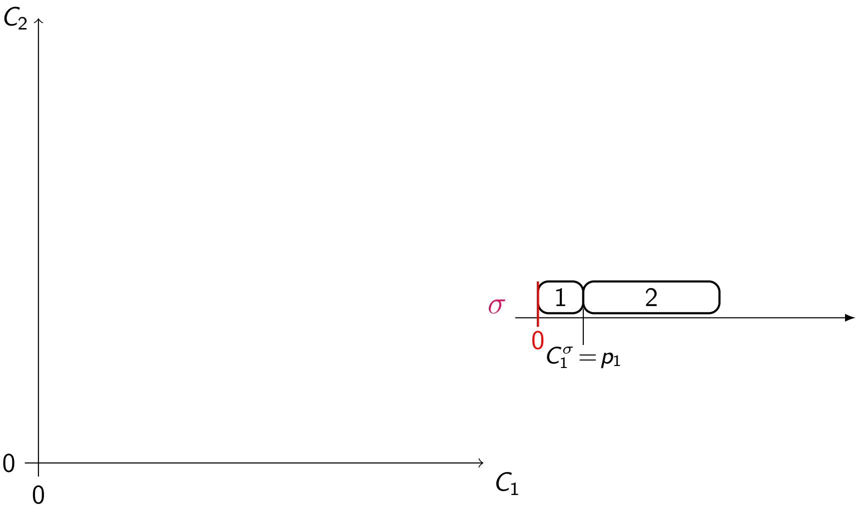

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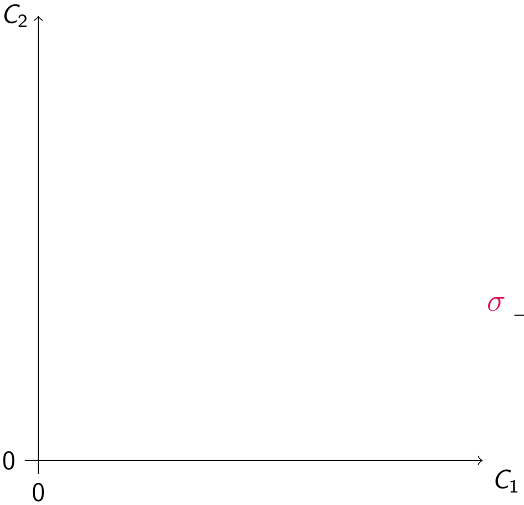

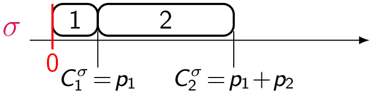

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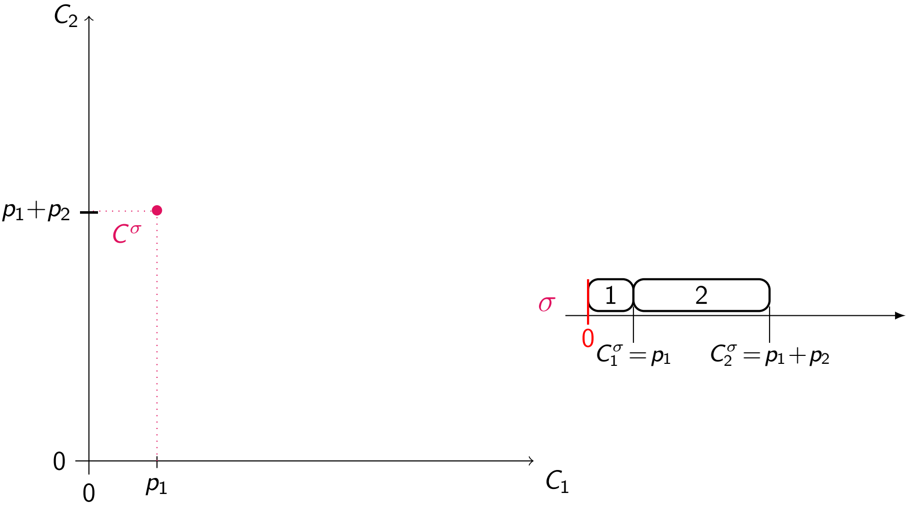

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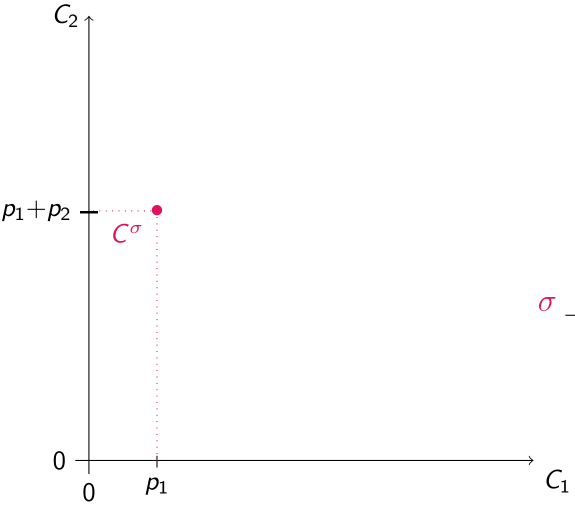

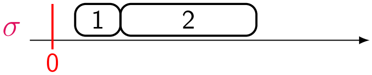

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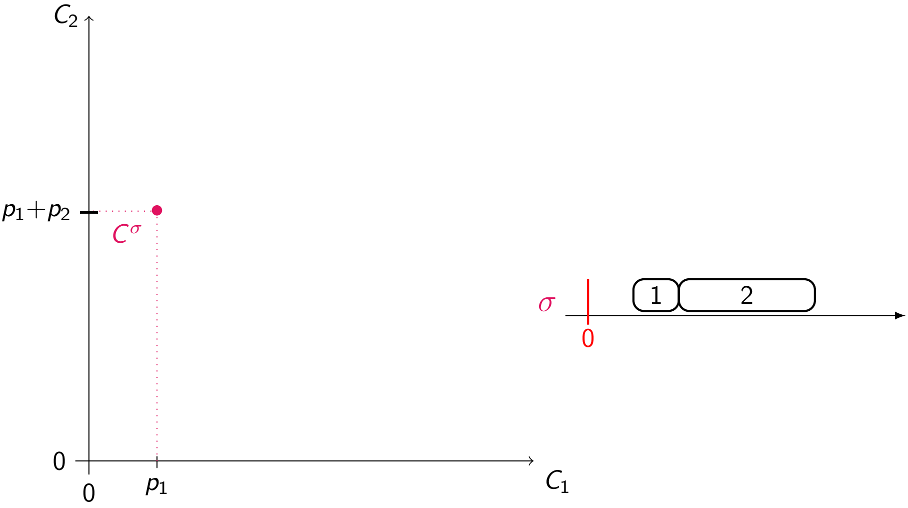

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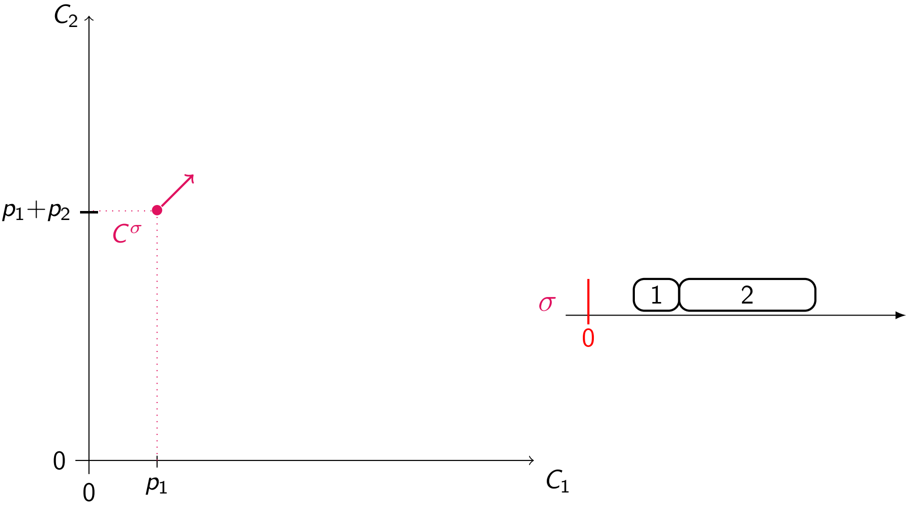

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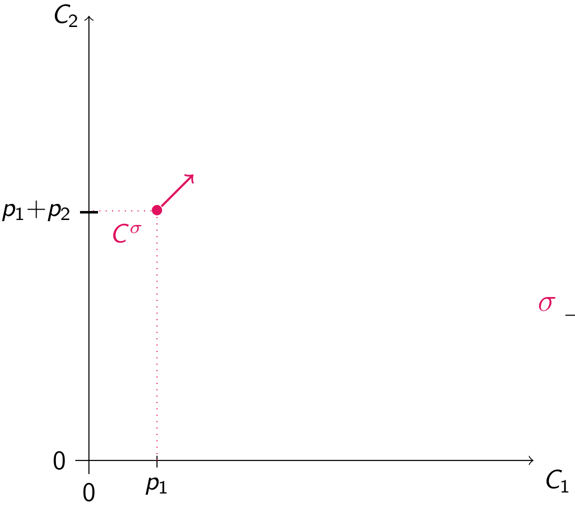

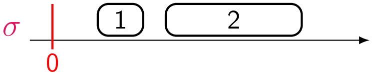

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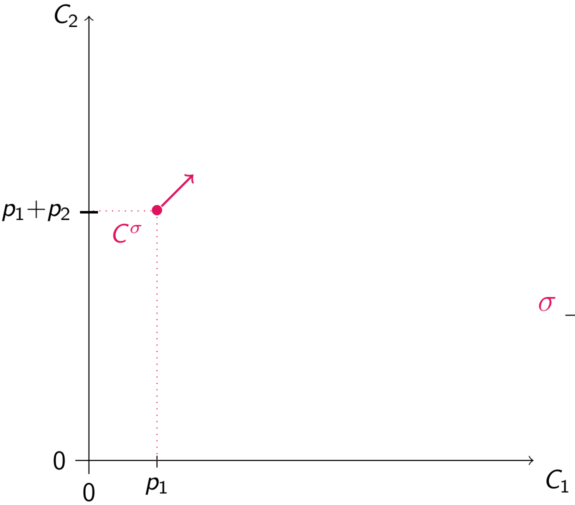

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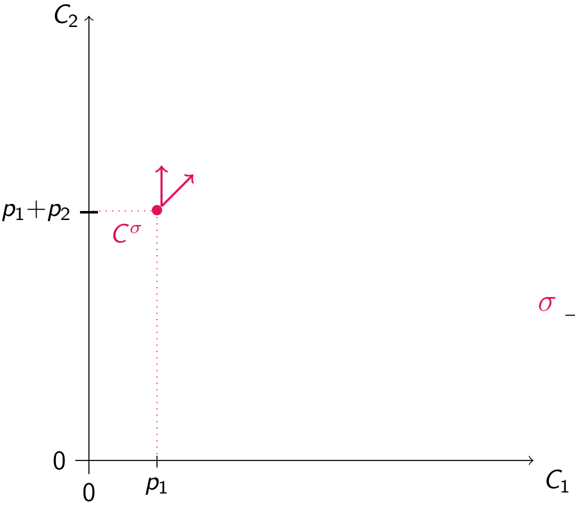

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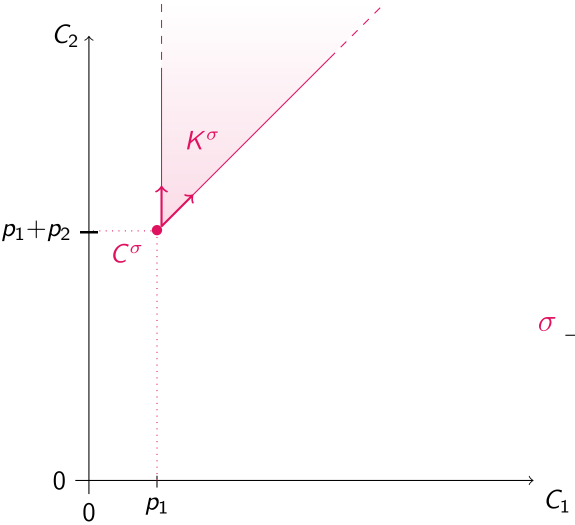

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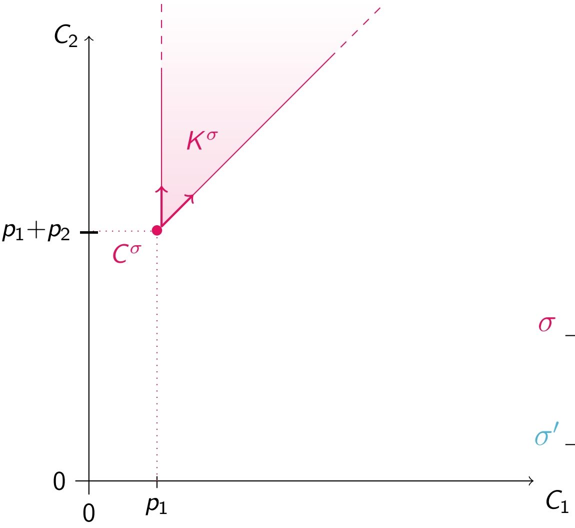

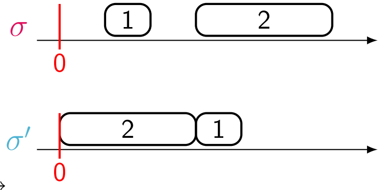

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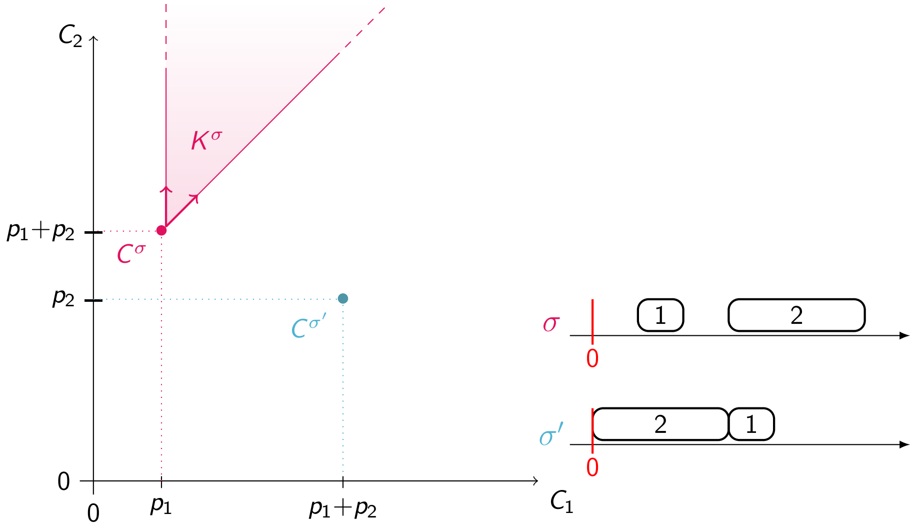

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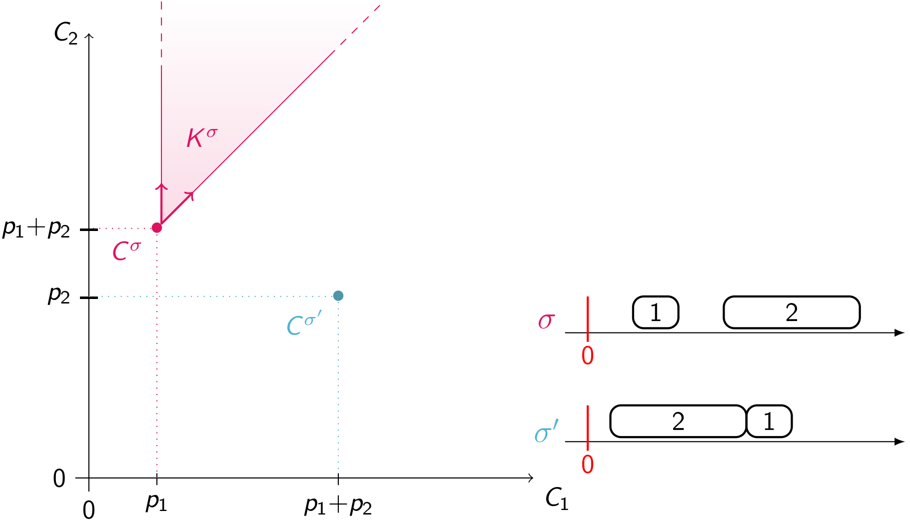

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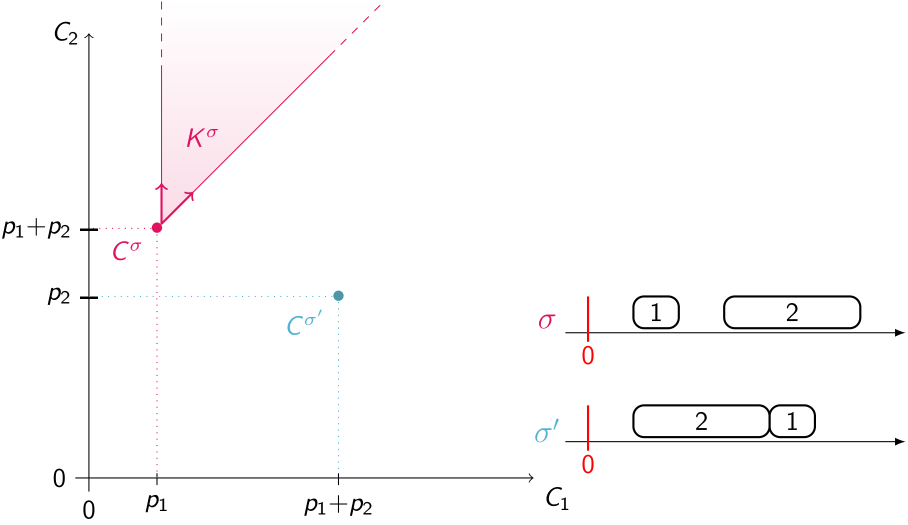

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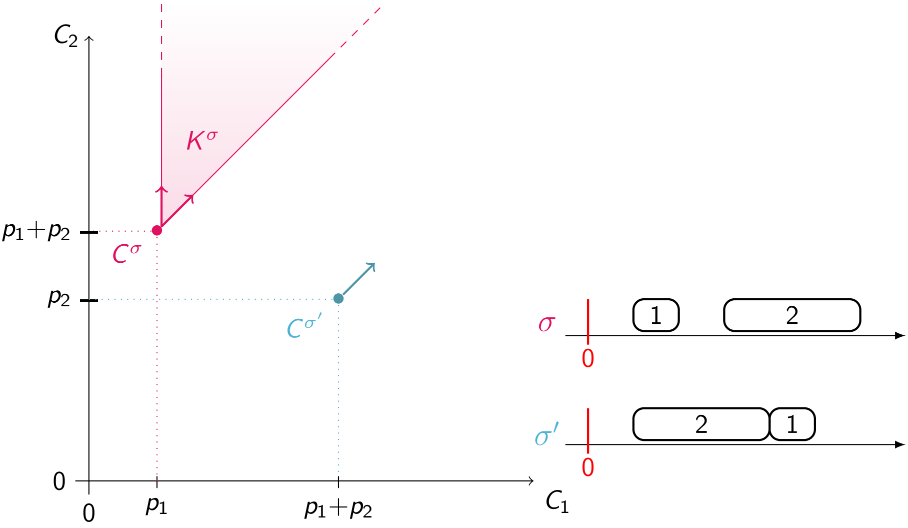

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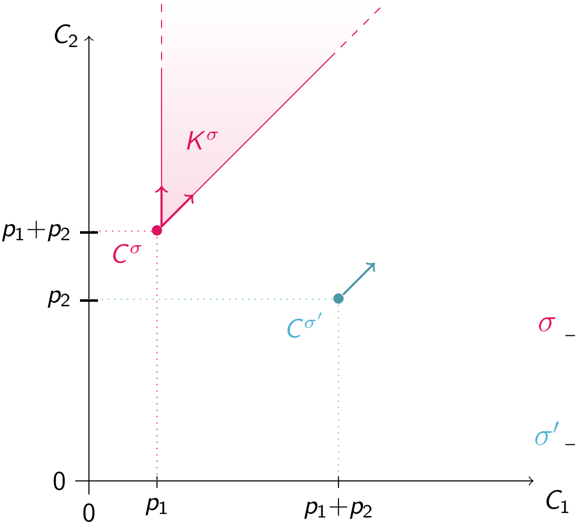

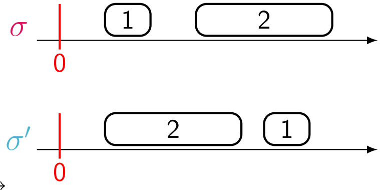

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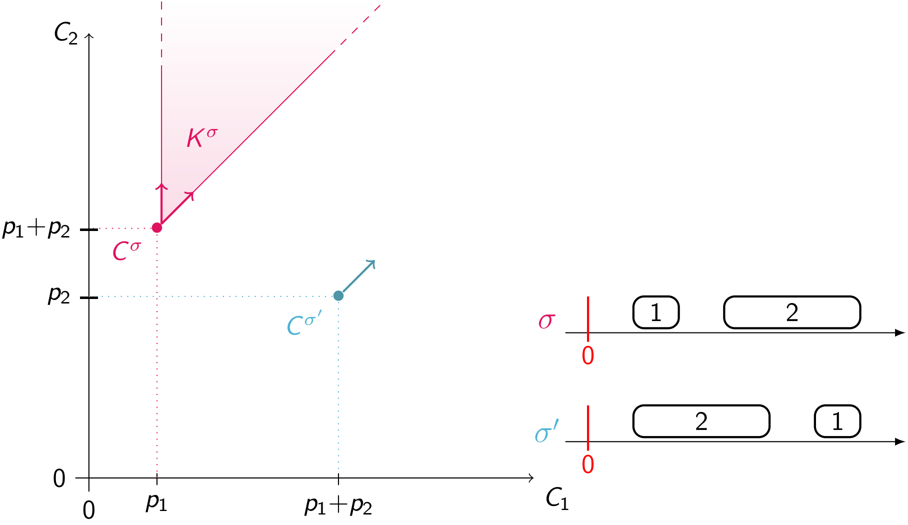

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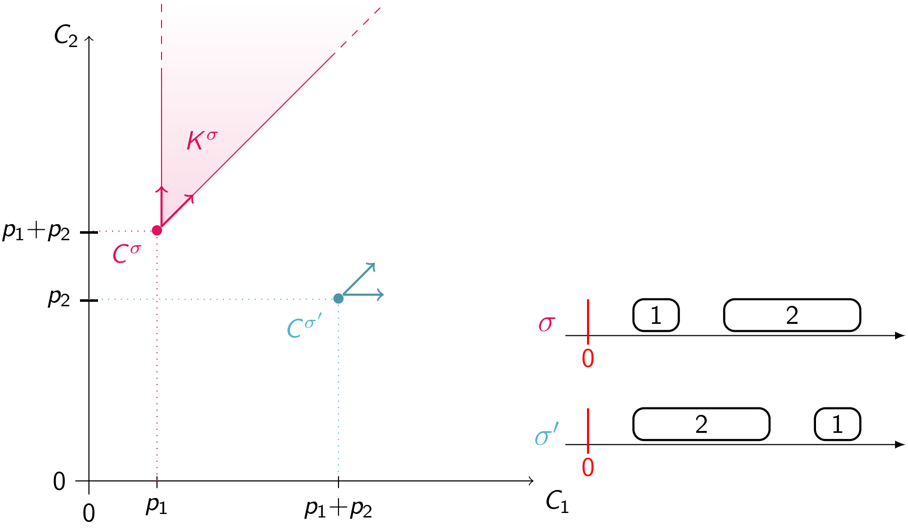

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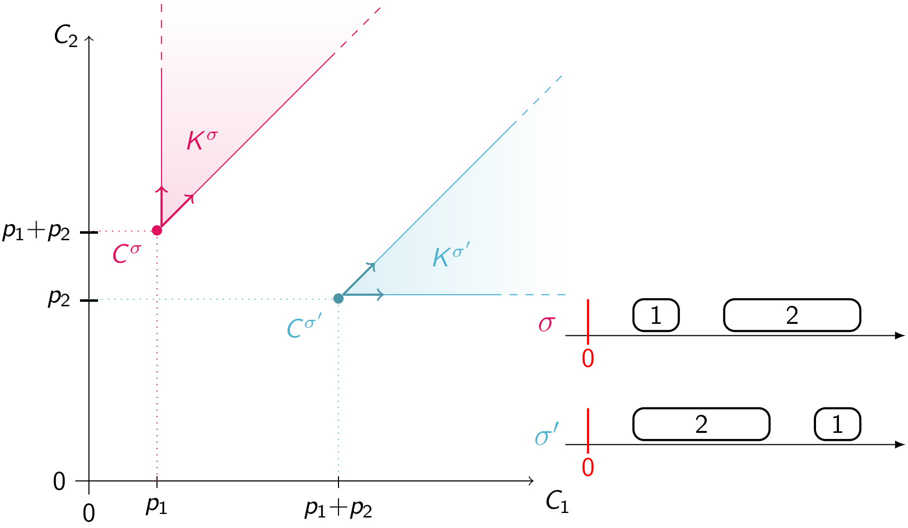

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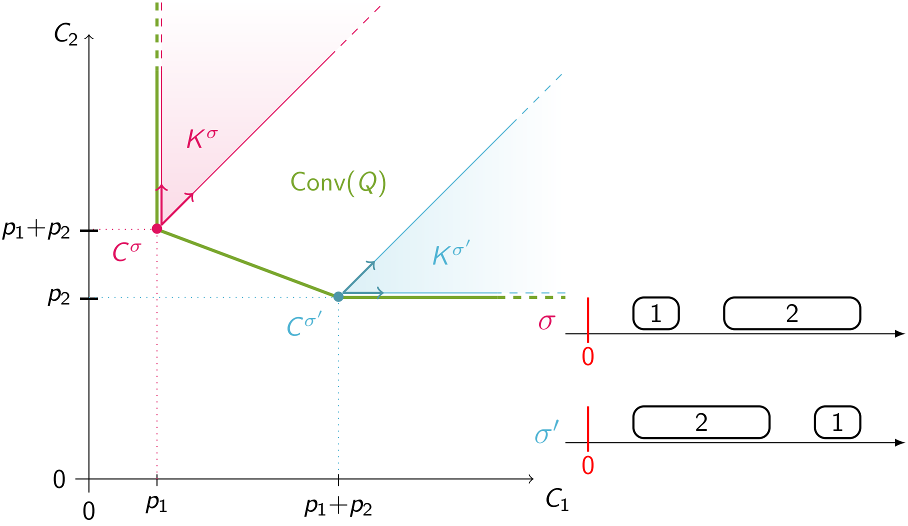
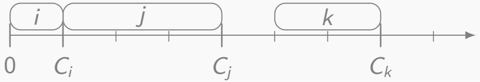


Non-overlapping inequalities for UCDDP

Queyranne's non-overlapping inequalities



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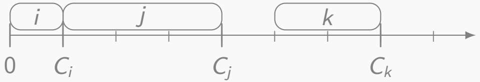

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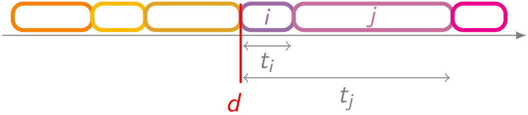

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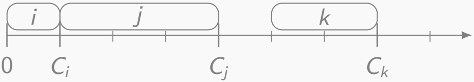

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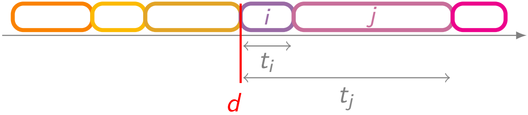

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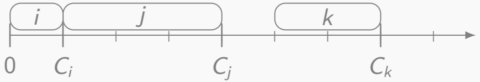

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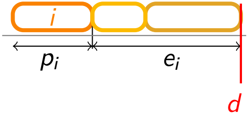

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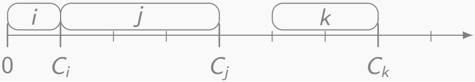

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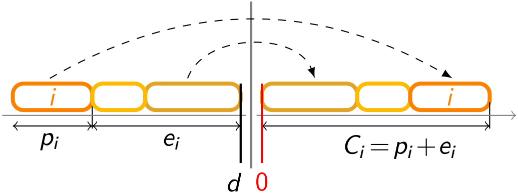

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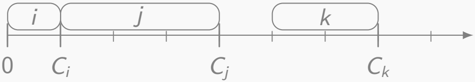

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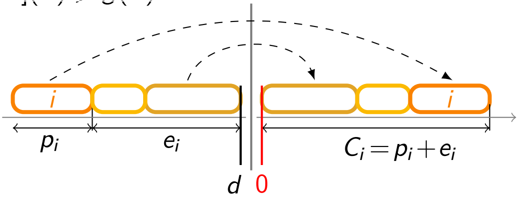

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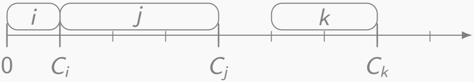

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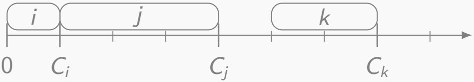

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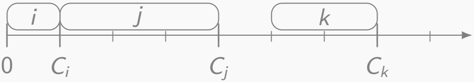

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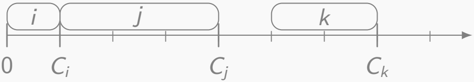

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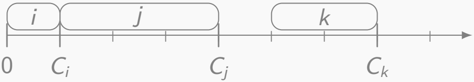

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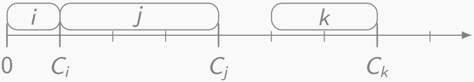

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Formulation F^3 for UCDDP

$$\forall (i,j) \in J^{<},\ \begin{aligned} X_{i,j} &\geqslant 0 && (x.1)\\ X_{i,j} &\leqslant \delta_i + \delta_j && (x.2)\\ X_{i,j} &\geqslant \delta_i - \delta_j && (x.3)\\ X_{i,j} &\geqslant 2 - \delta_i - \delta_j && (x.4) \end{aligned}$$

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$$\forall S \in \mathcal{P}(J), \quad \sum_{i \in S} p_i\, e_i \geqslant \sum_{(i,j) \in S^<} p_i\, p_j\, \frac{\delta_i + \delta_j - X_{i,j}}{2} \qquad (S1)$$

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$$P^3 = \left\{ (e,t,\delta,X) \;\middle|\; \begin{array}{ll}
\forall j \in J, 0 \leqslant \delta_j \leqslant 1 & (\delta) \\
\forall j \in J,\ e_j \geqslant 0 \quad (e.0) \qquad e_j \leqslant M\,\delta_j \quad (e.1) & \forall j \in J,\ t_j \geqslant 0 \quad (t.1) \qquad t_j \leqslant M\,(1-\delta_j) \quad (t.2) \\
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$$F^3 : \min \left\{ \sum_{j\in J} \alpha_j\, e_j + \beta_j\, t_j \;\middle|\; (e,t,\delta,X) \in \mathrm{extr}(P^3) \text{ and } \delta \in \{0,1\}^J \right\}$$

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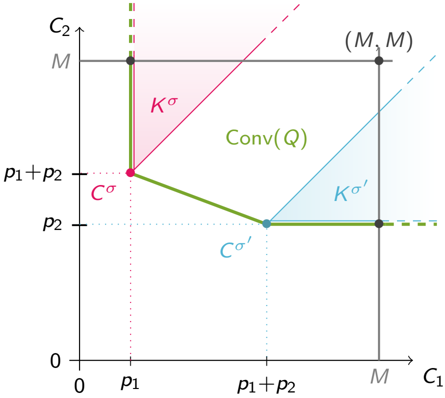
- validity proof
  - is not based on a geometrical proof
  - must be compatible with additional inequalities



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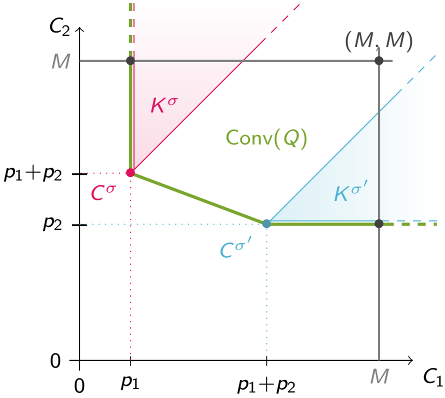

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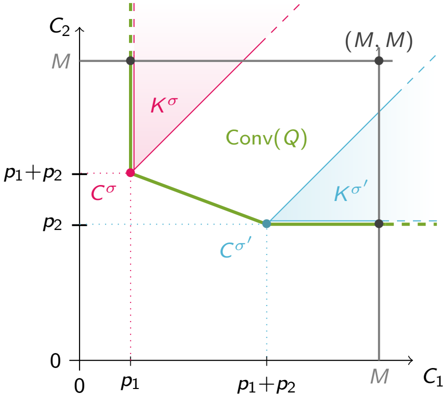

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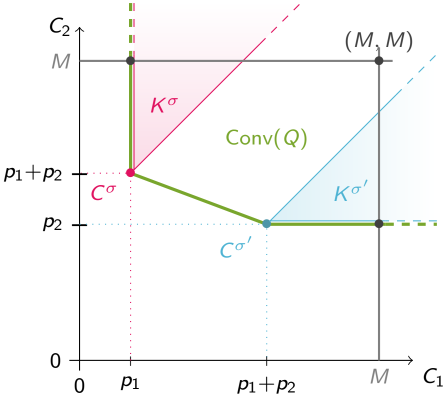

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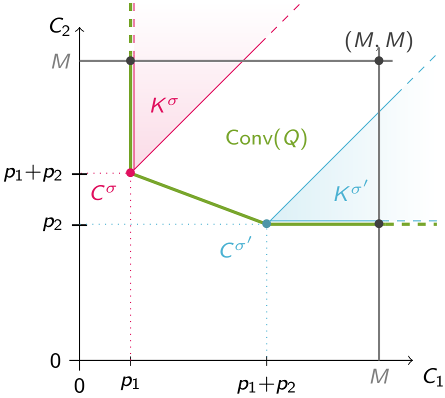
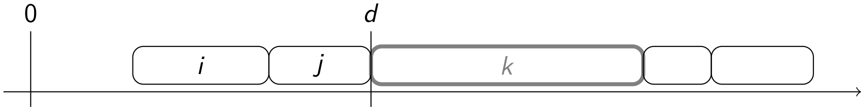


How to adapt the formulation for the general case ?

- unrestrictive case: d-blocks are dominant



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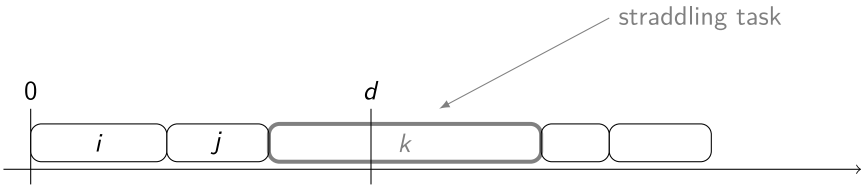

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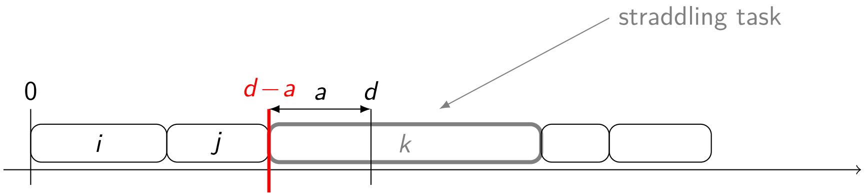

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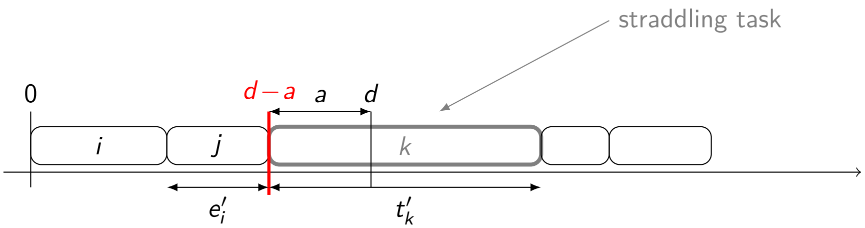
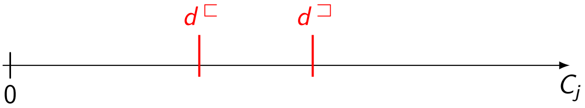


Interest of a flexible reference point?

Common due window problem:

→ a due window $[d^{\sqsubset}, d^{\sqsupset}]$ instead of a due date d



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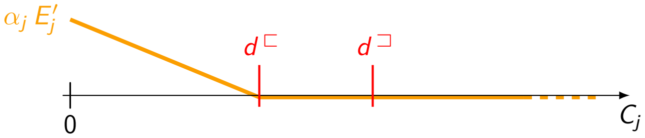

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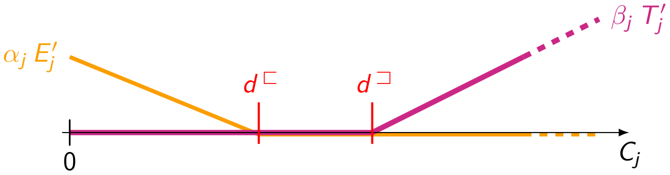

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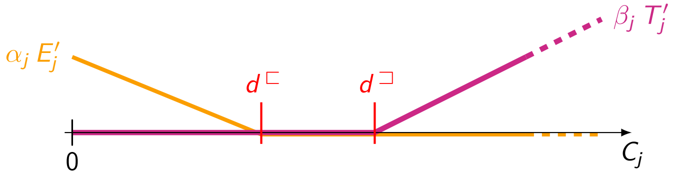

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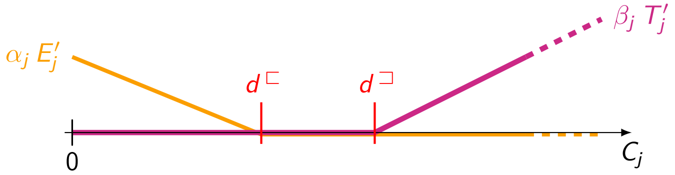

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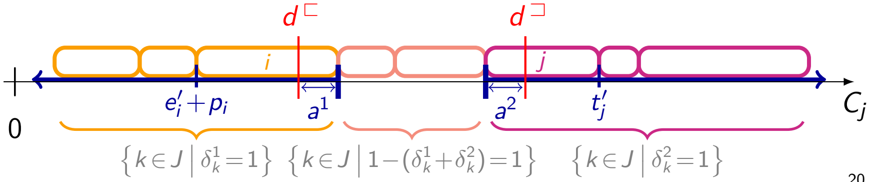

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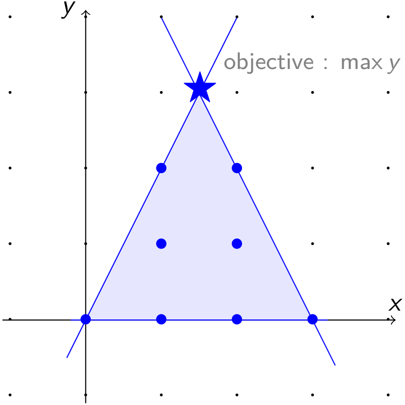

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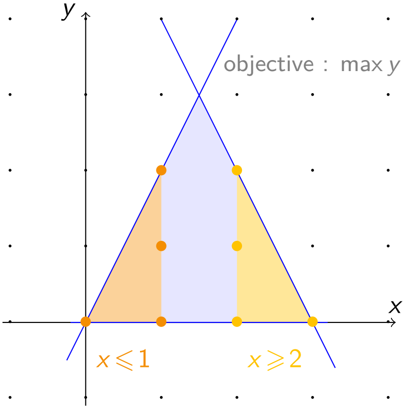

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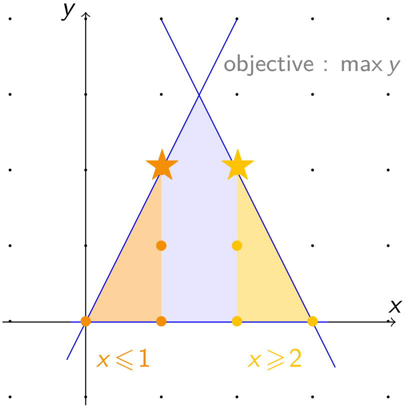

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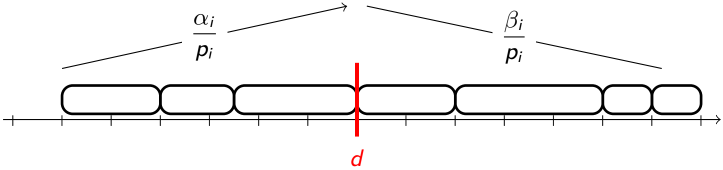
Why it is not so efficient? poor linear relaxation value

Outline

1. Introduction
2. Focus 1: A formulation for UCDDP using natural variables
3. Interlude : a first attempt to eliminate dominated solutions
4. Focus 2: dominance inequality
5. Conclusion and perspectives

A compact MIP formulation for UCDDP...

... using the V-shaped dominance property



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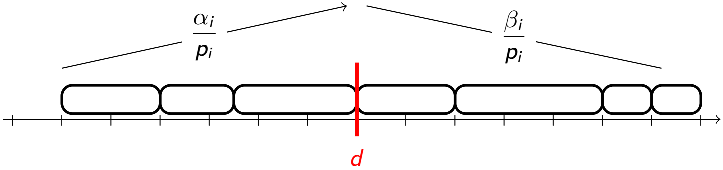

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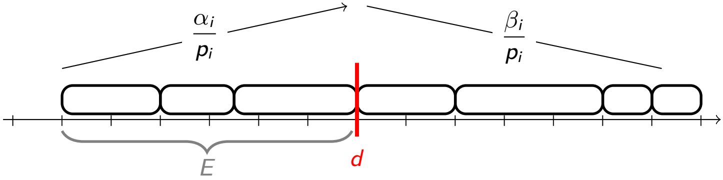

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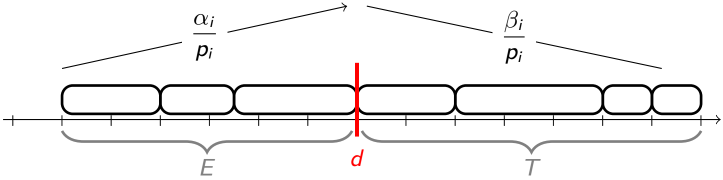

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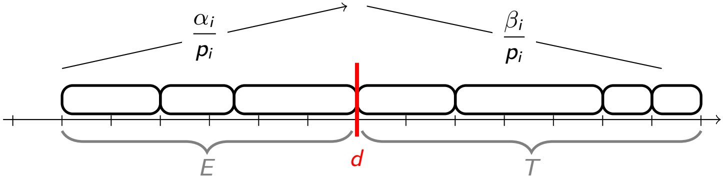

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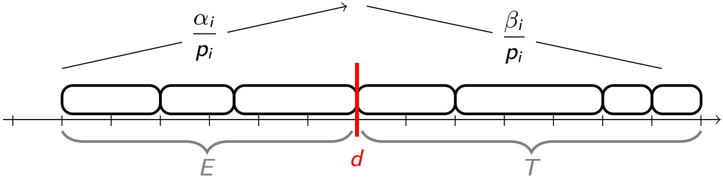

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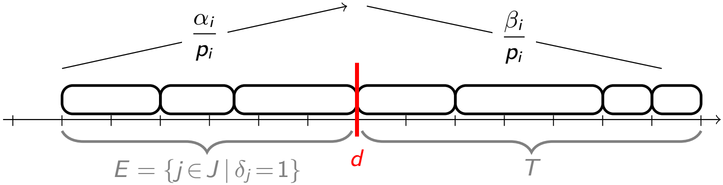

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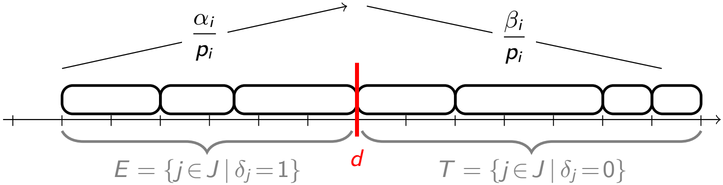

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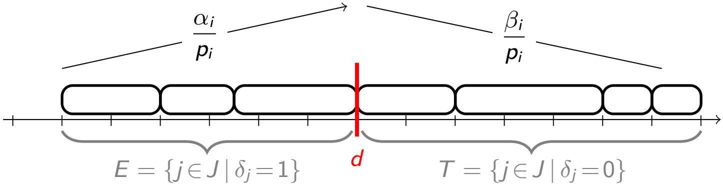

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- formulate UCDDP as a **MIP** $F^2 : \min_{(\delta,X) s.t. (X.1-X.4)} h_{\alpha,\beta}(\delta, X)$

  where $h_{\alpha,\beta}$ is a linear function depending on α and β

Link between F^2 and the Cut Polytope

in Formulation F^2 | in the complete graph K_n

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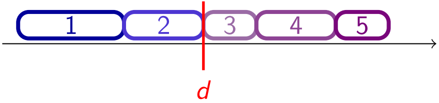
δ variables $\longleftrightarrow$ vertices



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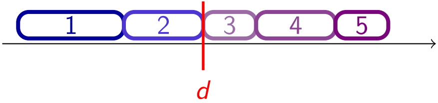

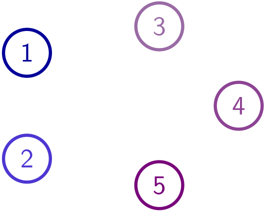

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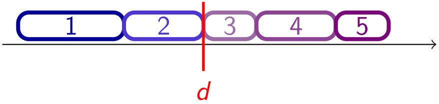

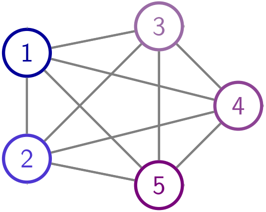

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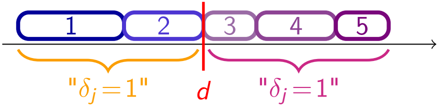

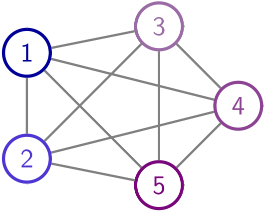

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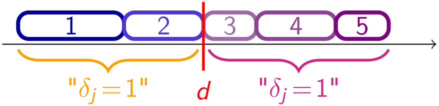

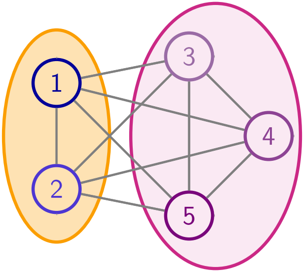

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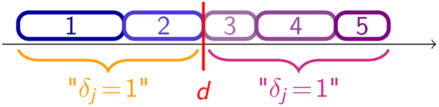

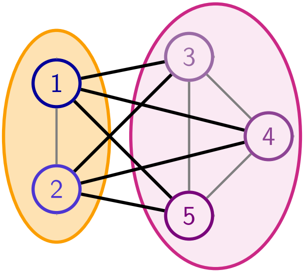

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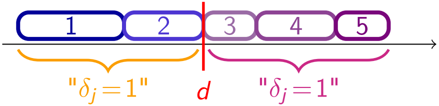
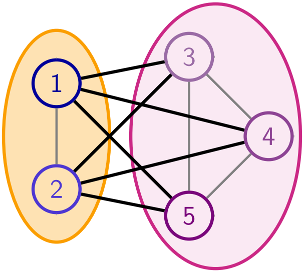
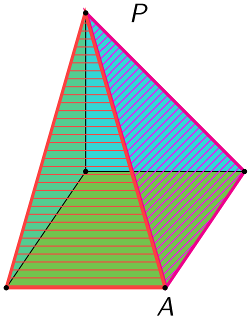
$(\delta, X)\in P^2$	$\longleftrightarrow$	$X \in \mathrm{CUT}_n$ the cut polytope for K_n





Idea

Eliminate an extreme point corresponding to a "bad" solution according to the objective value,



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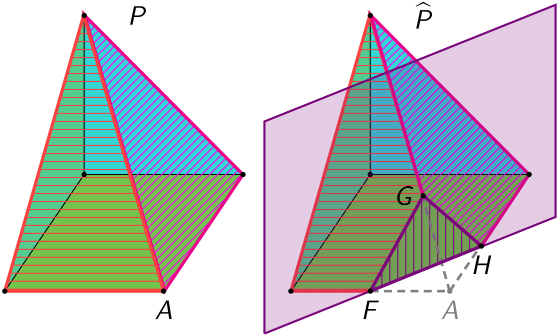

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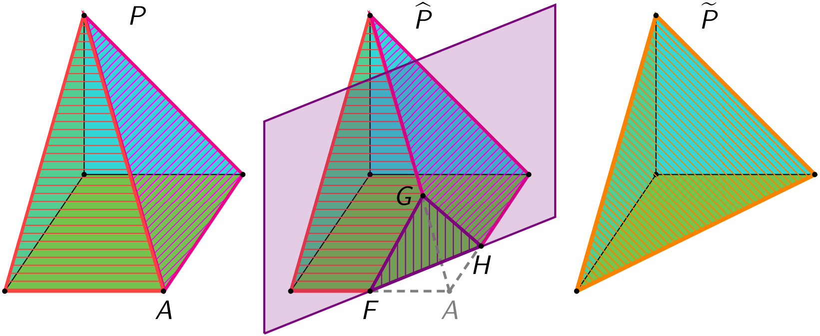
Eliminate an extreme point corresponding to a "bad" solution according to the objective value, using **facet defining** inequalities to avoid the apparition of new extreme points



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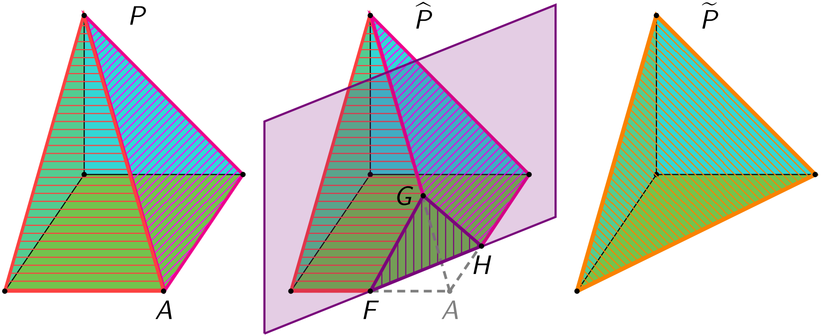
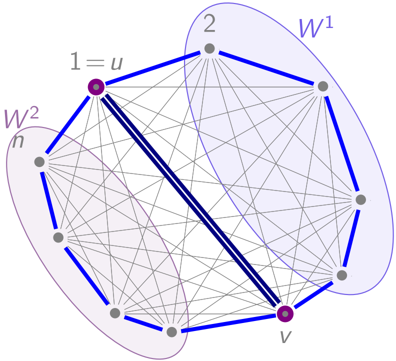


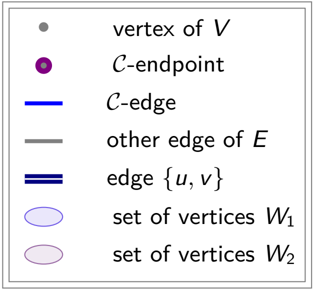
Application to P^2, elimination of $(\delta, X) = (0, 0)$,

$$\widetilde{P}^n_{\delta,X} = \text{conv}\left\{(\delta, X) \in \{0,1\}^J \times \{0,1\}^{J^<} \,\middle|\, (X.1 - X.4) \text{ and } \delta \neq 0\right\}$$

Example of a new facet defining inequality family

for $\mathcal{C}$ an hamiltonian cycle in K_n, $\underline{\delta_u+\delta_v} - \underline{X_{u,v}} + \underline{X(\mathcal{C})} \geqslant 2$





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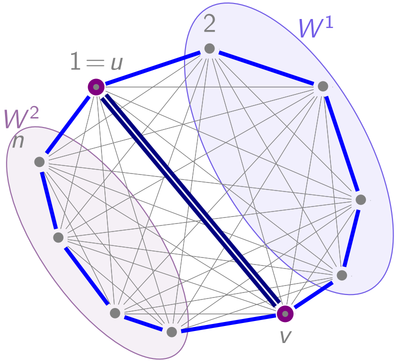

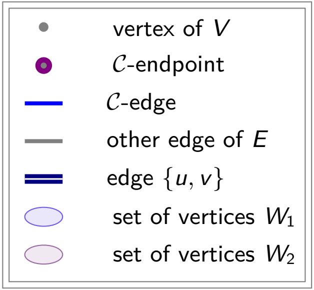

Too many and too various inequalities appear → change of strategy

Outline

Neighborhood based dominance properties : generic idea

Remark: If a solution is dominated by one of its neighbors,
then it is not an optimal solution.

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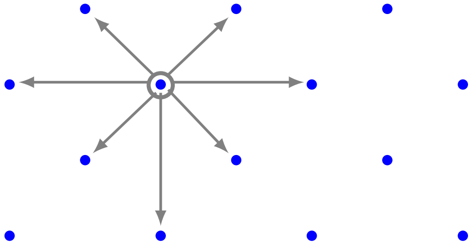
Our approach:

- define a neighborhood based on operations
- translate the associate dominance property by linear inequalities

Neighborhood: solution-centered vs operation-centered point of view



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*= consider **all** the neighbors of **one** given solution*

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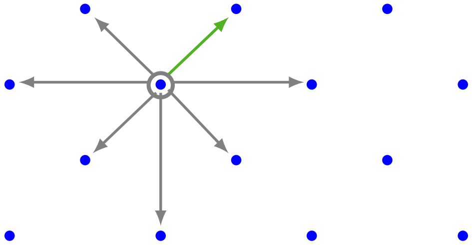

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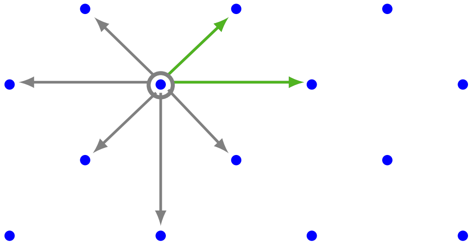

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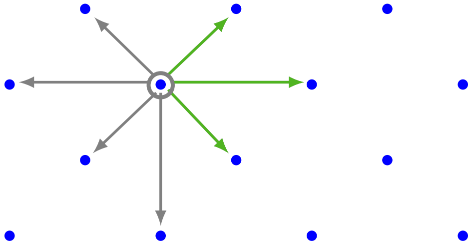

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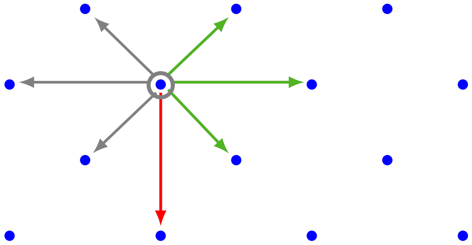

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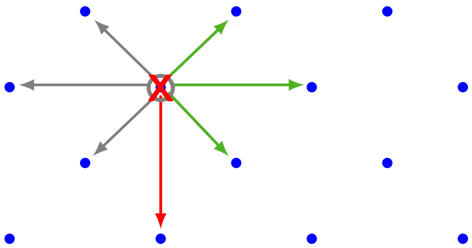

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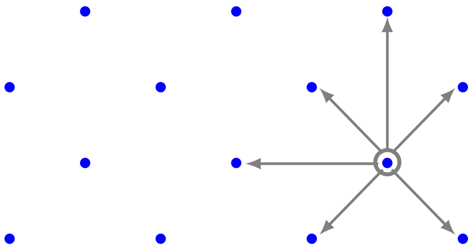

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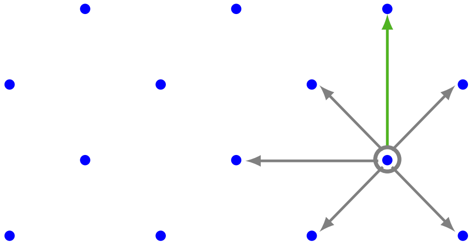

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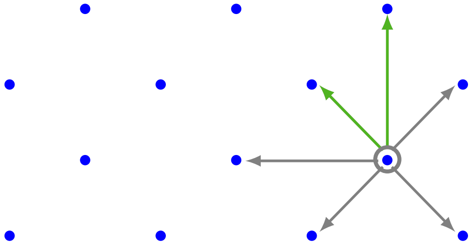

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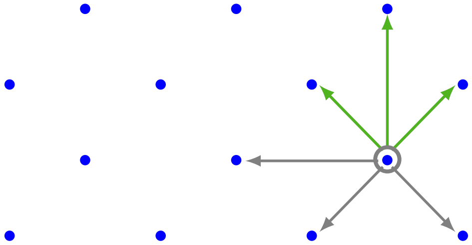

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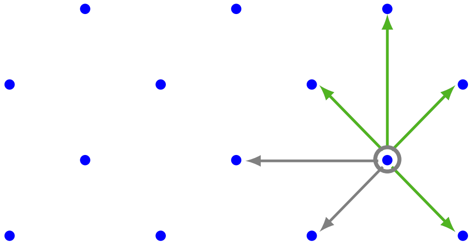

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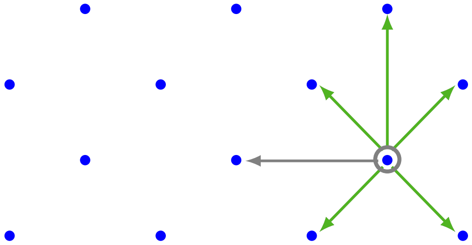

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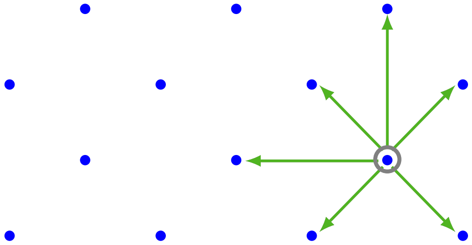

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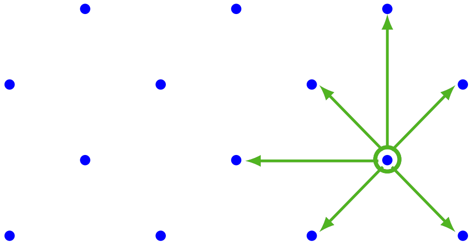

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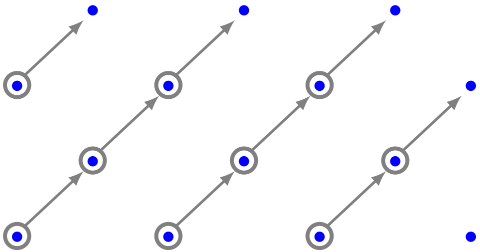

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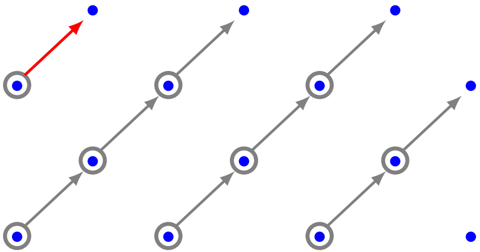

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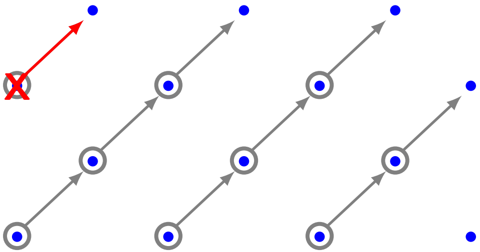

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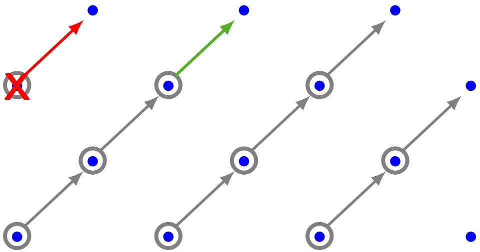

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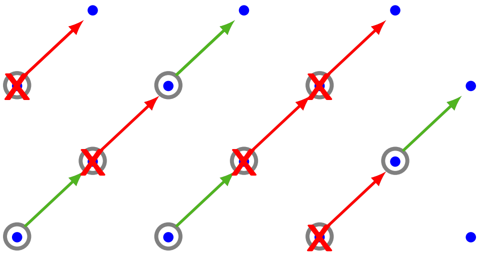

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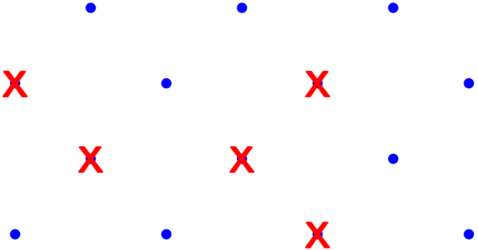

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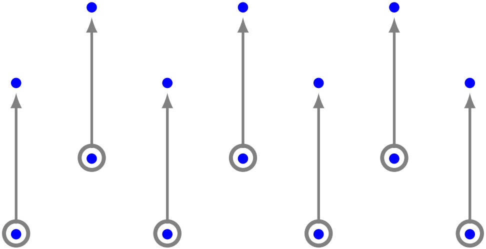

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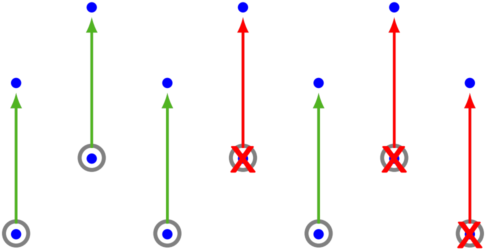
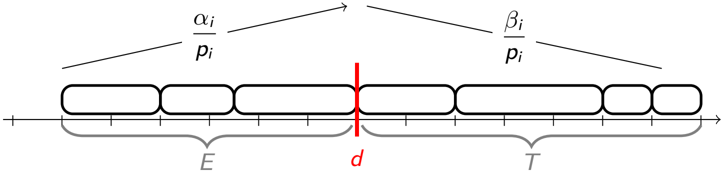


Operation-centered

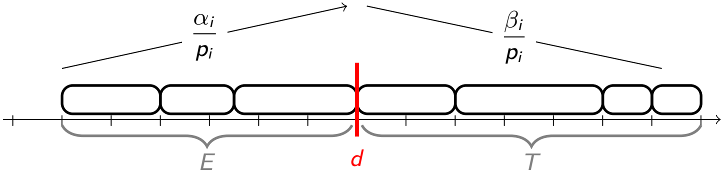
= *consider **one** given type of neighbor for **all** the solutions*

Structure of a V-shaped d-block



- introduce notations for any $u \in J$,

Structure of a V-shaped d-block



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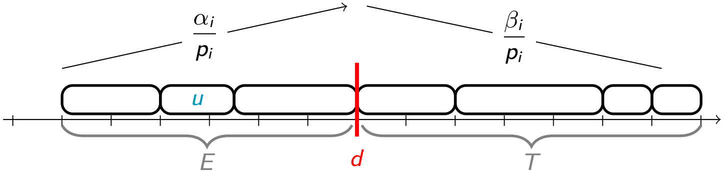

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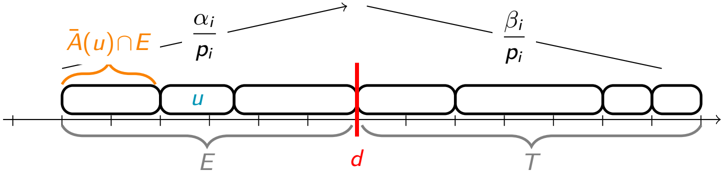

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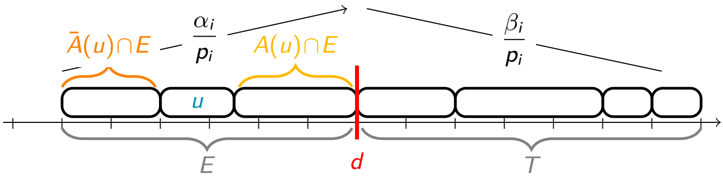

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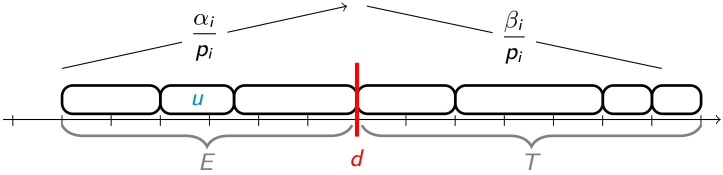

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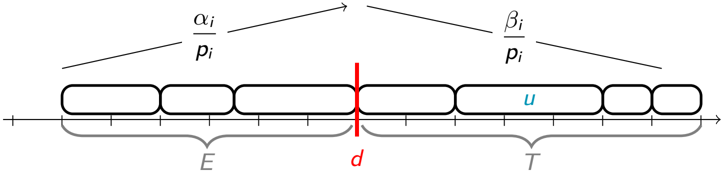

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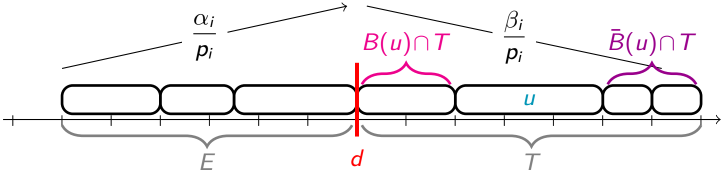

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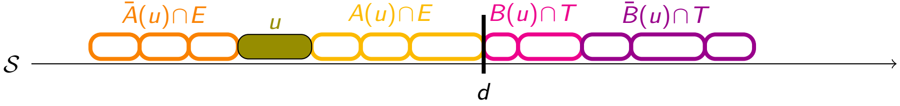
Cost variation induced by the insertion of an early task u



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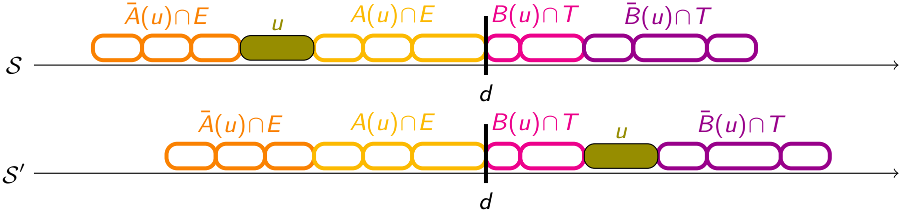

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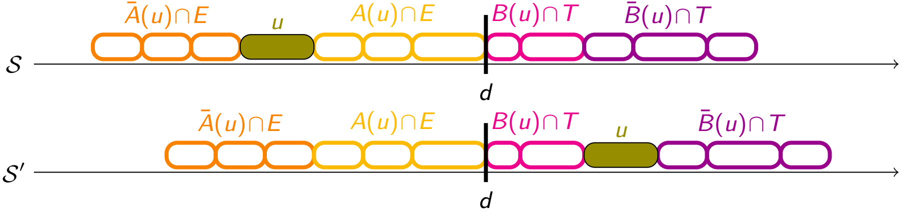
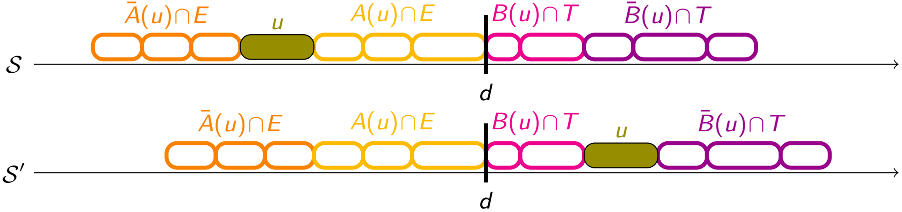


$\text{cost}(\mathcal{S}') = \text{cost}(\mathcal{S})$

Cost variation induced by the insertion of an early task u



$$\text{cost}(\mathcal{S}') = \text{cost}(\mathcal{S}) - \alpha_u \, p\big(\underline{A(u) \cap E}\big)$$

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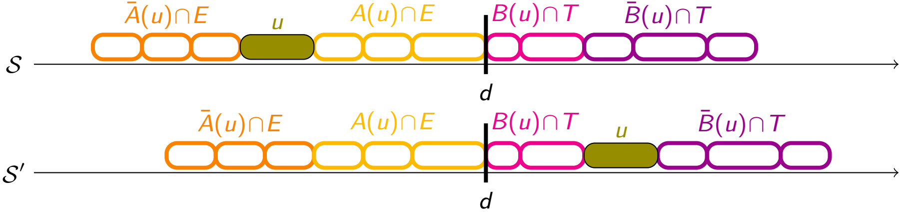

$$\text{cost}(\mathcal{S}') = \text{cost}(\mathcal{S}) - \alpha_u \, p\big(\underline{A(u) \cap E}\big) + \beta_u \Big(p\big(\underline{B(u) \cap T}\big) + p_u\Big)$$

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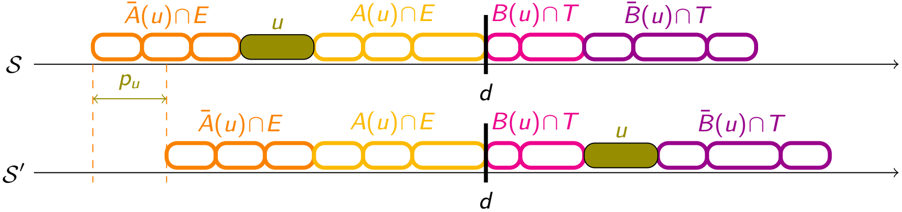

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$$- p_u\, \alpha\big(\bar{A}(u) \cap E\big)$$

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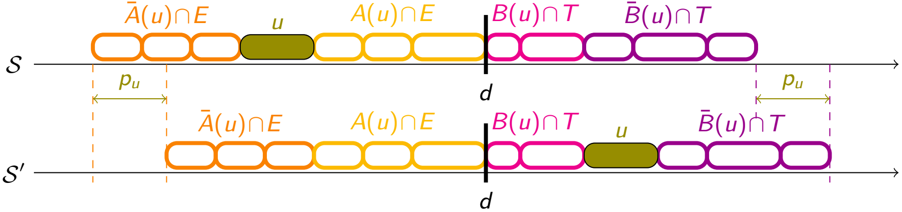

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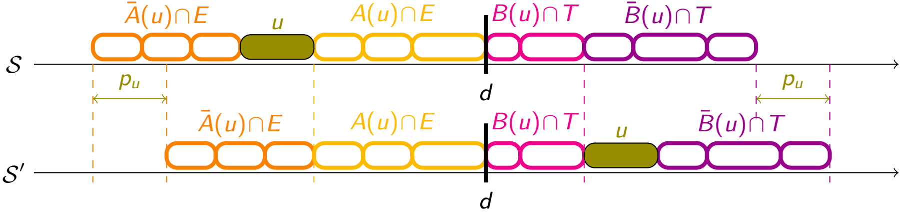

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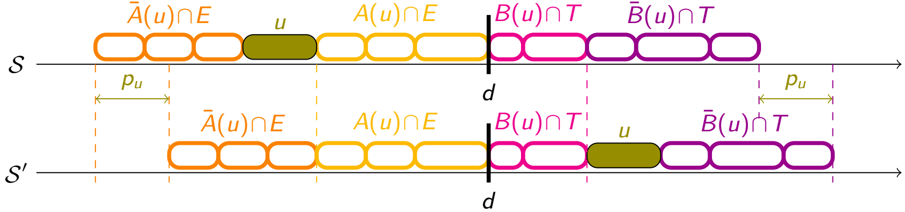

$$\Delta_u^{early}(E,T) = -\alpha_u\, p\big(A(u)\cap E\big) + \beta_u \Big(p\big(B(u)\cap T\big) + p_u\Big)$$
$$- p_u\, \alpha\big(\bar{A}(u)\cap E\big) + p_u\, \beta\big(\bar{B}(v)\cap T\big)$$

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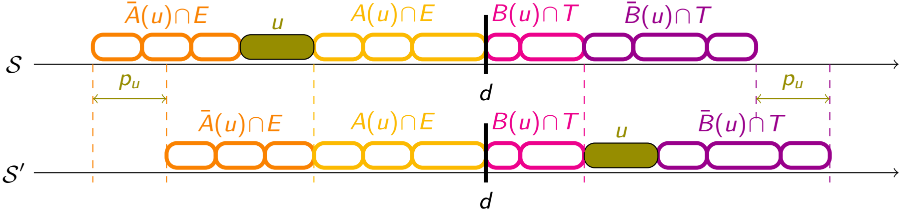
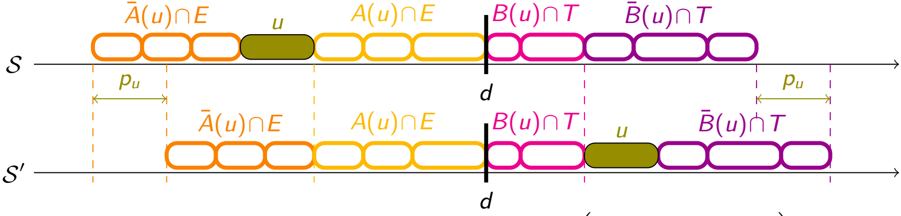


$$\Delta_u^{early}(E,T) = -\alpha_u\, p\big(A(u)\cap E\big) + \beta_u \Big(p\big(B(u)\cap T\big) + p_u\Big) - p_u\, \alpha\big(\bar{A}(u)\cap E\big) + p_u\, \beta\big(\bar{B}(v)\cap T\big)$$

dominance constraint
for the early-insert of $u \in J$ $\boxed{\Delta_u^{early}(E,T) \geqslant 0 \text{ if } u \in E}$

Cost variation induced by the insertion of an early task u



$$\Delta_u^{early}(E,T) = -\alpha_u\, p\big(\underline{A(u)\cap E}\big) + \beta_u \Big(p\big(\underline{B(u)\cap T}\big) + p_u\Big)$$
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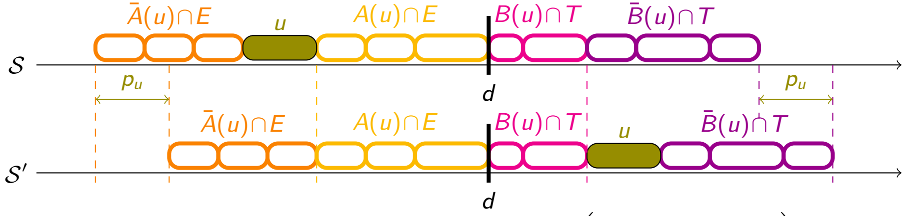
dominance constraint for the early-insert of $u \in J$ $\boxed{\Delta_u^{early}(E,T) \geqslant 0 \text{ if } u \in E}$

Similarly: dominance constraint for the tardy-insert of $v \in J$ $\boxed{\Delta_v^{tardy}(E,T) \geqslant 0 \text{ if } v \in T}$

Cost variation induced by the insertion of an early task u



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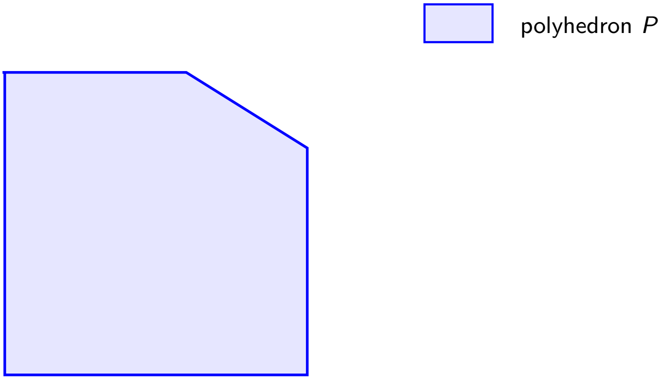
dominance constraint for the early-insert of $u \in J$: $\Delta_u^{early}(E,T) \geqslant 0$ if $u \in E$

Similarly:

dominance constraint for the tardy-insert of $v \in J$: $\Delta_v^{tardy}(E,T) \geqslant 0$ if $v \in T$

dominance constraint for the swap of $u \in J$ and $v \in J$: $\Delta_{u,v}^{swap}(E,T) \geqslant 0$ if $(u,v) \in E \times T$

Unusual inequalities



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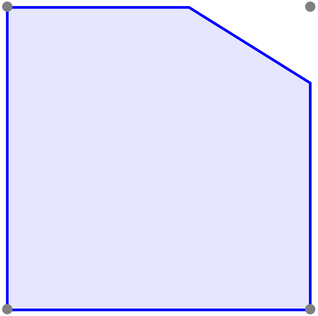

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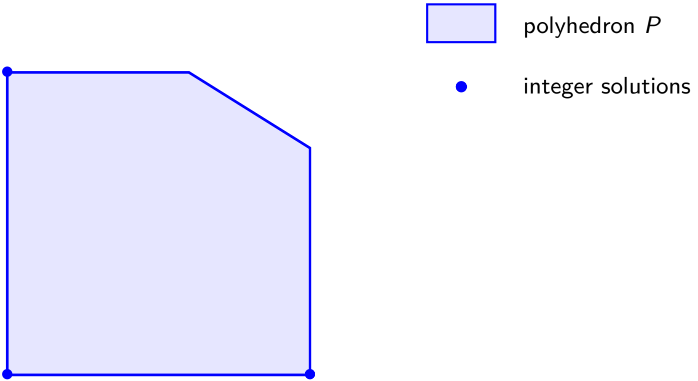

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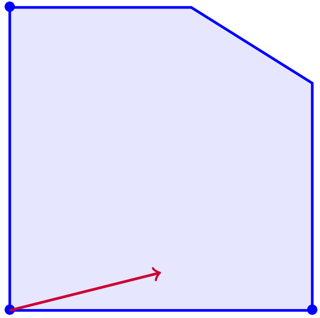

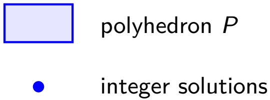

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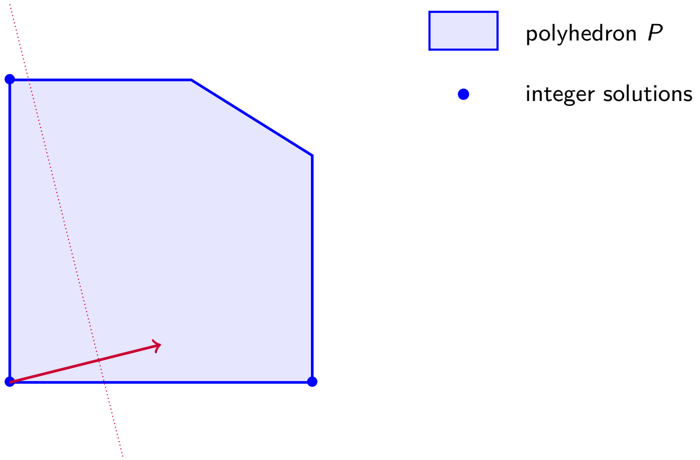

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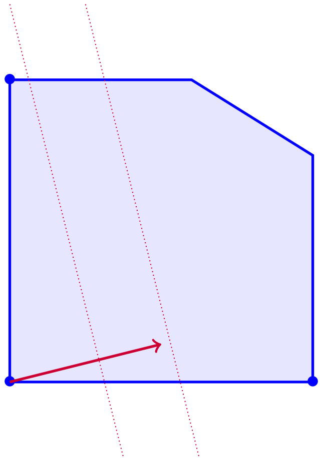

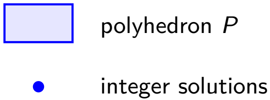

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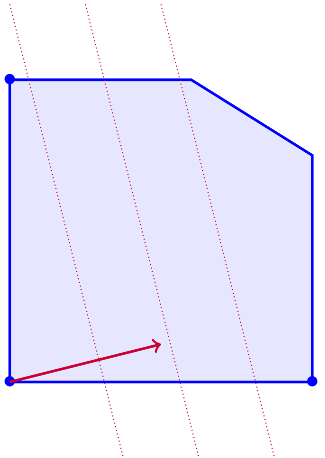
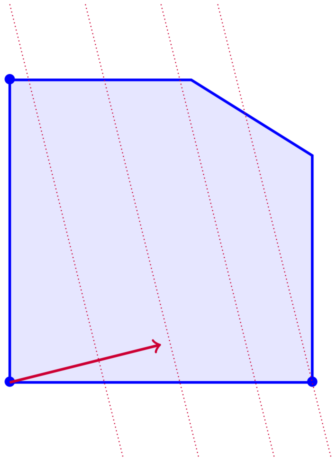
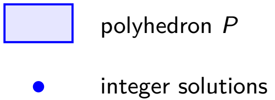


polyhedron P

integer solutions

Unusual inequalities





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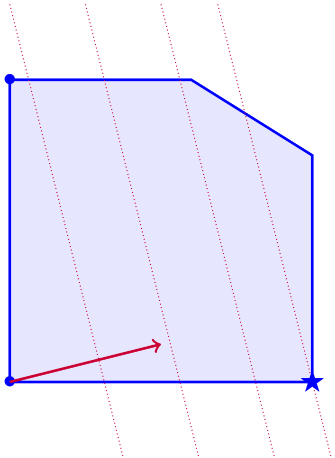

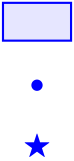

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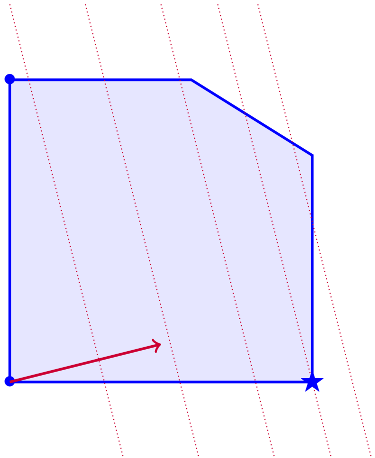

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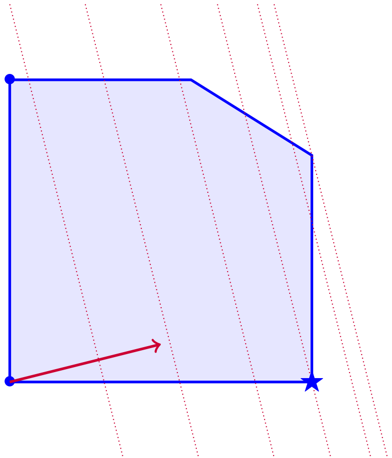

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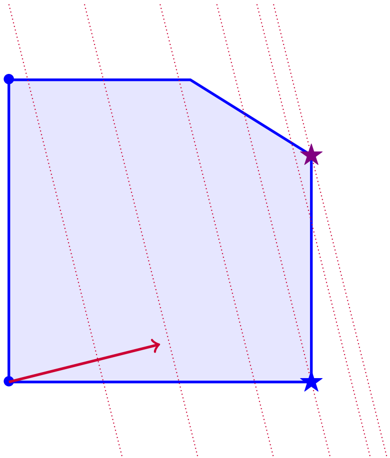
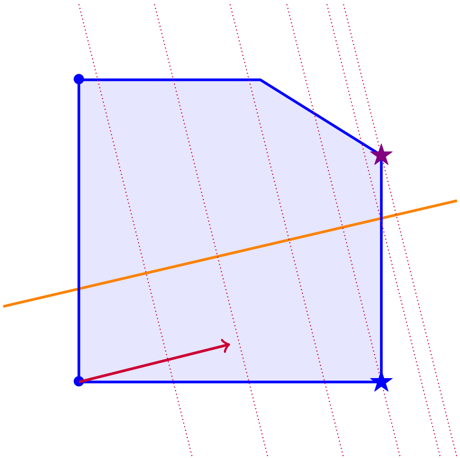
Unusual inequalities



- polyhedron P
- integer solutions
- best integer solution
- best fractionnal solution in P

Unusual inequalities



polyhedron P

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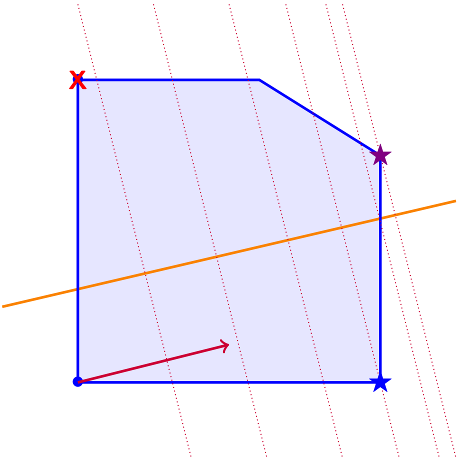
a dominance inequality

Unusual inequalities



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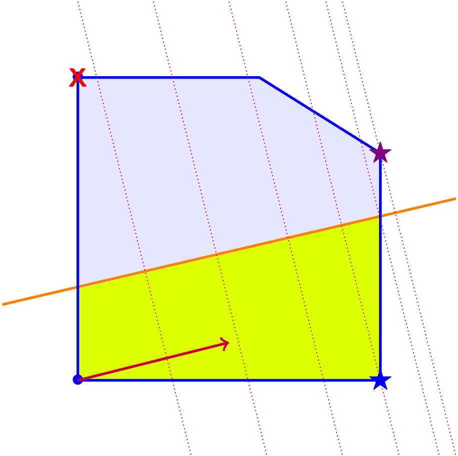

- polyhedron P
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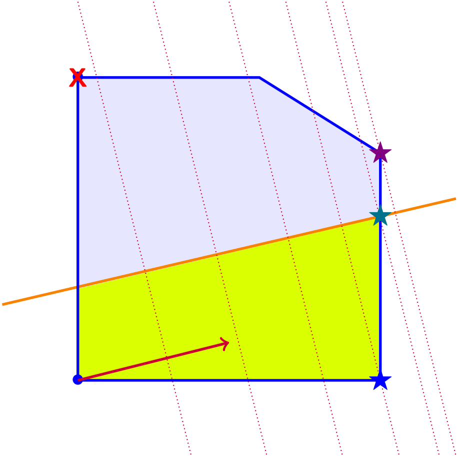

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Conclusion on dominance inequalities

Insert and swap inequalities allow to:

→ reinforce F^3 in a different way as usual strengthen inequalities
no improvement of the linear relaxation value

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Conclusion on dominance inequalities

Insert and swap inequalities allow to:

- → reinforce F^3 in a different way as usual strengthen inequalities
  no improvement of the linear relaxation value
- → solve UCDDP up to size 180 within one hour
  (instead of size 60 without them)
- → illustrate the dominance inequality concept

Outline

1. Introduction
2. Focus 1: A formulation for UCDDP using natural variables
3. Interlude : a first attempt to eliminate dominated solutions
4. Focus 2: dominance inequality
5. Conclusion and perspectives

Done during the thesis

- providing formulation F^3 for UCDDP, F^4 for CDDP
- establishing two lemmas about non-overlapping inequalities
- proving validity of F^3 and F^4
- finding a separation algorithm for non-overlapping inequalities in F^3 and F^4
- providing another formulation F^2 for UCDDP
- implementing and testing F^2, F^3 and F^4
    - ↪ writing a journal paper submitted to DAM, *currently accepted*

- proposing a framework to "transpose" facet defining inequalities
- testing formulation F^2 when known facet defining inequalities are added
- using PORTA on small-dimensional "non-trivial cuts" polytopes
- identifying some family of facet defining inequalities for arbitrary dimension

- proposing a new kind of inequality to improve some MIP formulations solving
- providing dominance inequalities for F^2 based on insert and swap operations
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+ listening and speaking in seminars or conferences, attending summer school, teaching, writing final report...

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Generic frameworks (tools to take away)

- About natural variables and non-overlapping inequality formulations:
  - → a methodology to formulate some scheduling problem
  - → a scheme of validity proof for such formulations
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  - $\rightarrow$ a scheme of validity proof for such formulations
  - $\rightarrow$ two key lemmas about non-overlapping inequalities
- About facet defining inequalities:
  - $\rightarrow$ a ready-to-use property to transpose facet defining inequalities

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- About natural variables and non-overlapping inequality formulations:
  - → a methodology to formulate some scheduling problem
  - → a scheme of validity proof for such formulations
  - → two key lemmas about non-overlapping inequalities
- About facet defining inequalities:
  - → a ready-to-use property to transpose facet defining inequalities
- About dominance inequalities:
  - → a theoretical framework
  - → a recipe to obtain a dominance inequality from a given operation

Perspectives

About dominance inequalities:

→ How do insert and swap inequalities improve formulation F^2?

→ Can we provide dominance inequalities useful for other combinatorial problems?

The end

Thank you for your attention