Layered controller synthesis for dynamic multi-agent systems

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November 30 2023

Our objectives

- A running example

https://perso.eleves.ens-rennes.fr/people/Emily.Clement/Videos/example_episodes/ex_0.mp4

	Timed Automata	Reinforcement Learning
Model	Abstract representation (acceleration)	
Waekness	Time of execution	Combinatorial or Continuous aspects

Our layered approach

- Our assumptions
  - **Our control:** the speed of (all) cars.

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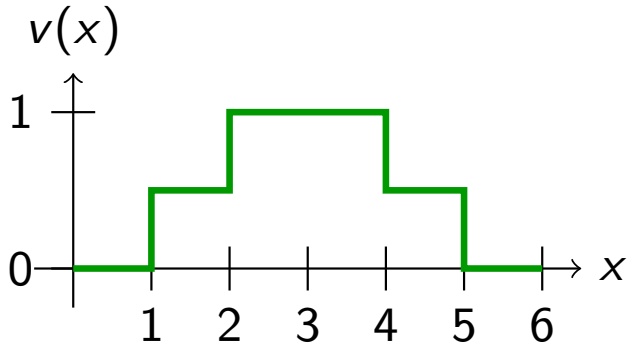

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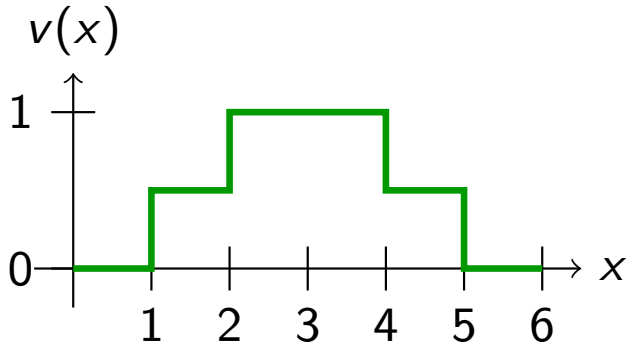

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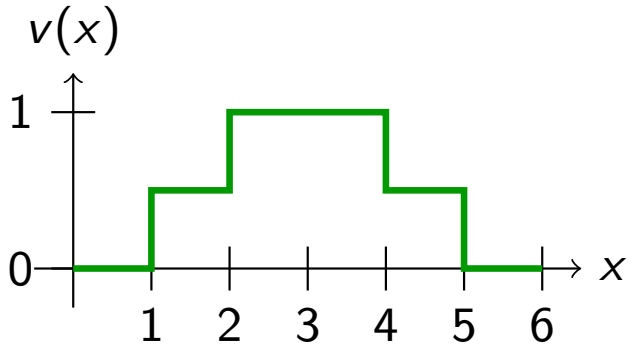

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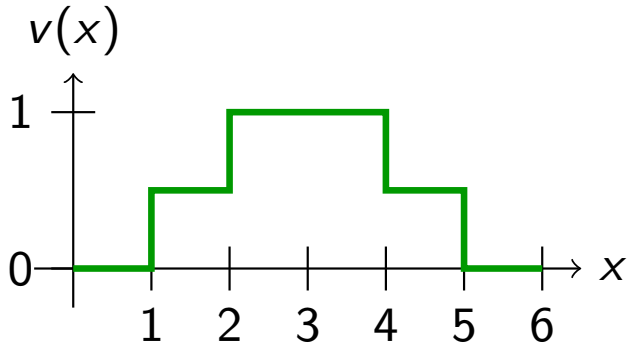

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SWA-SMT Solver

***Stage 1**: Reachability algorithm on a simplified ISWA model*

SWA

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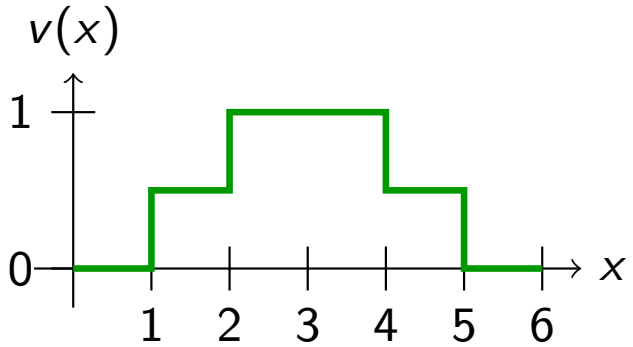

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***Stage 1**: Reachability algorithm on a simplified ISWA model*

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***Stage 2**: Refine the model of the speed*

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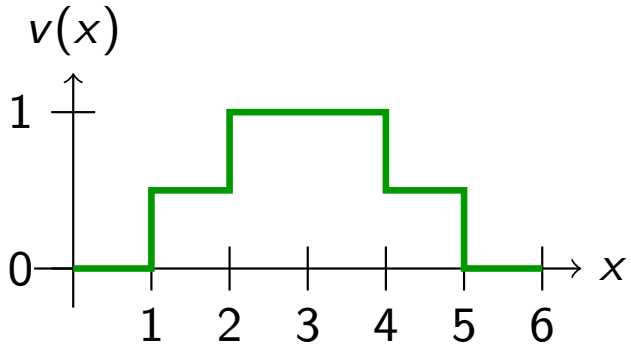

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SWA-SMT Solver

***Stage 1**: Reachability algorithm on a simplified ISWA model*

SWA

***Stage 2**: Refine the model of the speed*

SMT

RL training

***Generate** a dataset for random initial positions*

Dataset

***Stage 3**: Train an RL algorithm with our dasaset*

RL

SWA-SMT solver

SWA solver

Stage 1: Reachability algorithm on system of ISWA

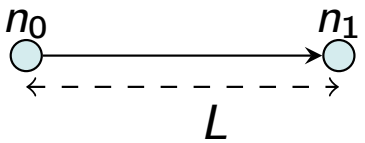
SWA

***Stage 2**: Model the accelleration and de-celeration*

SMT

Model for a car traffic

- A point in $\mathbb{R}^2$: a node 

- A section $s_{[n_0,n_1],L}$ of the road: 

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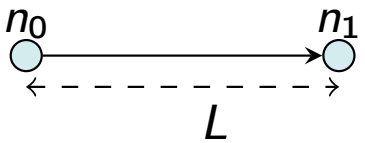
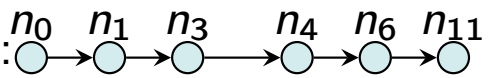
- A section $s_{[n_0,n_1],L}$ of the road: 

- A path: p_0 : 

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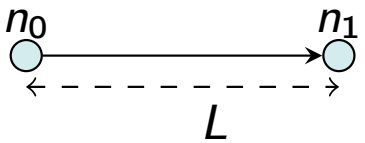
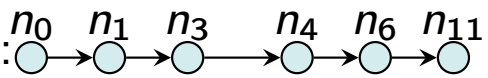
- A section $s_{[n_0,n_1],L}$ of the road: 

- A path: p_0 : 

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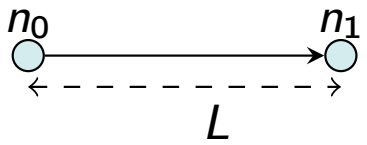
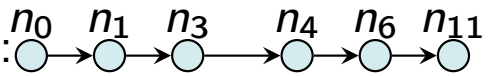
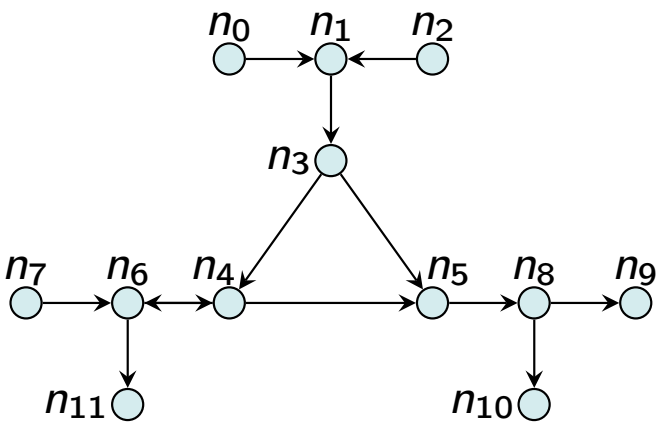
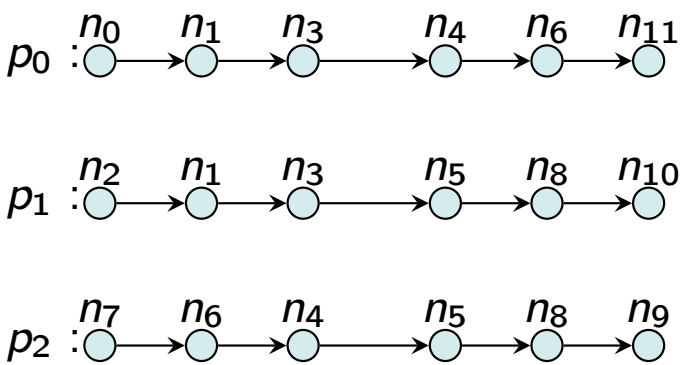
- A section $s_{[n_0,n_1],L}$ of the road: 

- A path: p_0 : 

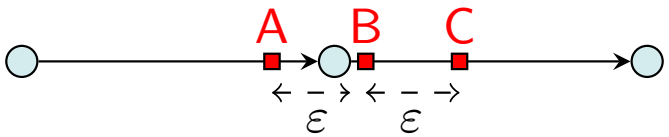
- Car: (position, speed, trajectory)
- A car traffic: c_0, c_1, c_2 are each assigned paths p_0, p_1, p_2:





Rules to model collision avoidance

- #1: security distance when driving in the same direction and between neighbouring sections



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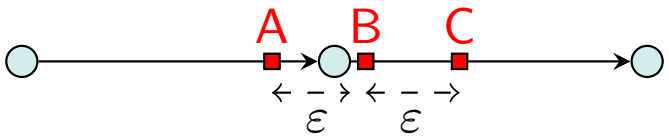

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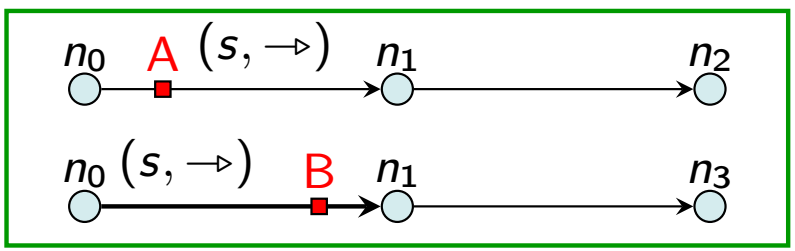

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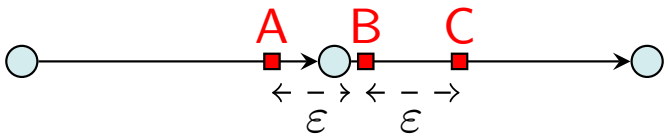

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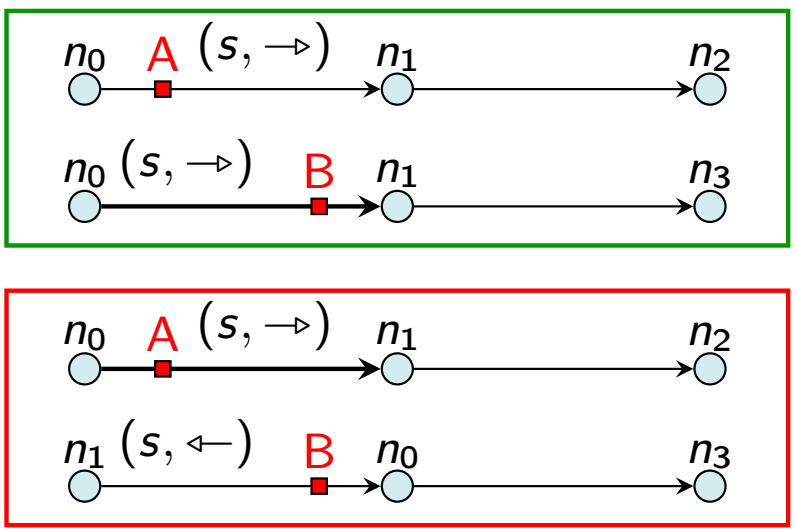

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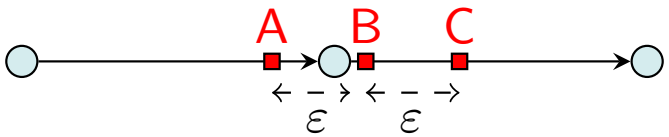

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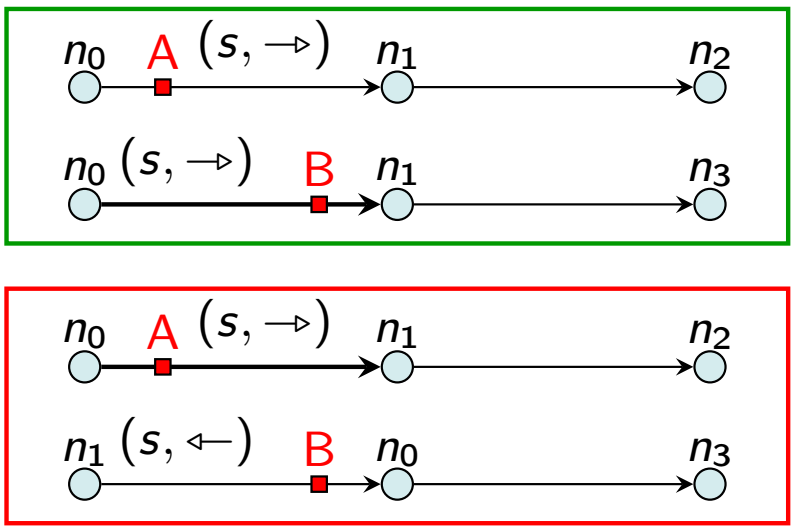

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- #3: No Overtaking between cars

What type of Timed Automata to use to model this?

- Needs
  - **Clocks of TA**: Monitor each car's progress.

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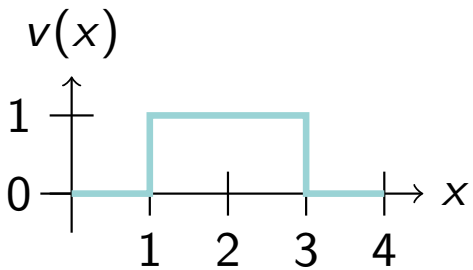

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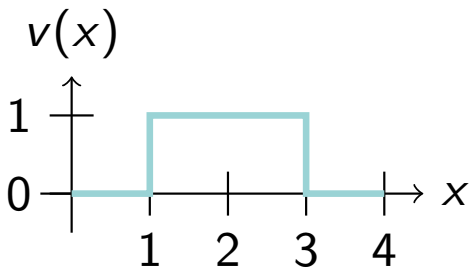

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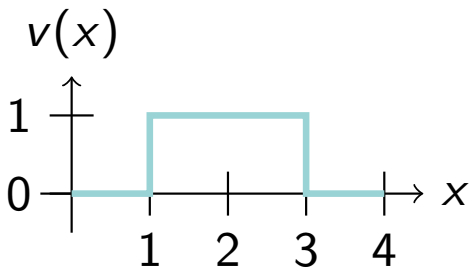
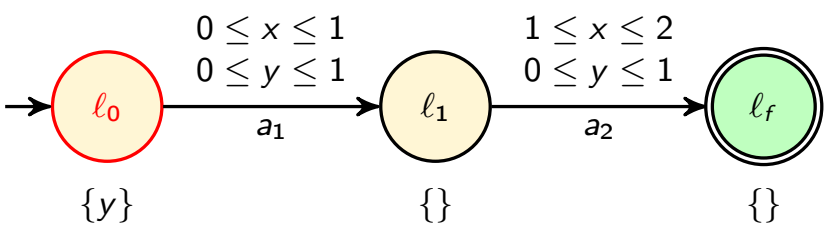
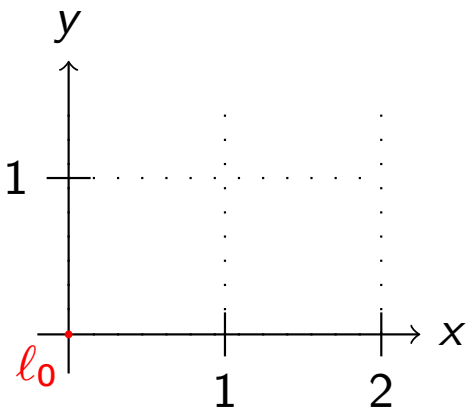


  - **Synchronised action**: Compute distance between each cars.
  - **FiFo channels**: A car cannot overtake another car.

Initialized Stopwatch Timed Automata with bounded channels

- Example of a two-clocks Stopwatch Timed Automata





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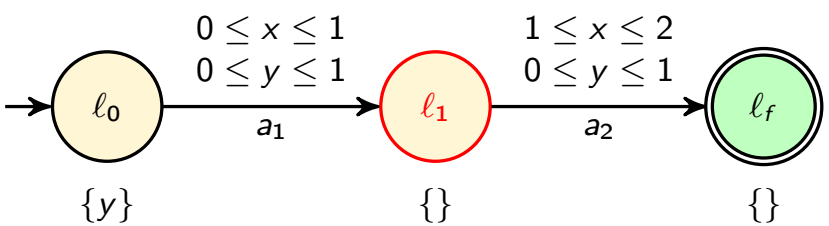

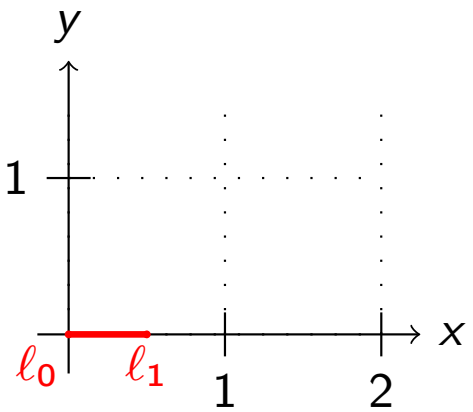

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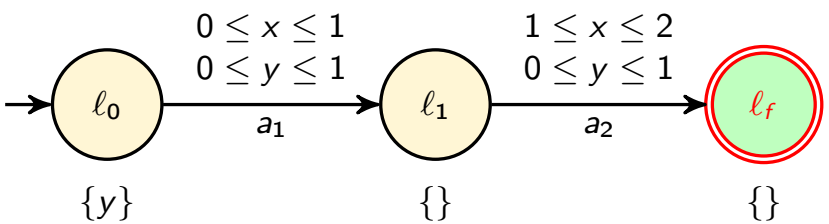

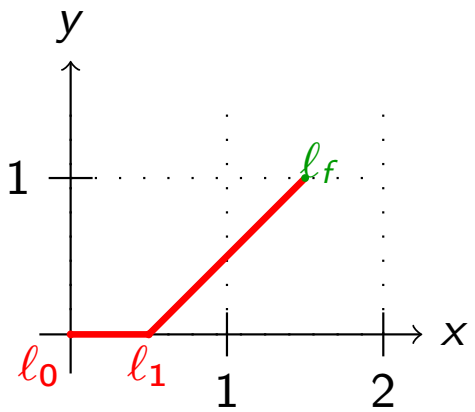

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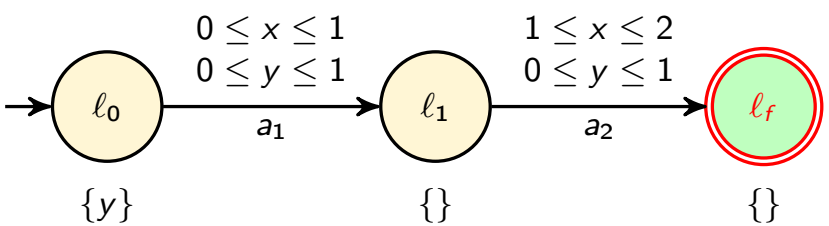

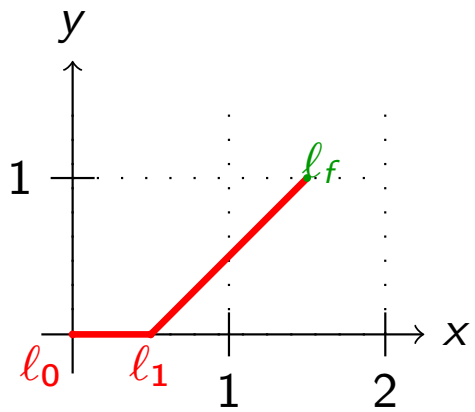

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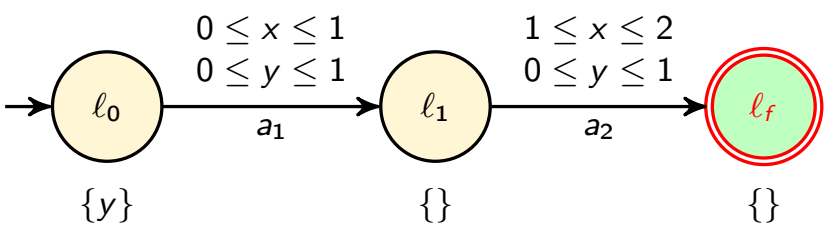

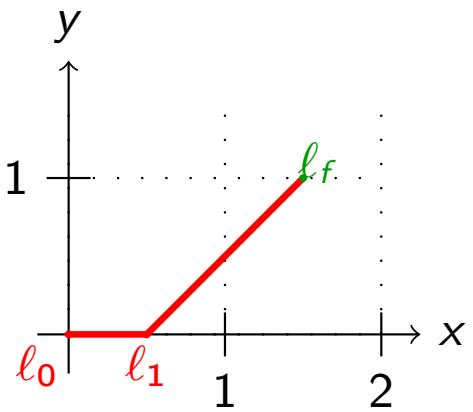

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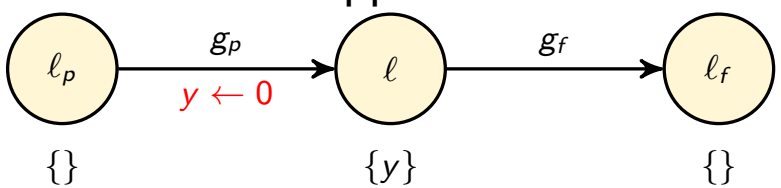
- Initialized Stopwatch Timed Automata
  - Reset the stopped clock in the previous or following transition:



  - **Reachability** becomes Decidable for this fragment of SWA.

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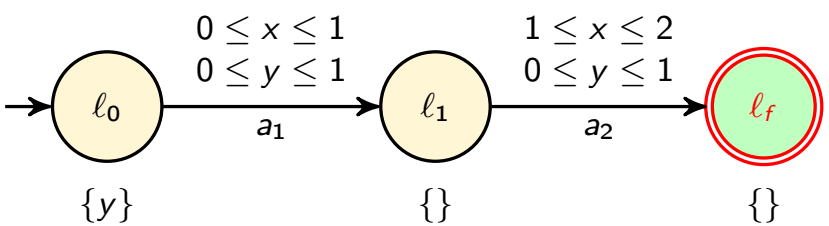

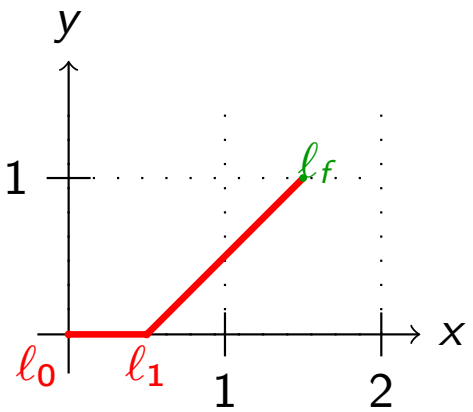

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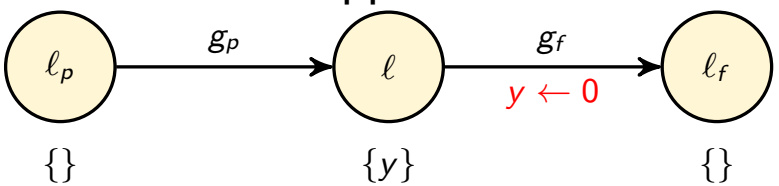

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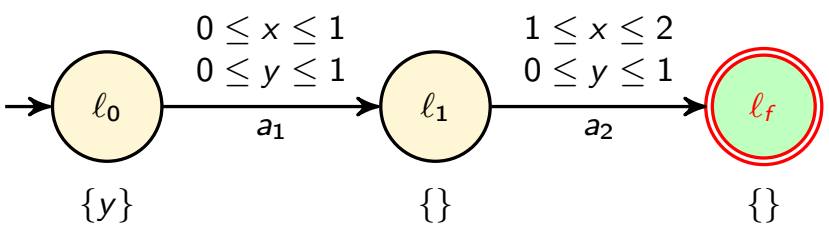

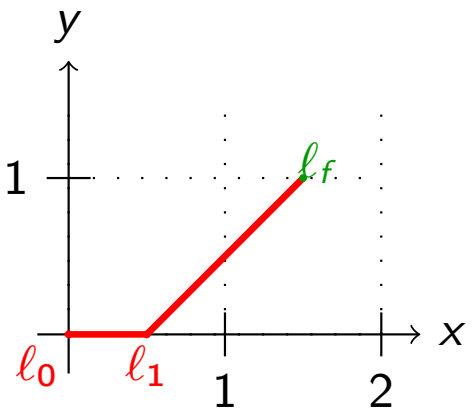

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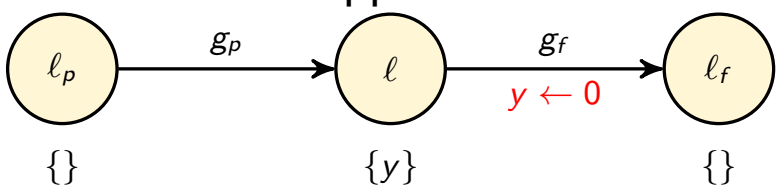

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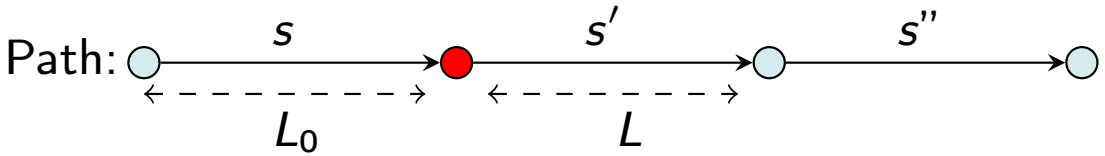
- Bounded channels
  - Channels: FiFo queue of symbols (actions) to be pushed/read

Model the car progress

- Car A progress along its paths



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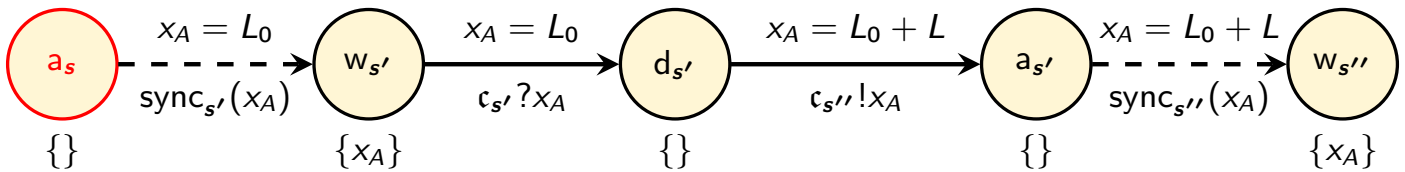

- ▷ **Clock** x_A: distance travelled along its paths
- ▷ **Stopwatches** $\{x_A\}$: the car A stops instantly.
- ▷ **Channels** $c_{s'}!x_A/c_{s'}?x_A$: respect the order of cars in a section $s \Rightarrow$ no overtaking.
- ▷ **Intersection**: use classical synchronized action to activate *intersection automata*

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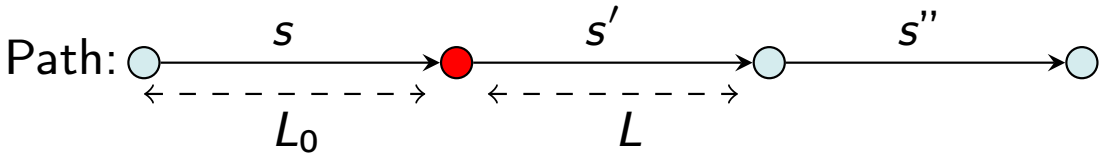

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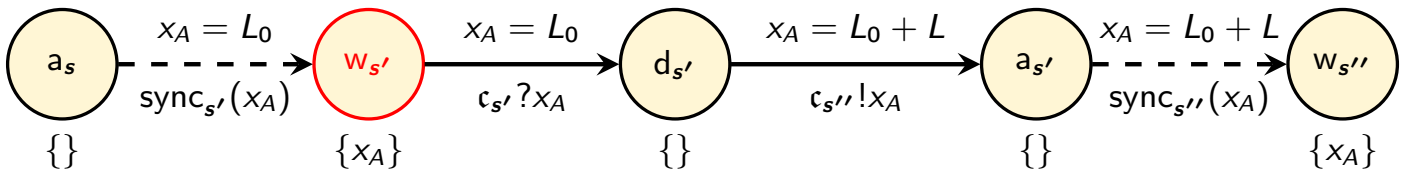

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- ▷ **Stopwatches** $\{x_A\}$: the car A stops instantly.
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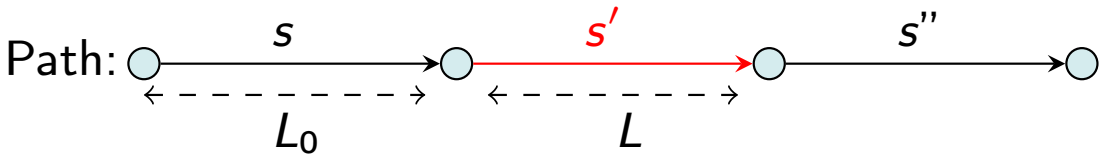
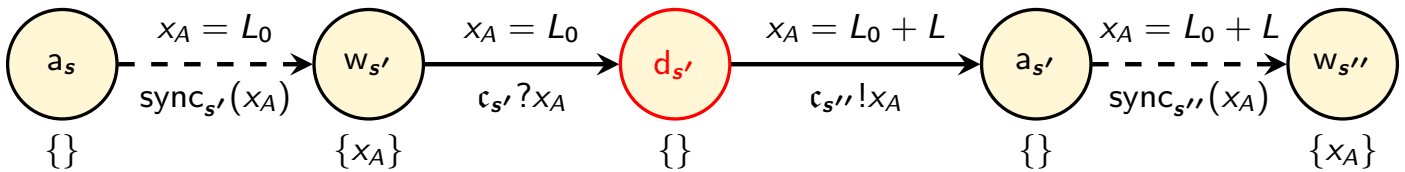
- ▷ **Intersection**: use classical synchronized action to activate *intersection automata*

Model the car progress

- Car A progress along its paths



- Car A Timed automaton:



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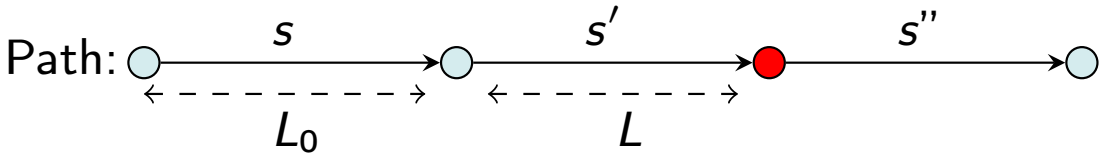

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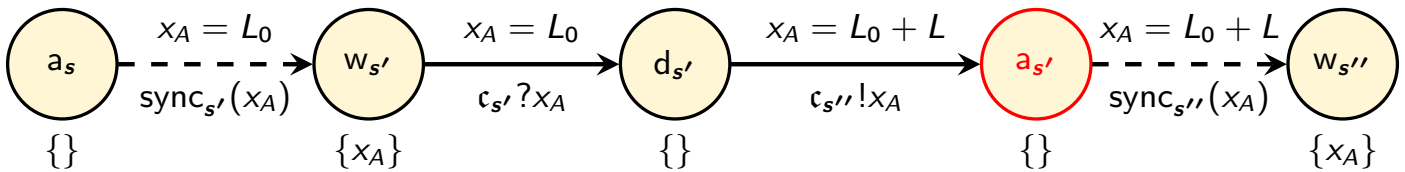

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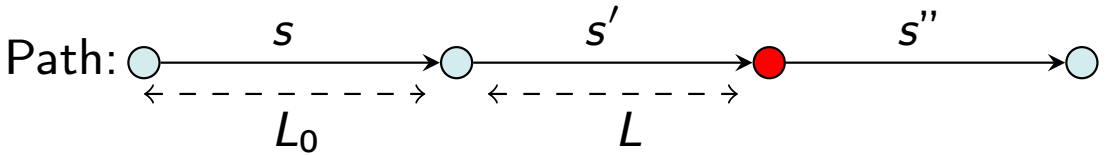

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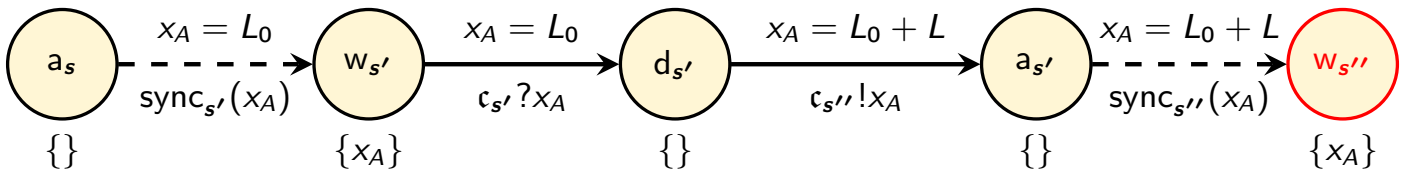

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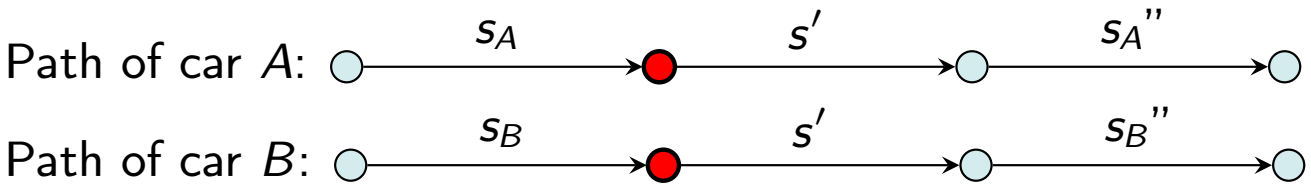

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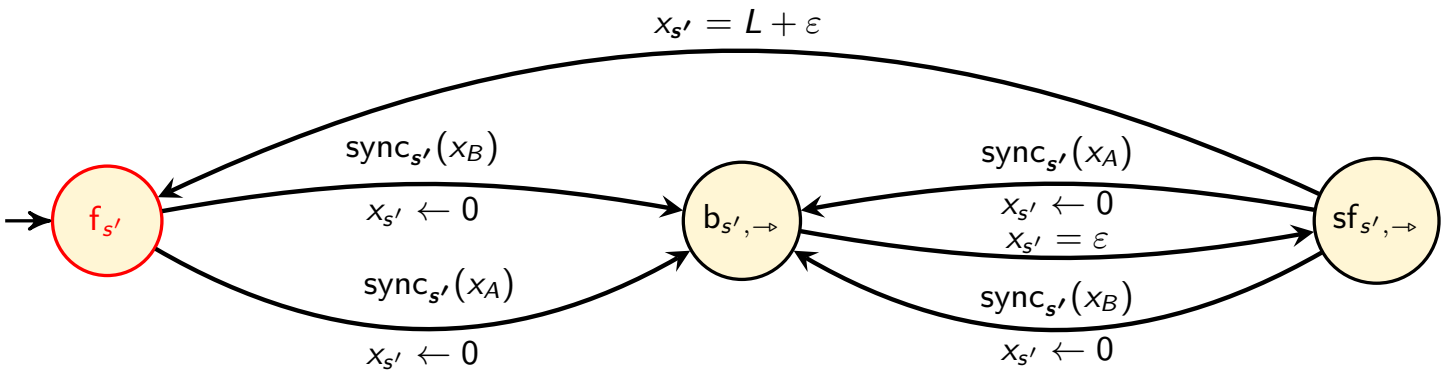

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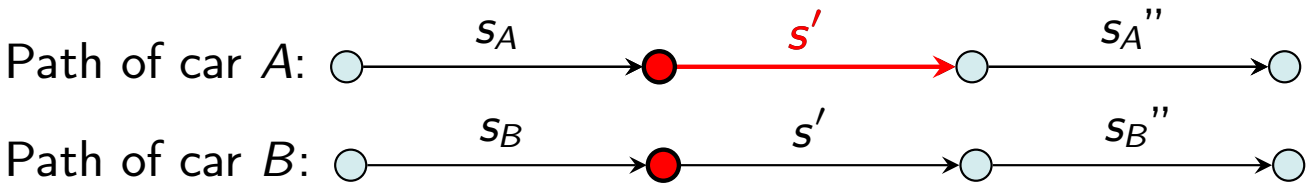

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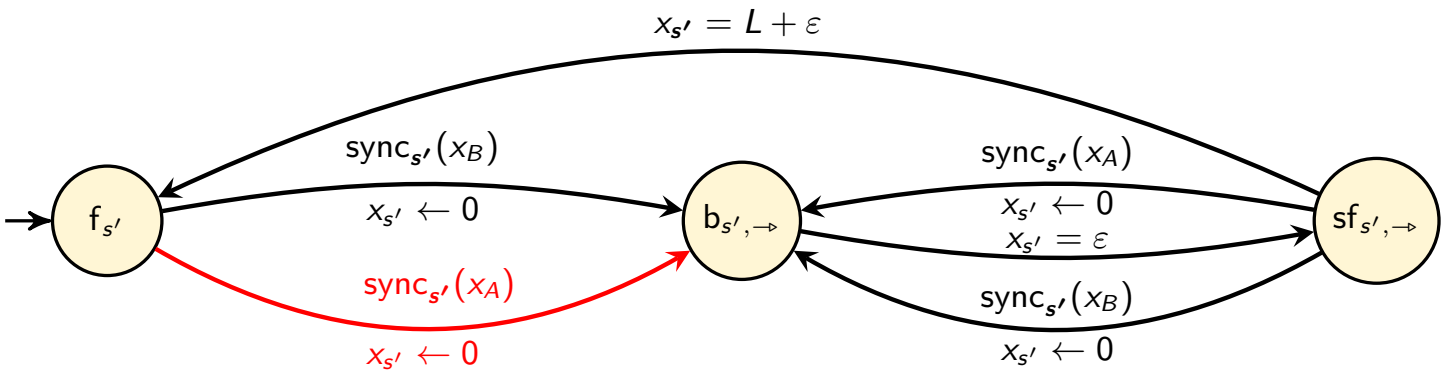

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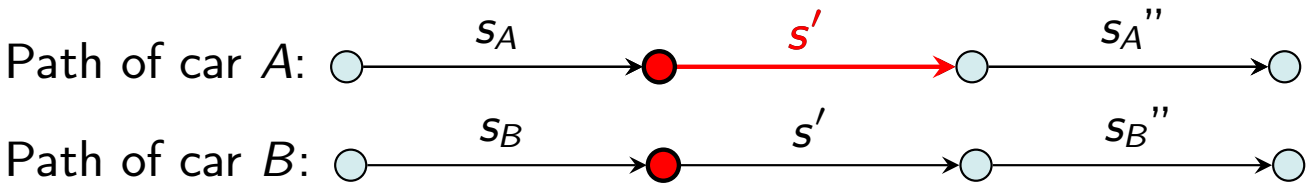

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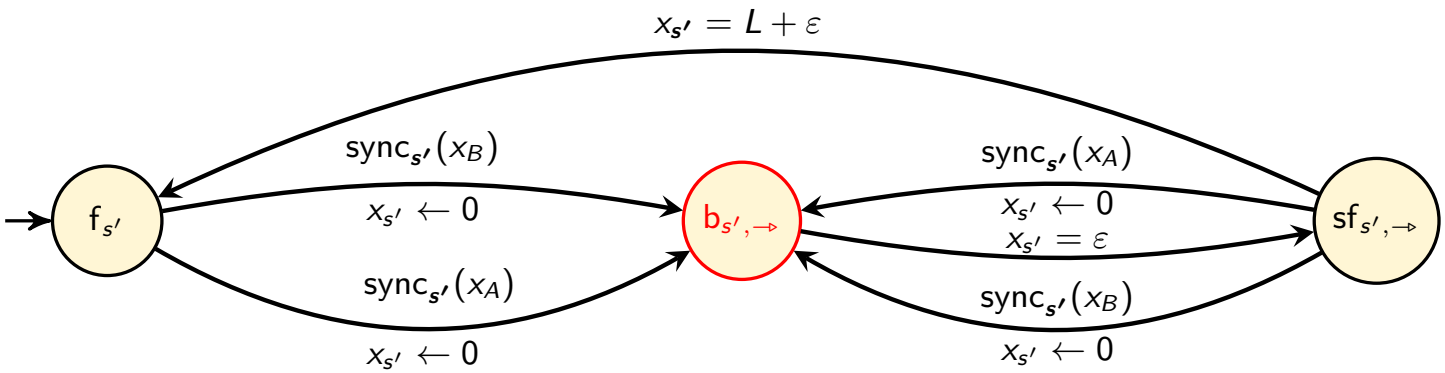

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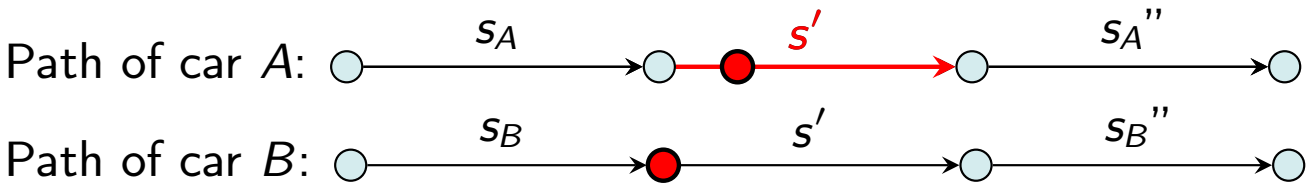

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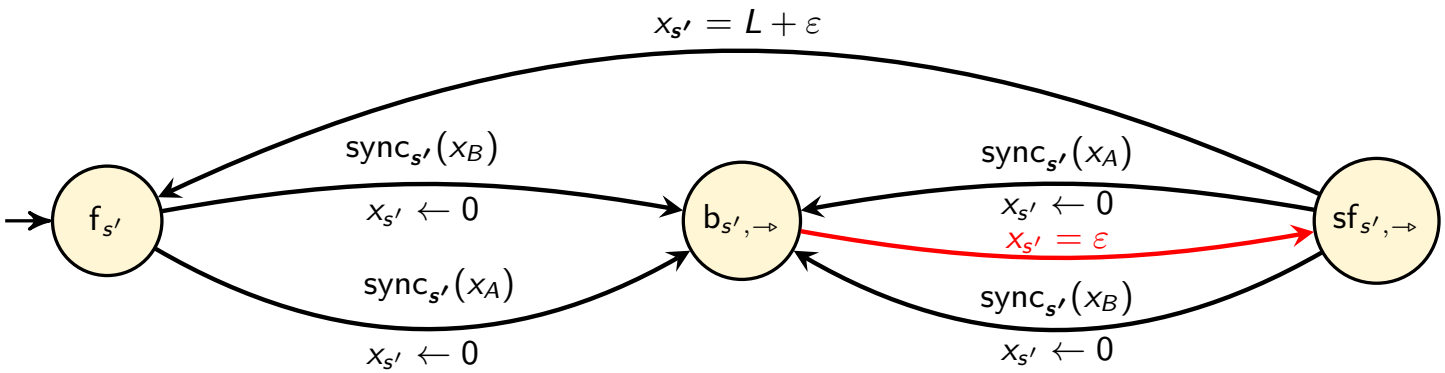

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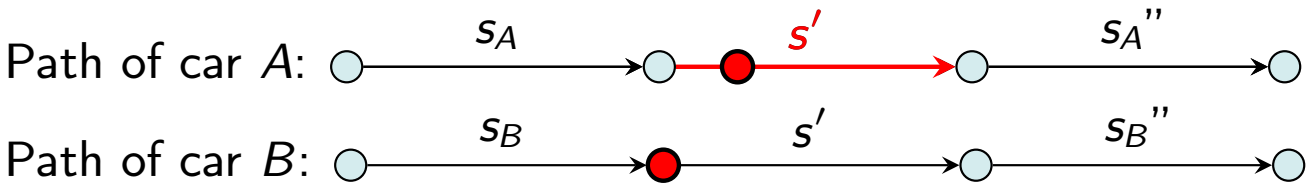

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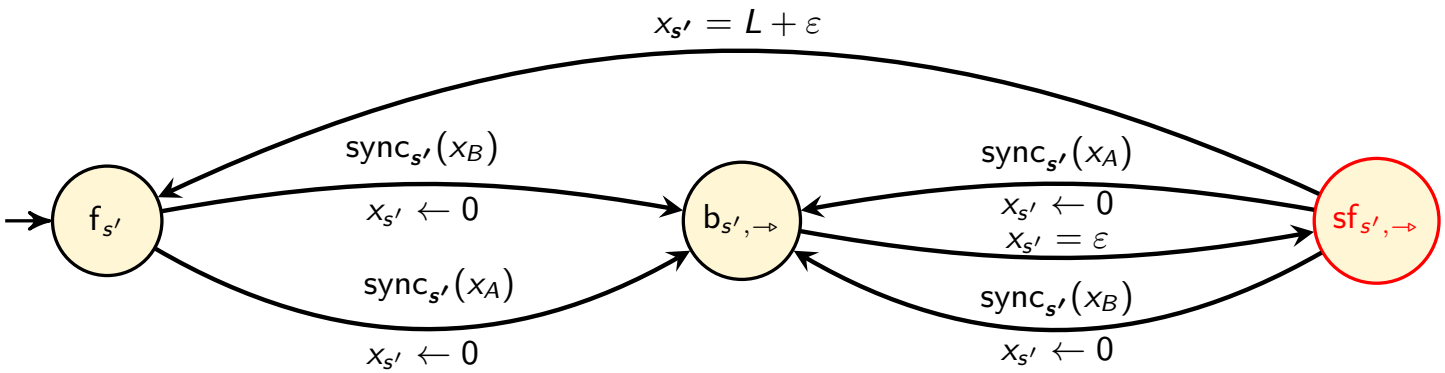

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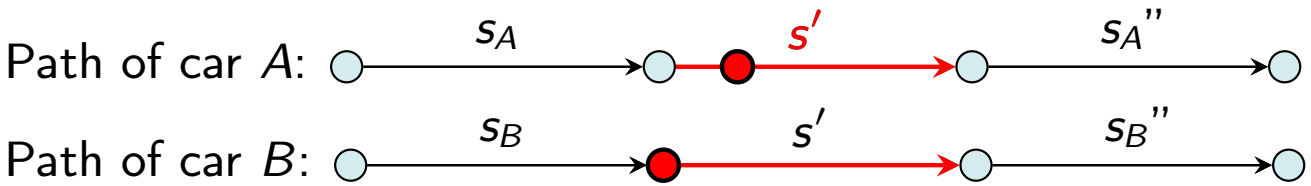

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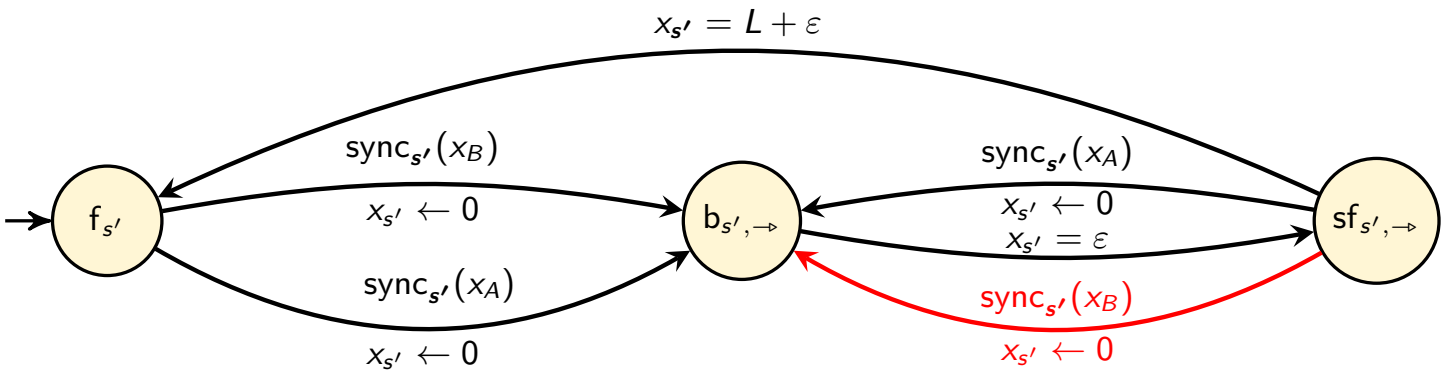

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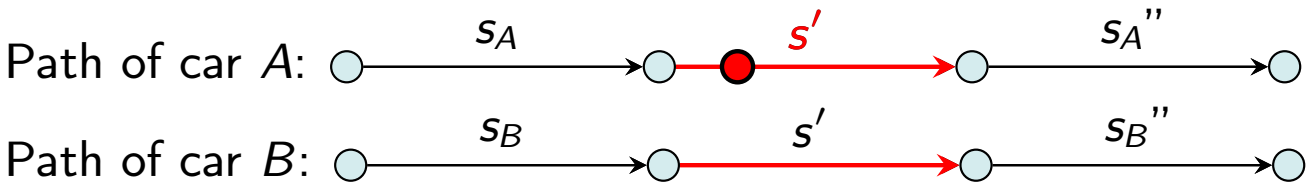

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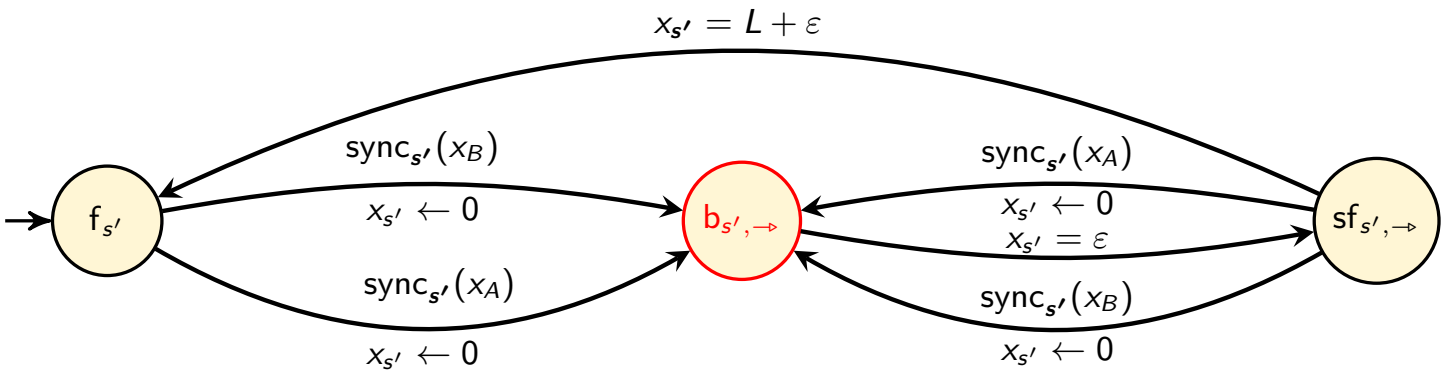

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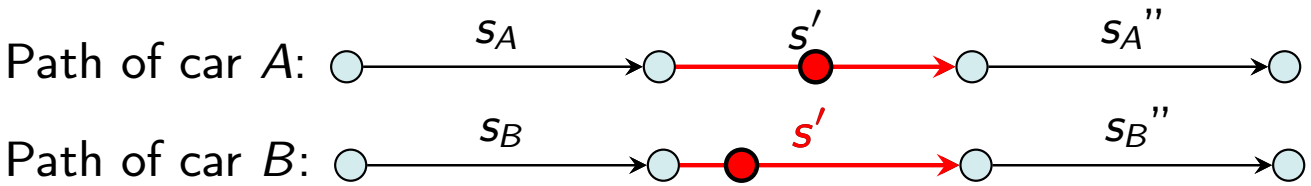

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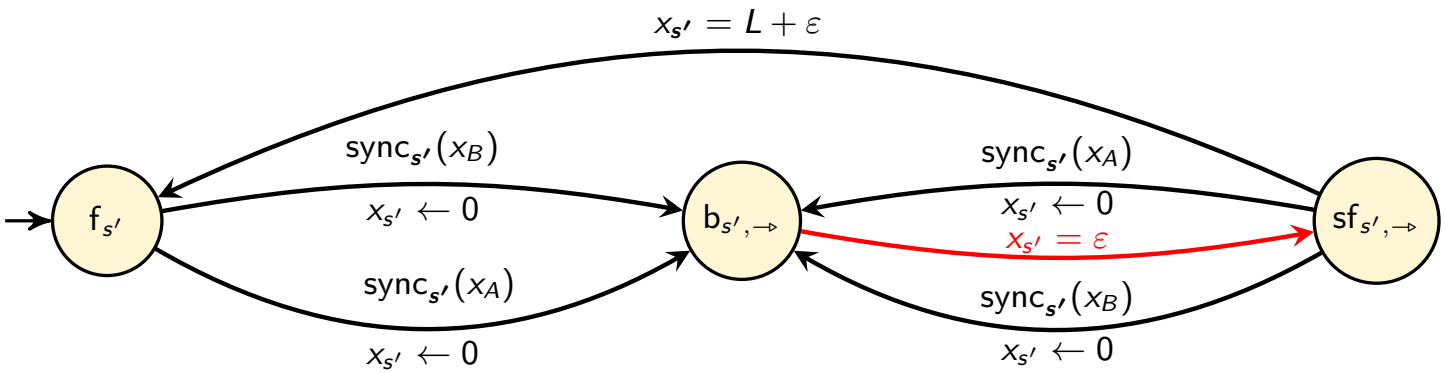

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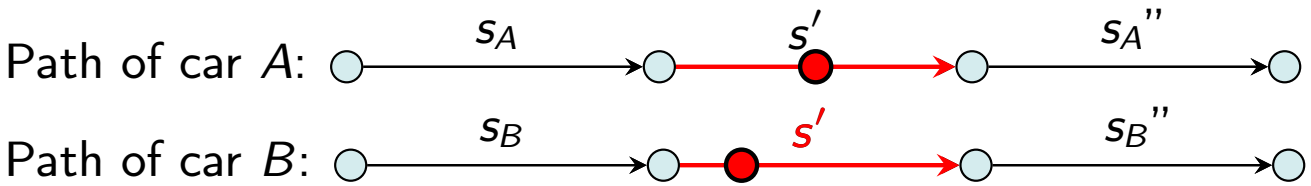

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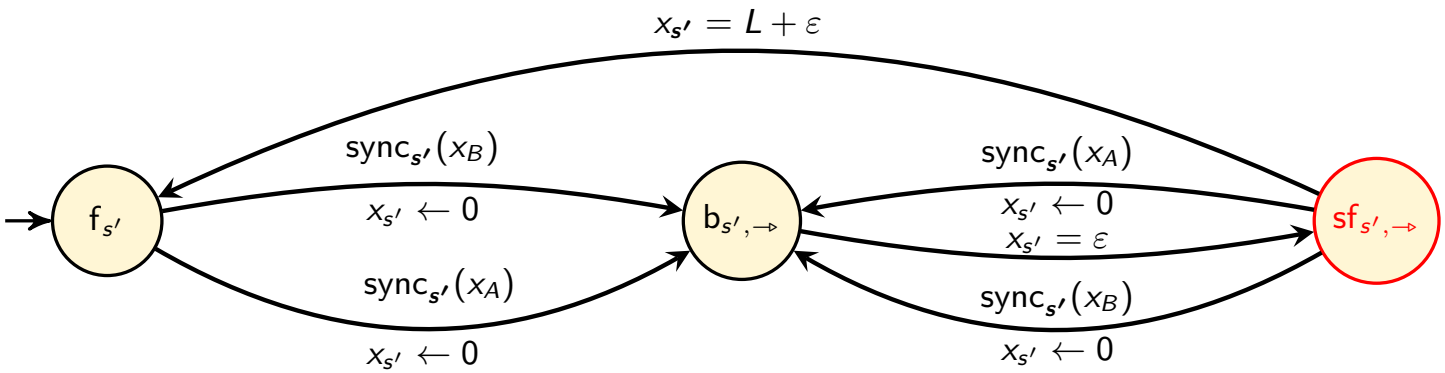

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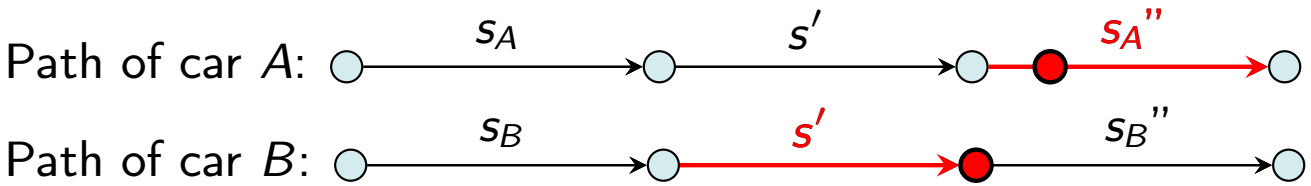

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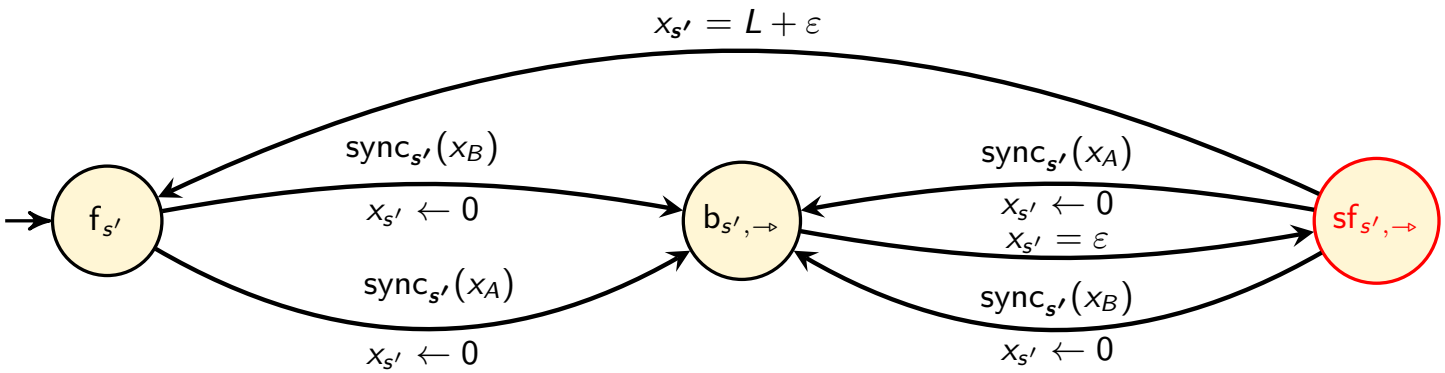

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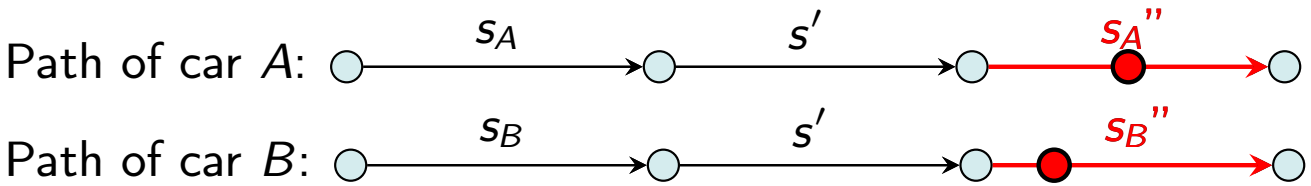

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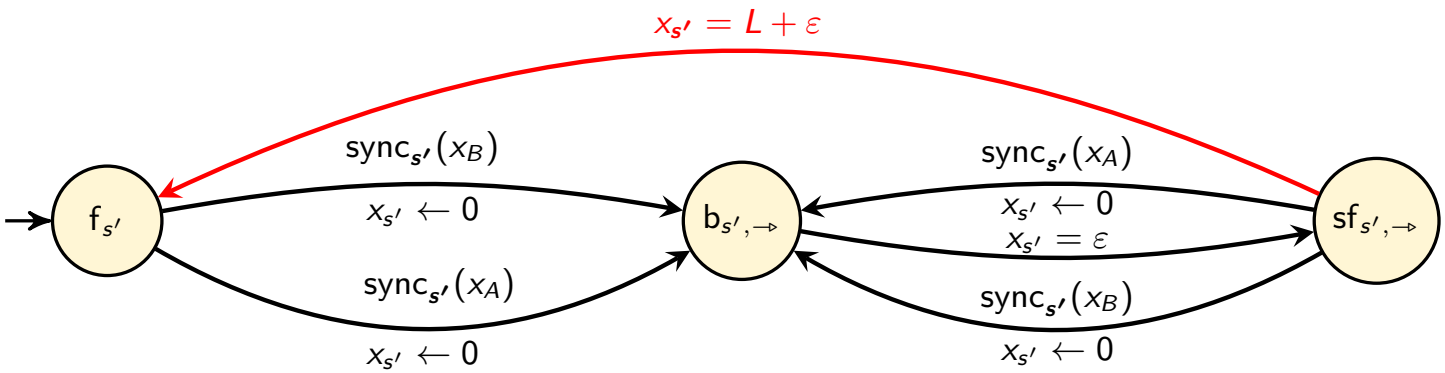

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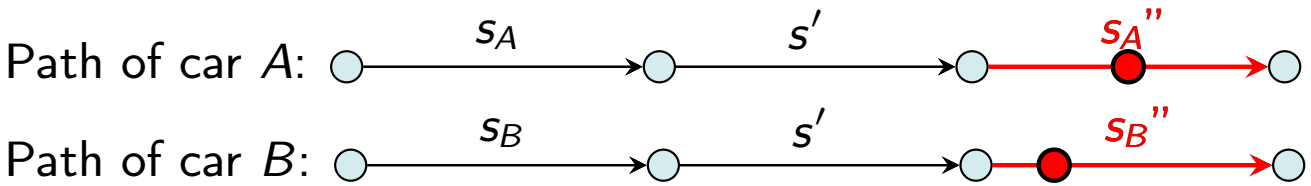

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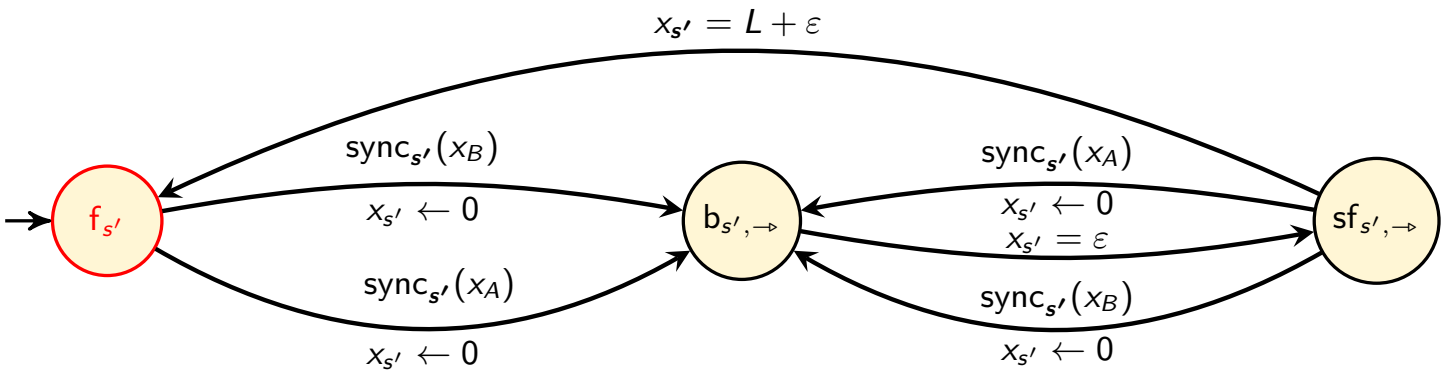
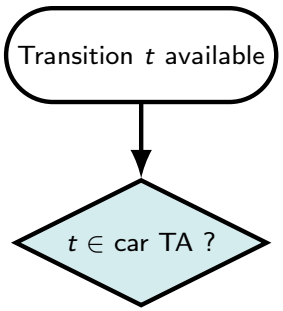


Our Algorithm: a DFS with an optimised succ function



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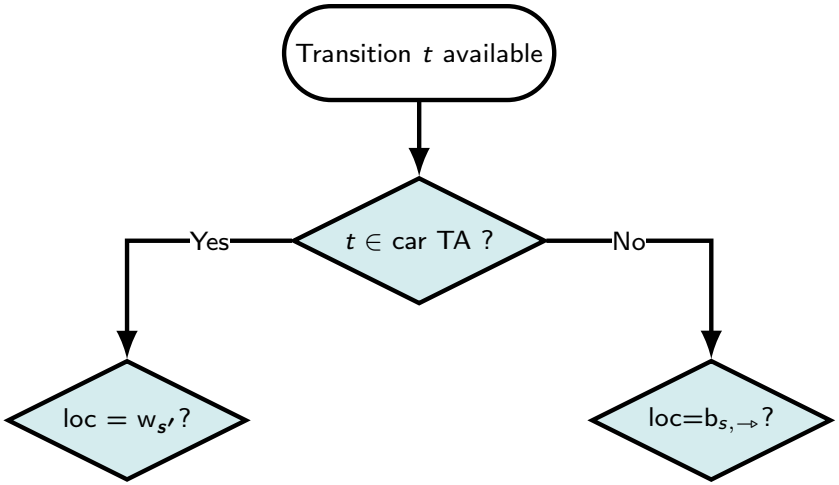

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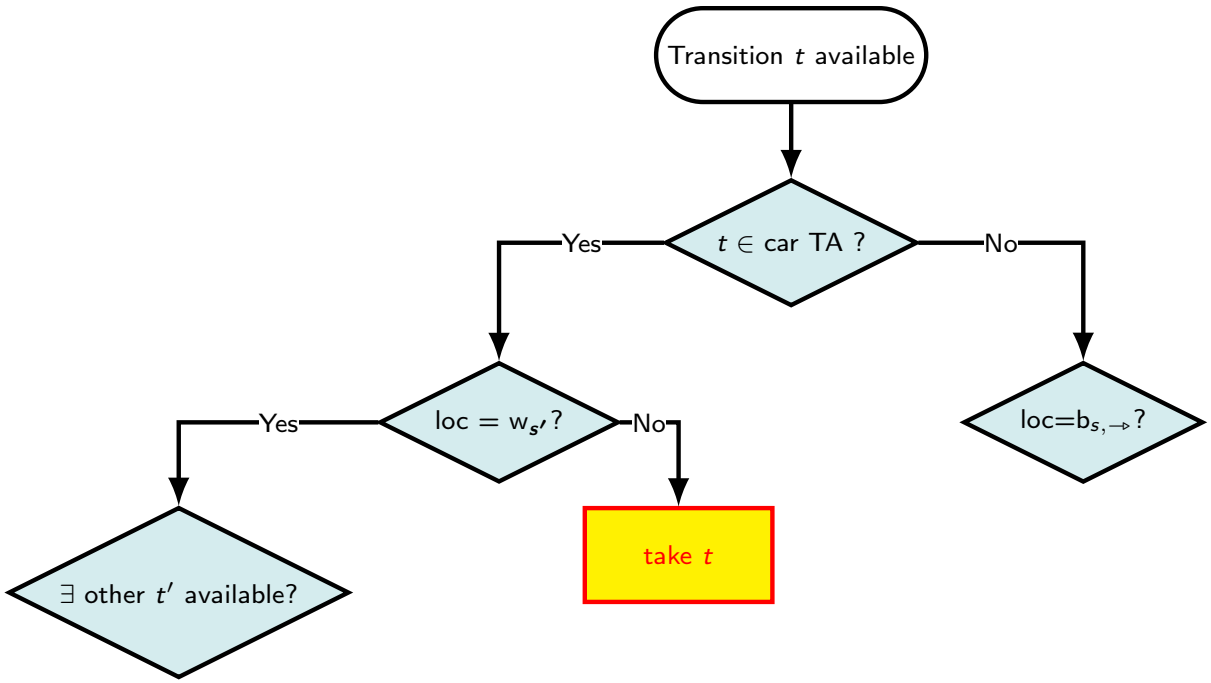

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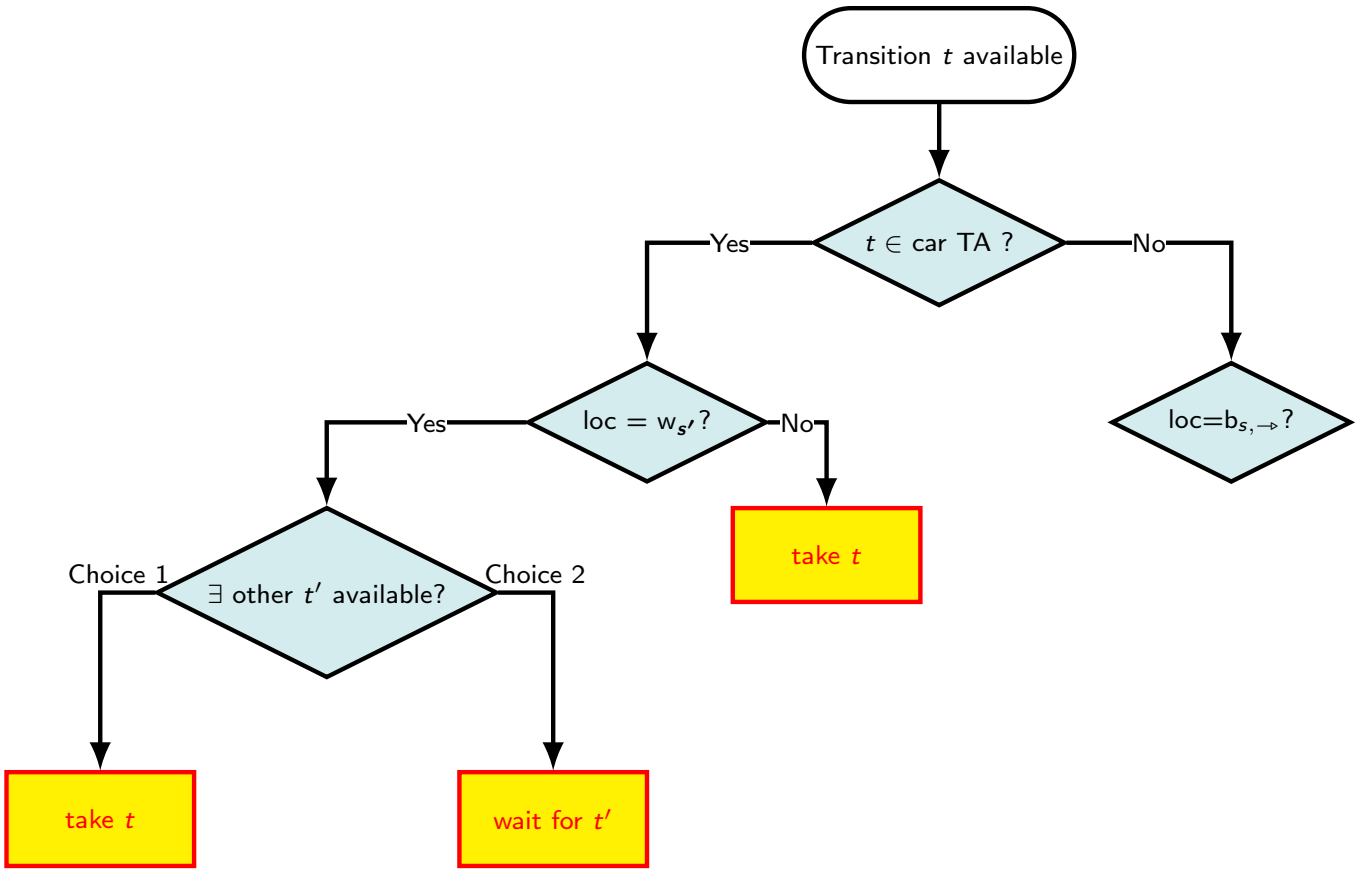

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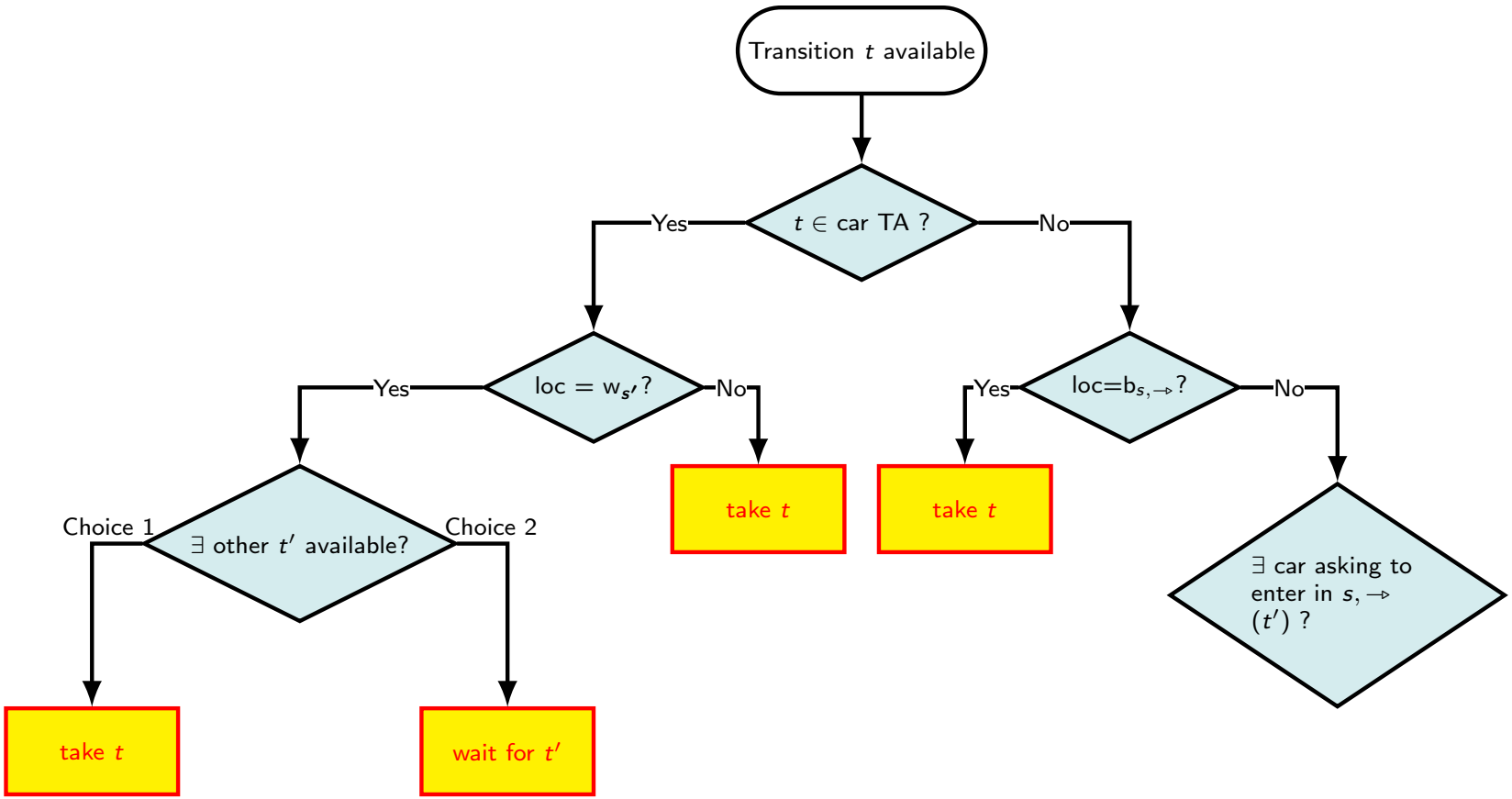

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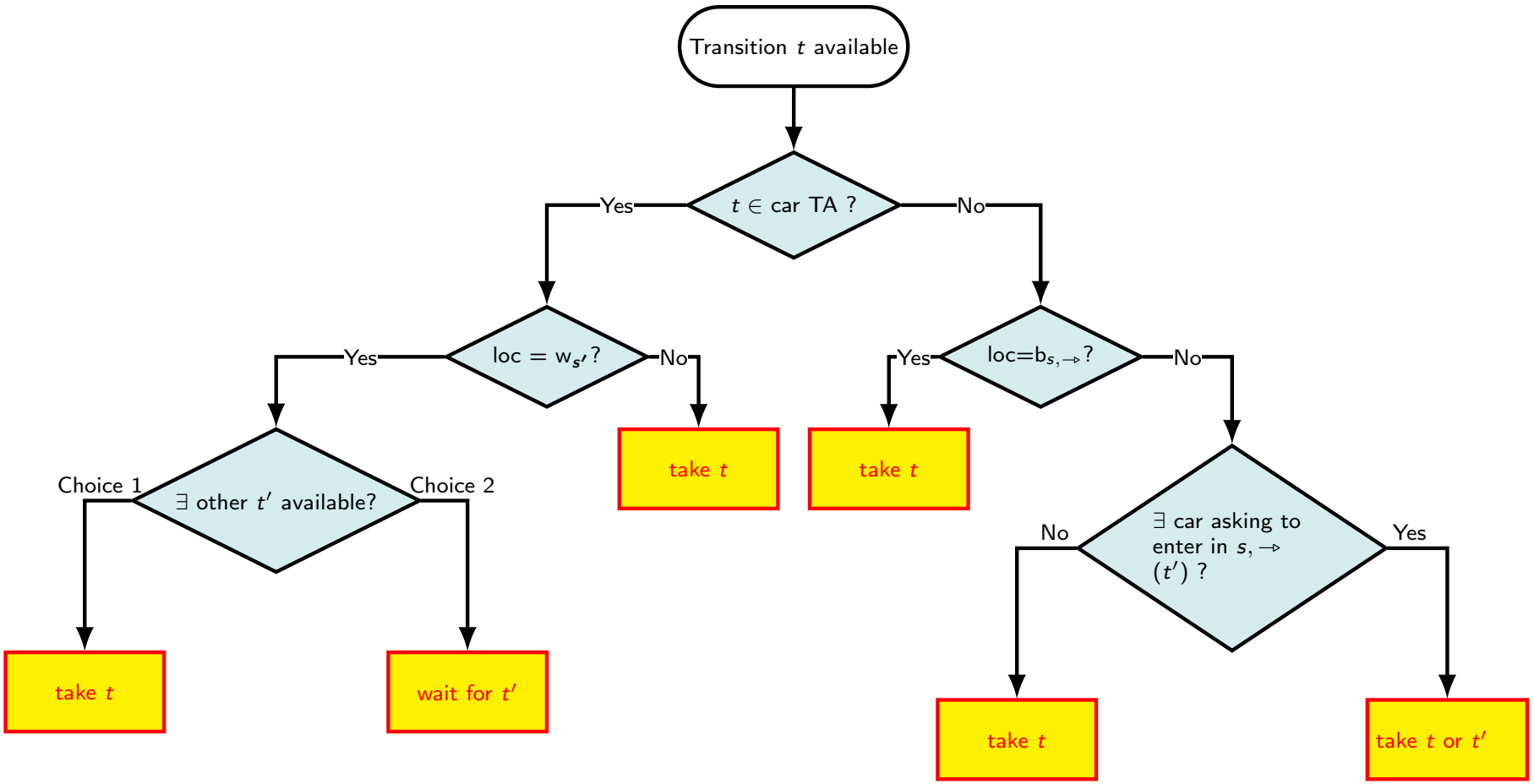


SWA-SMT solver

SMT solver

Stage 1: Reachability algorithm on system of ISWA

SWA

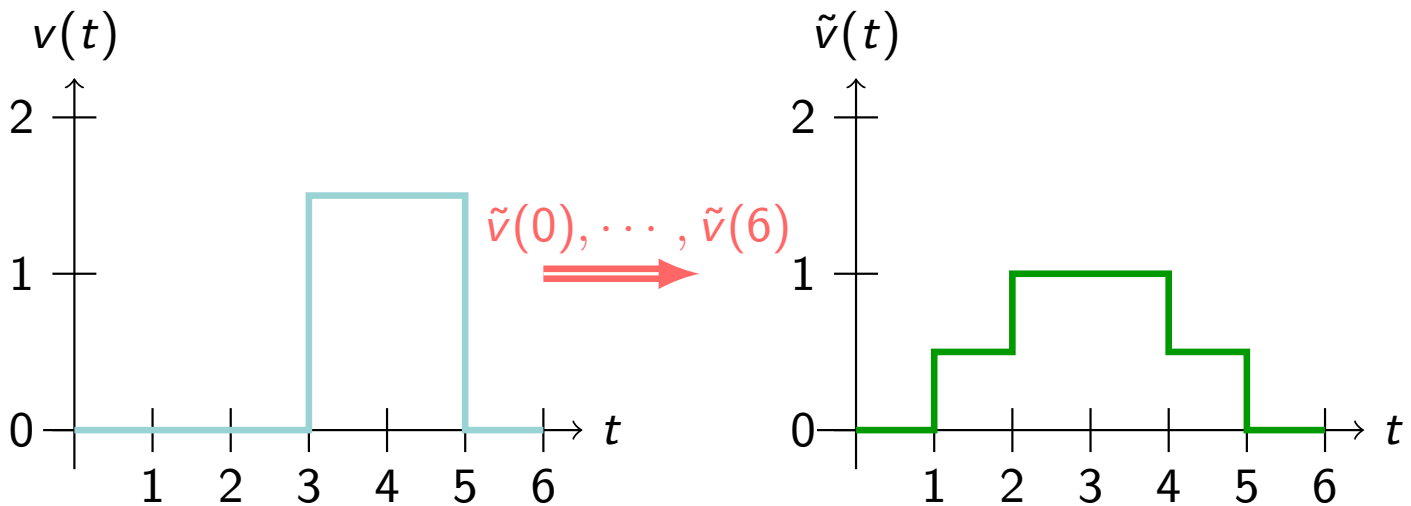
***Stage 2**: Model the accelleration and de-celeration*

SMT

New model for speed graph

- A constant piecewise affine function
  - A more realistic model that takes into account the **dynamic of the system**
  - **Different** car speeds
  - **Bounds** on deceleration and acceleration

$$\begin{aligned} v_i(t) &\Rightarrow \tilde{v}_i(0), \cdots, \tilde{v}_i(k-1) \\ x(t) &\Rightarrow \tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l) \end{aligned}$$



How to preserve security distance ?

- New positions/speeds
  - $\tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l)$
  - $\tilde{v}_i(0), \cdots, \tilde{v}_i(k-1)$

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- Example of SMT solver's inequalities

For each step k :

- $\tilde{v}_i(k) - d_{\max} \leq \tilde{v}_i(k+1) \leq \tilde{v}_i(k) + a_{\max}$

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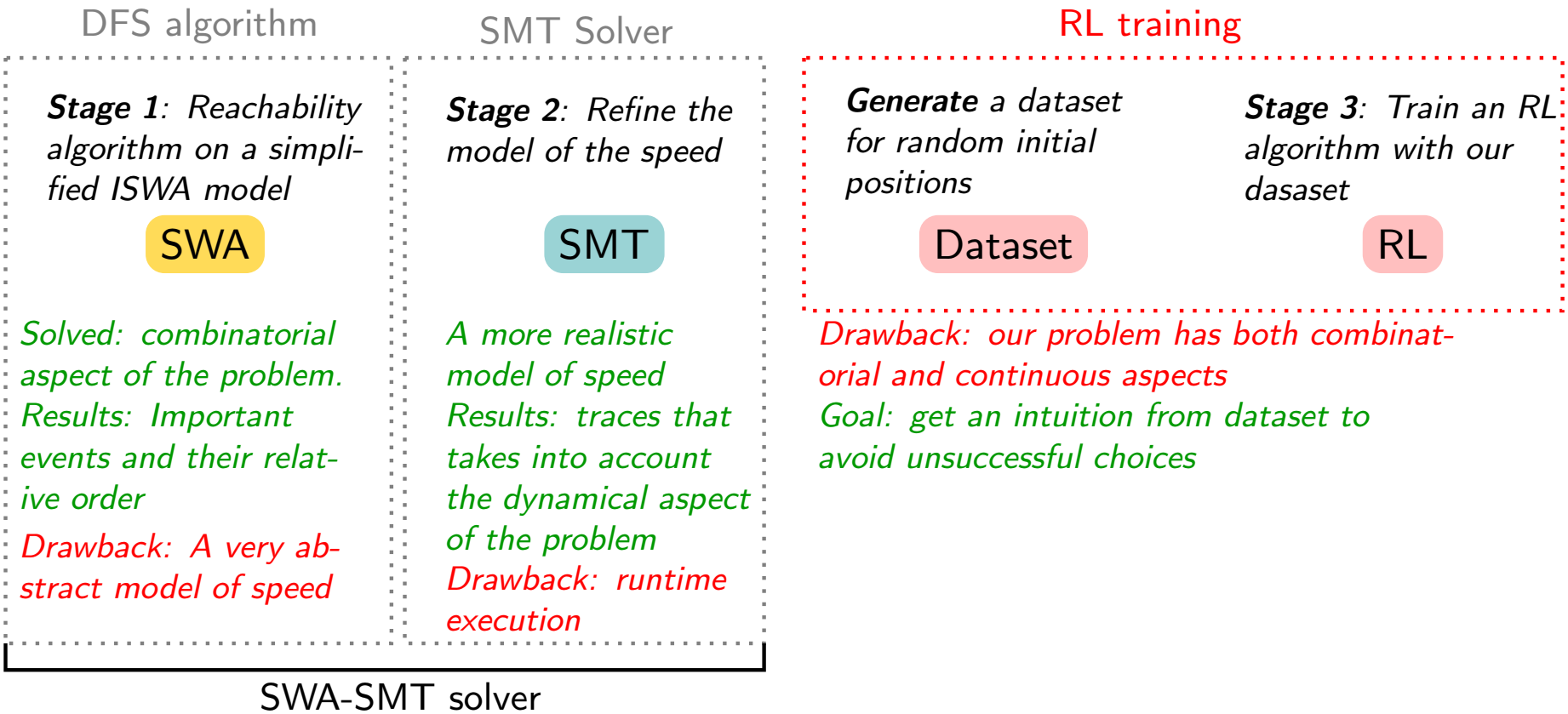
Why use of SMT solver?



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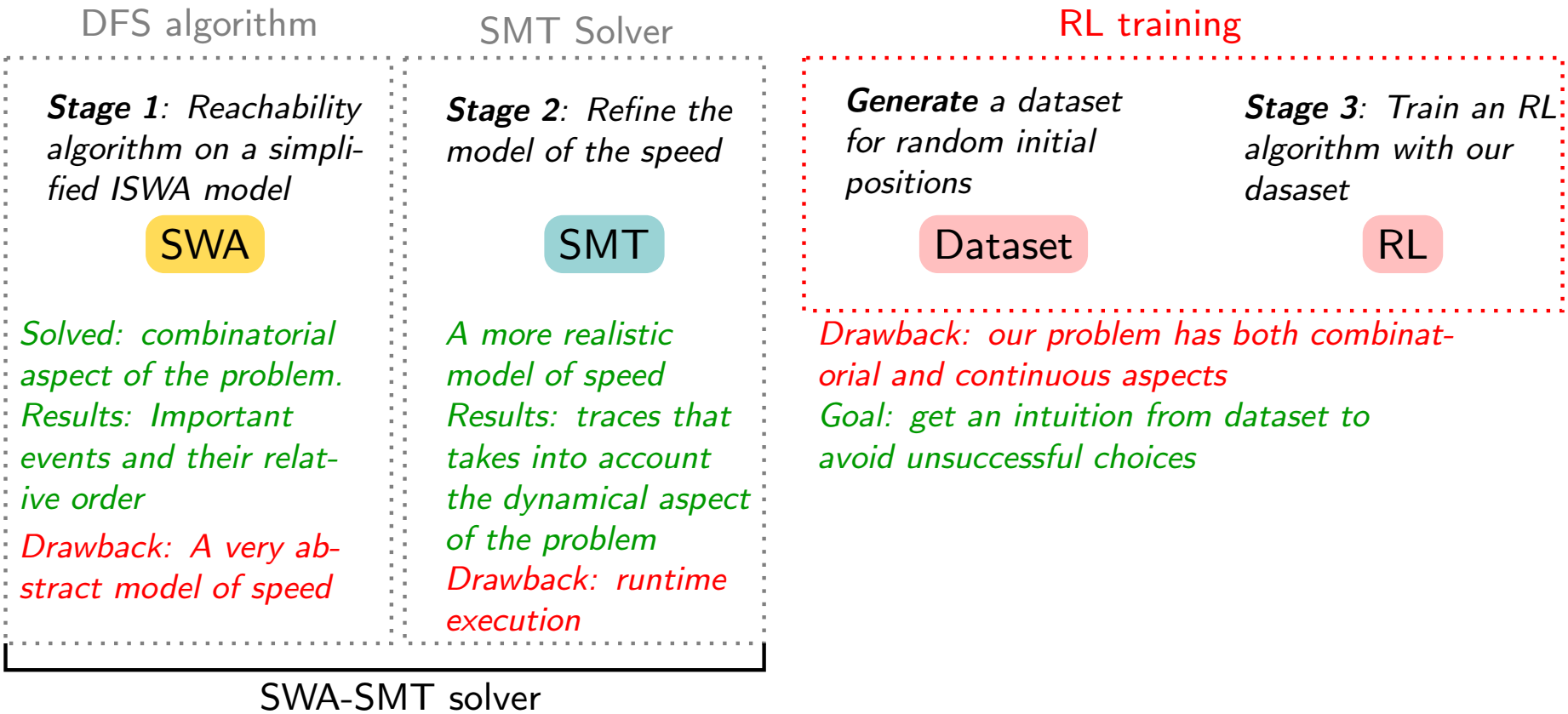
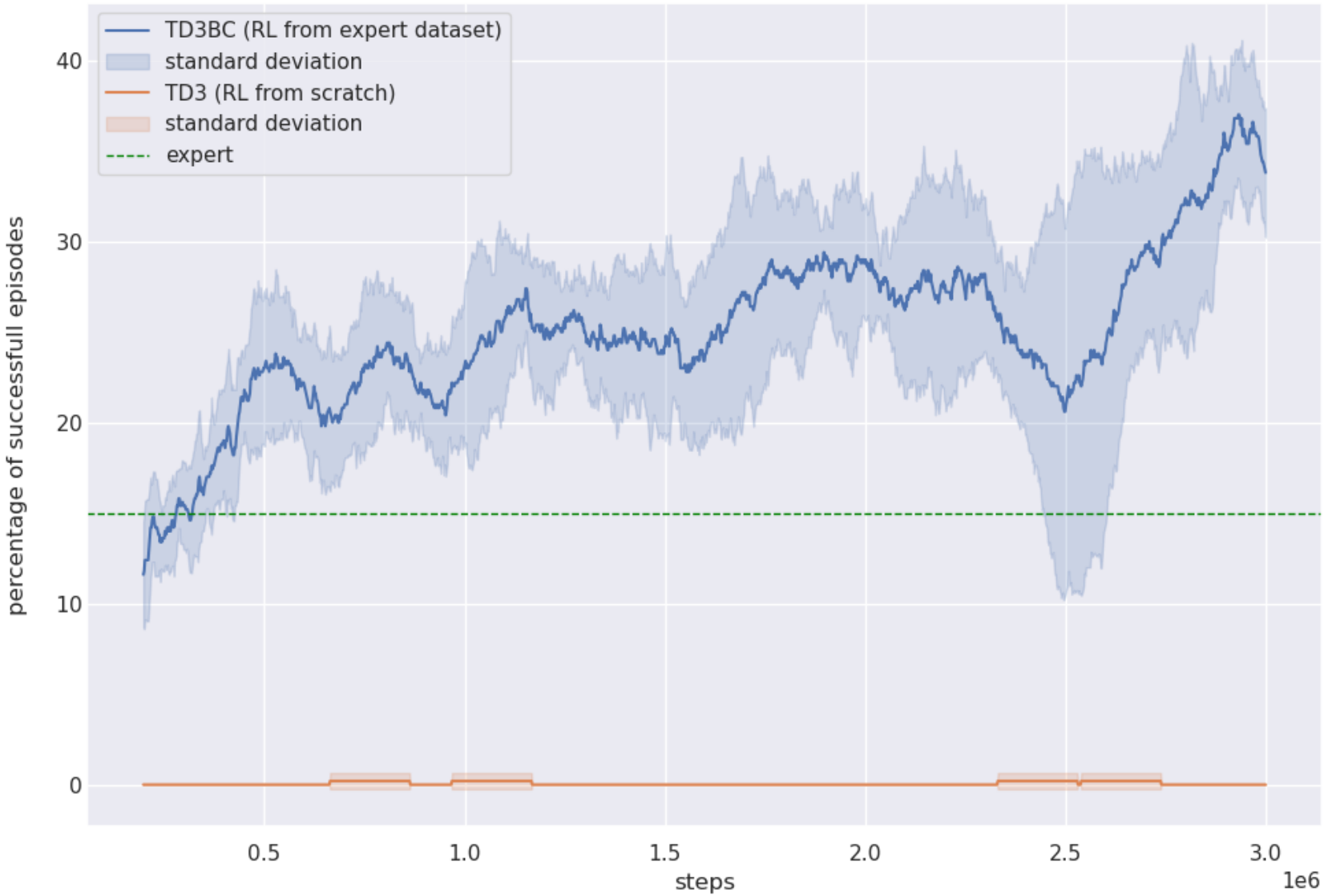
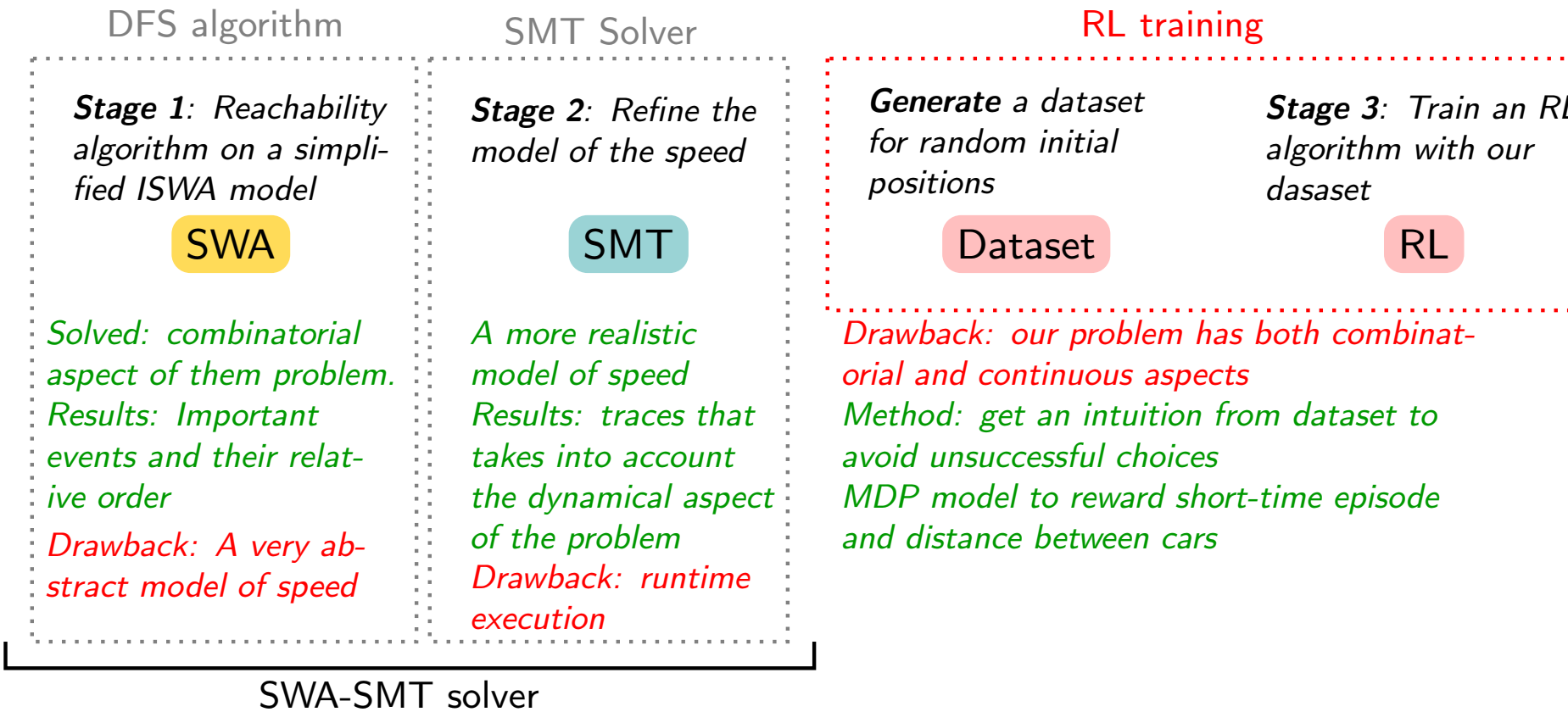


- RL training dataset
  - Create random initial positions/speeds for cars
  - Generate traces with the SWA-SMT solver

Results with SWA-SMT solver, post SWA-SMT solver RL and single RL training



Steps of the layered method



Conclusion

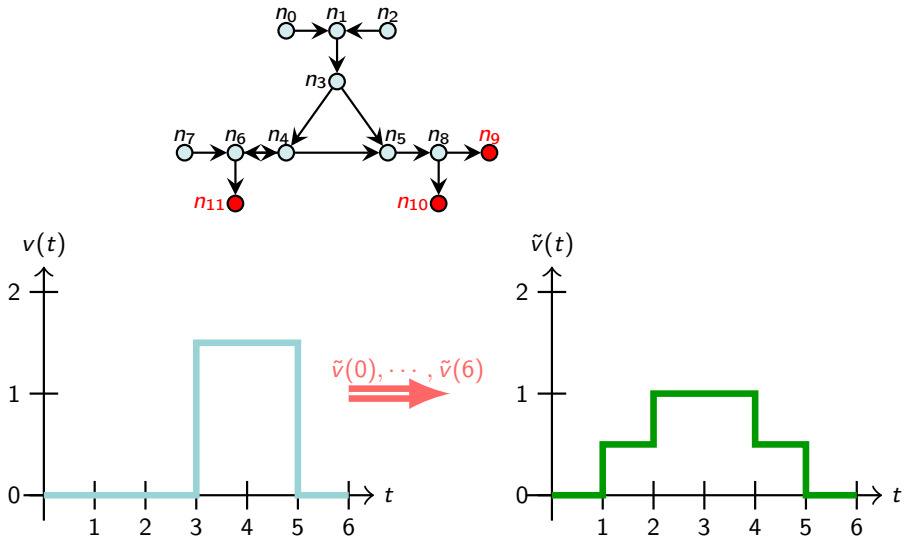
- SWA-SMT Solver

Automata-based model

Efficient algorithm
Abstract model with unrealistic speed model

Piecewise-affine speed graph

Bounded accelleration and deceleration
Different speed
SMT solver to model and solve the distance constraints



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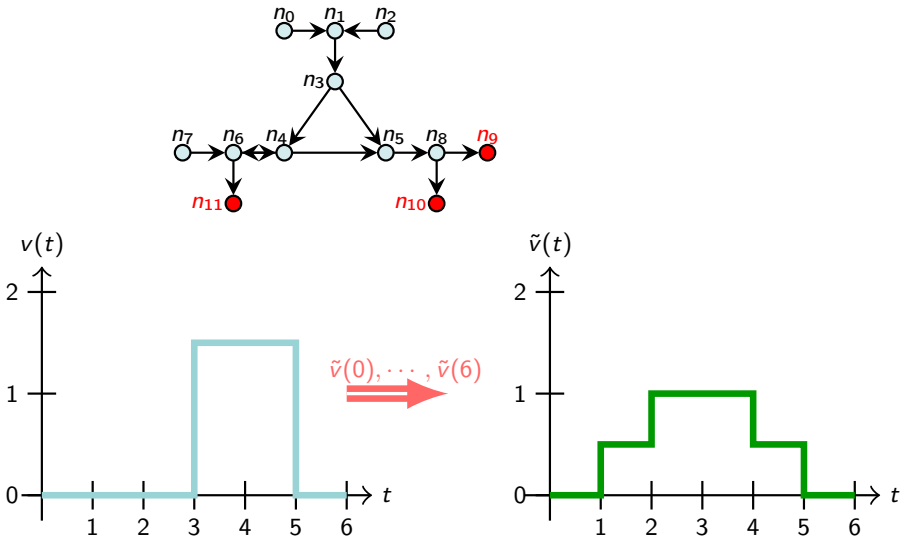

- RL training

Dataset

Trace generated with SWA-SMT solver
Random positions & speeds

Performance of RL (helped with SWA-SMT solver)

Better than single RL
Better than SWA-SMT solver
Runtime: $\sim$ 2 days

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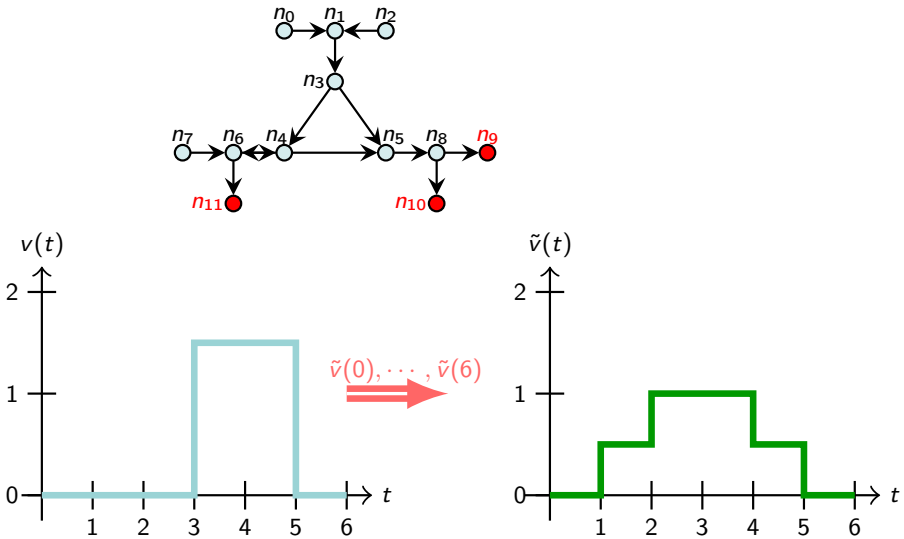

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- Future work: Decentralized multi-agent systems

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- Markov Decision Process
  - **Deterministic running example**: deterministic transition function.

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$$\underset{i}{\left(\mathrm{pos}_{i,c}, v_{i,c}\right)} \dashrightarrow \underset{i+1}{\left(\mathrm{pos}_{i,c} + v_i, v_{i,c} + \mathrm{acc}_{i,c}\right)}$$

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  - **Trajectories** s_i, Obs_i, act_i

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  - **State** s_i. For each section s, if a car c is in s: $v_{i,c}, \mathrm{pos}_{i,c}, \mathrm{id_c}, 1$
  - **Action** act_i: $(\mathrm{acc}_{i,c})_{c \in \mathrm{Cars}}$

$$\underset{i}{(\mathrm{pos}_{i,c}, v_{i,c})} \dashrightarrow \underset{i+1}{(\mathrm{pos}_{i,c} + v_i, v_{i,c} + \mathrm{acc}_{i,c})}$$

  - **Trajectories** s_i, Obs_i, act_i
  - **Reward**:
    - $+2000$ if goals are achieved
    - -100 if distance rules are not respected
    - $\nearrow$ with speed
    - $\nearrow$ with the increase of distance between cars