

Layered controller synthesis for dynamic multi-agent systems

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Introduction

Dynamic multi-agent system's verification

- A running example

https://perso.eleves.ens-rennes.fr/people/Emily.Clement/Videos/example_episodes/ex_0.mp4

| | Timed Automata | Reinforcement Learning |
|----------|---|--|
| Model | Abstract representation (acceleration) | |
| Weakness | Time of execution | Combinatorial or Continuous aspects |

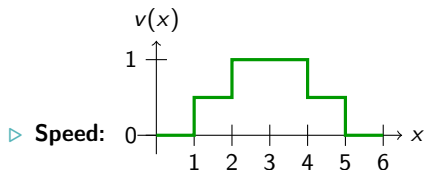
Our layered approach

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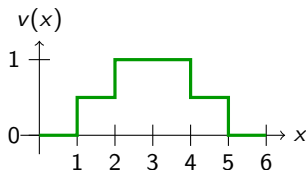
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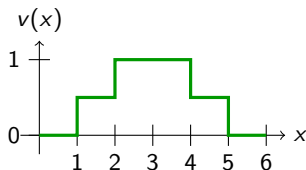
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- ▶ **Paths of cars:** fixed **trajectories**, fixed final & initial **positions**.

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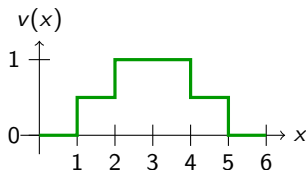
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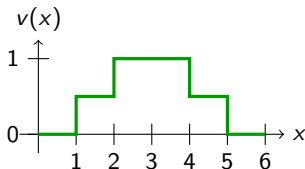
SWA-SMT Solver

Stage 1: Reachability algorithm on a simplified ISWA model

SWA

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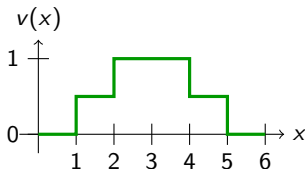
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SWA-SMT Solver

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RL training

Generate a dataset for random initial positions

Dataset

Stage 3: Train an RL algorithm with our dataset

RL

SWA-SMT solver


SWA solver

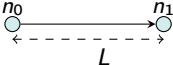
***Stage 1: Reachability
algorithm on system
of ISWA***


SWA

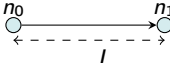
***Stage 2: Model the
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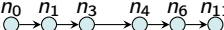
SMT


▷ A point in \mathbb{R}^2 : a node n_0 

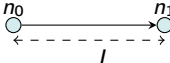
▷ A section $s_{[n_0, n_1], L}$ of the road: 

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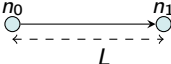
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▷ Car: (position, speed, trajectory)

Model for a car traffic

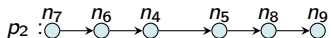
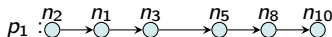
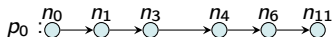
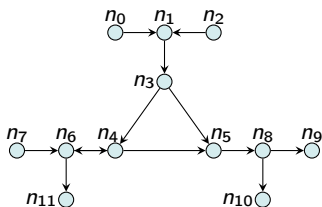
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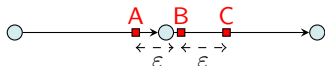
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▶ A car traffic: c_0, c_1, c_2 are each assigned paths p_0, p_1, p_2 :

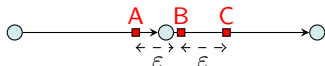


- #1: security distance when driving in the same direction and between neighbouring sections

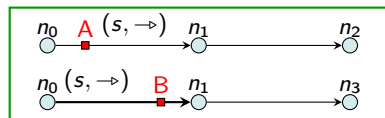


Rules to model collision avoidance

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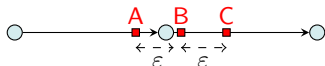


- #2: cars cannot share a section if driving in **opposite** direction

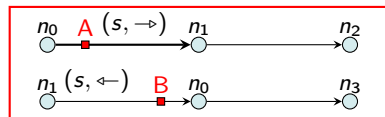
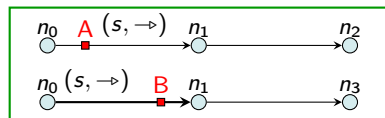


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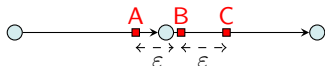


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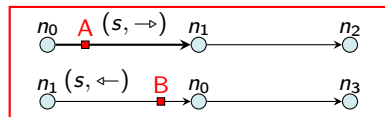
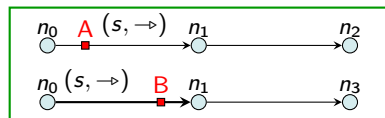


Rules to model collision avoidance

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- #3: No Overtaking between cars

What type of Timed Automata to use to model this?

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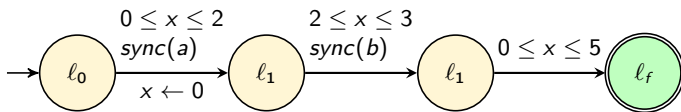
- ▷ **Clocks of TA:** Monitor each car's progress.



- ▷ **Stopwatch Timed Automata:**
- ▷ **Synchronised action:** Compute distance between each cars.
- ▷ **FiFo channels:** A car cannot overtake another car.

What is a Timed Automaton (and its variants) ?

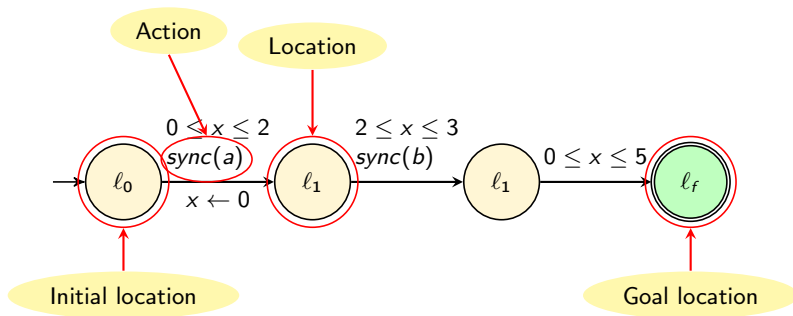
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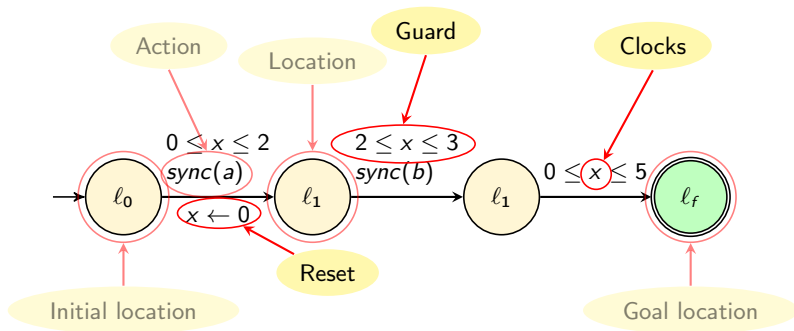
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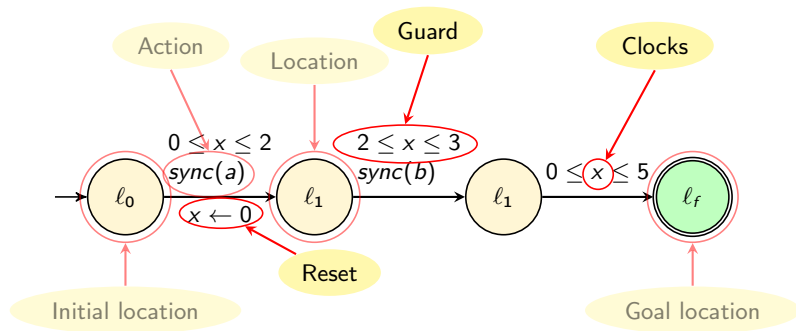
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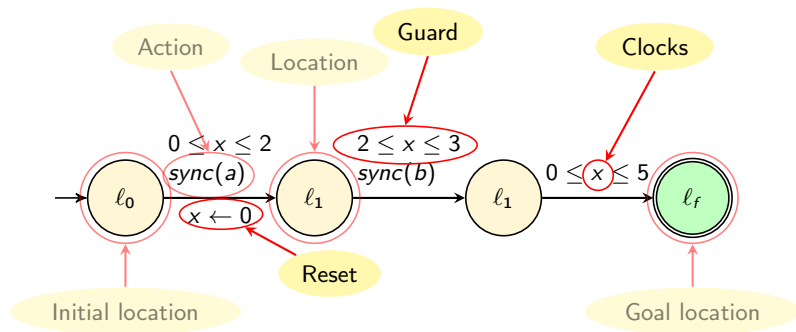


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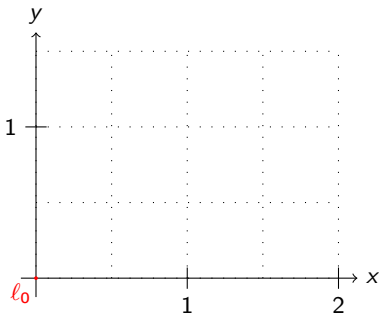
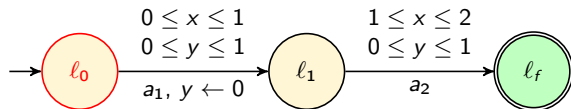


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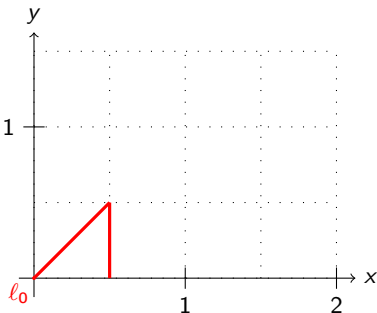
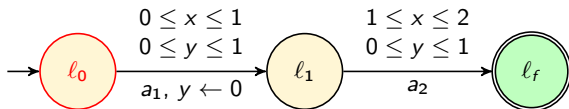
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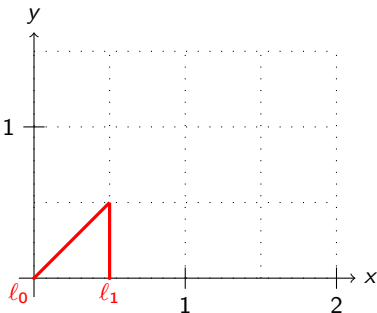
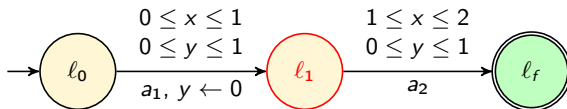
- A run of a two-clock Timed Automaton



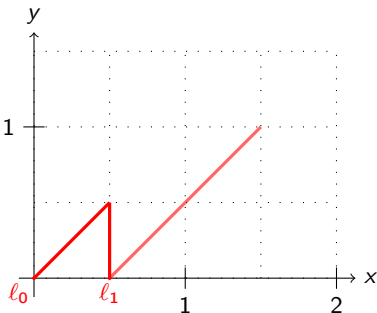
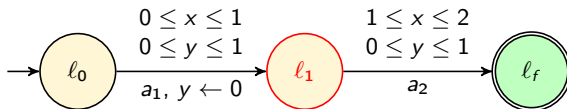
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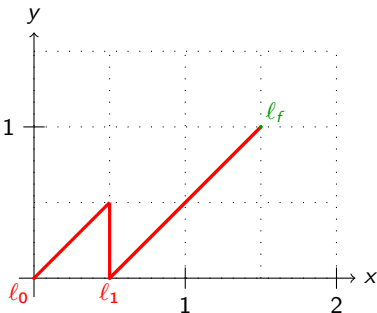
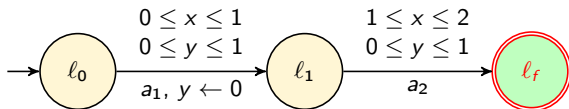
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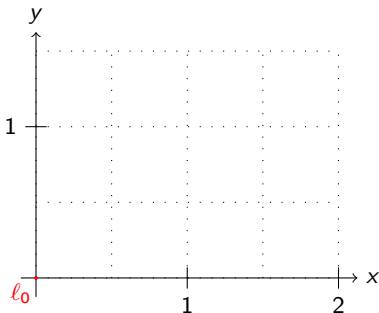
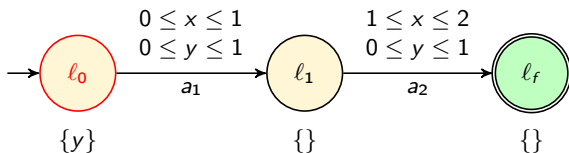
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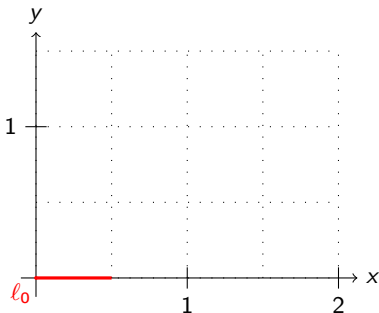
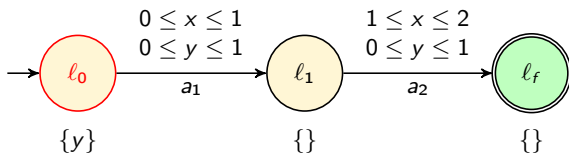
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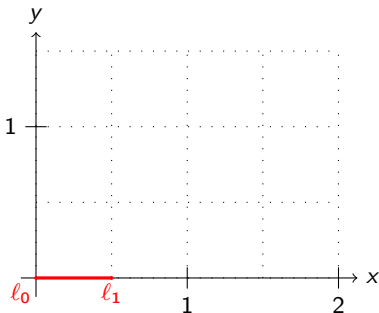
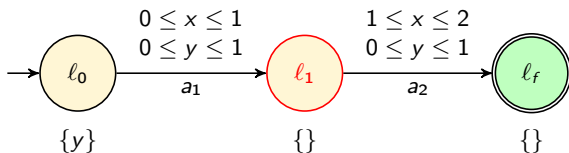
- A run with a (two-clock) stopwatch timed automaton (ISWA)



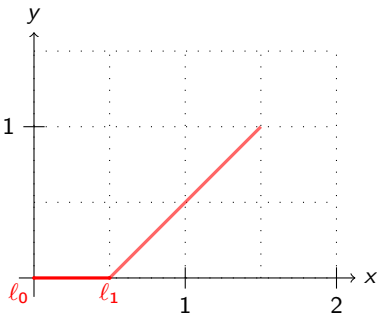
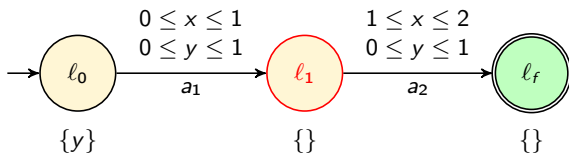
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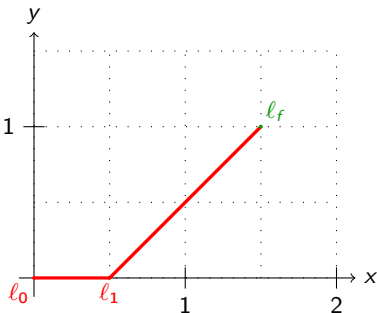
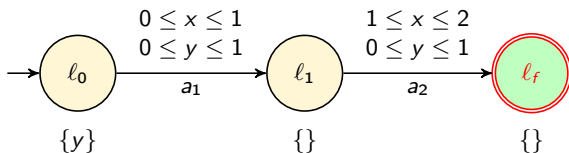
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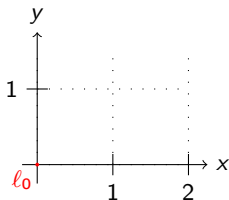
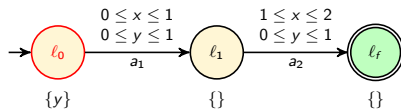
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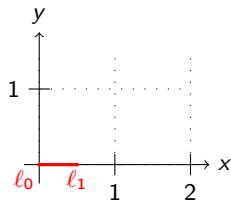
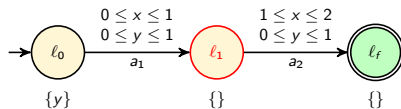
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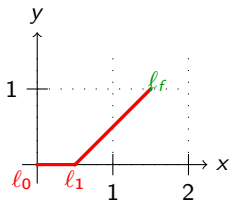
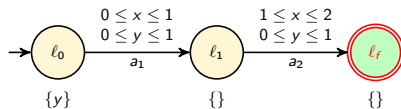
- Example of a two-clocks Stopwatch Timed Automata



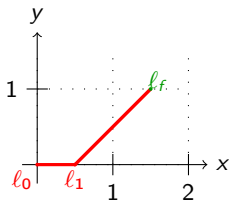
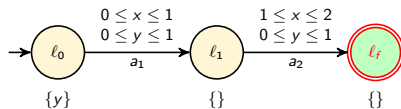
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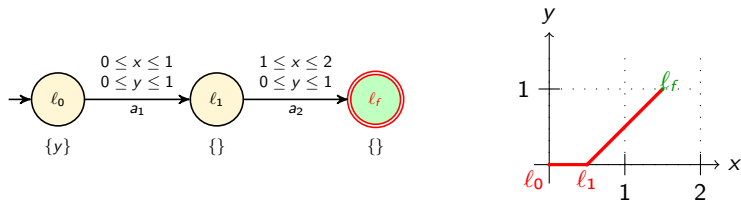


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- ▷ Reachability is **Undecidable** in general cases.

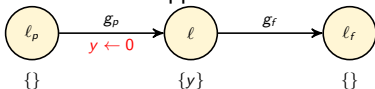
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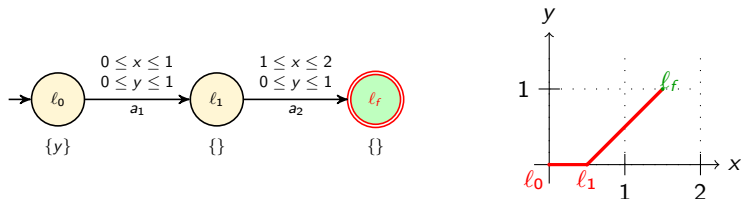
- Initialized Stopwatch Timed Automata

▷ Reset the stopped clock in the previous **or** following transition:



▷ **Reachability** becomes **Decidable** for this fragment of SWA.

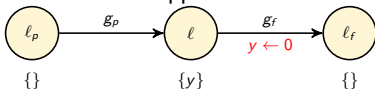
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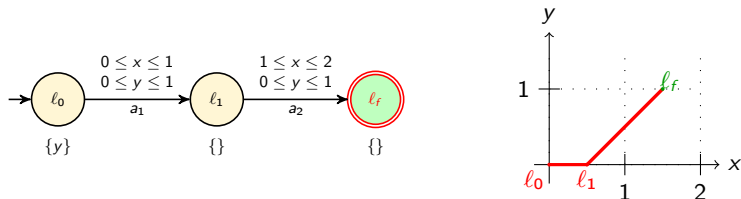
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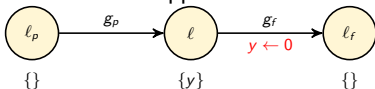
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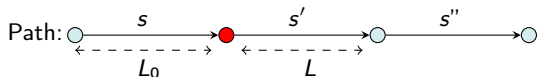


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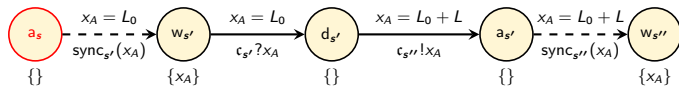
- Bounded channels

▷ Channels: FiFo queue of symbols (actions) to be pushed/read

- Car A progress along its paths

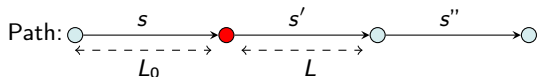


- Car A Timed automaton:

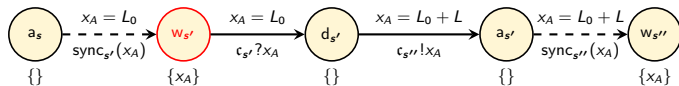


- ▷ **Clock** x_A : distance travelled along its paths
- ▷ **Stopwatches** $\{x_A\}$: the car A stops instantly.
- ▷ **Channels** $c_{s'}!x_A/c_{s'}?x_A$: respect the order of cars in a section $s \Rightarrow$ no overtaking.
- ▷ **Intersection**: use classical synchronized action to activate *intersection automata*

- Car A progress along its paths

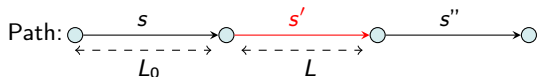


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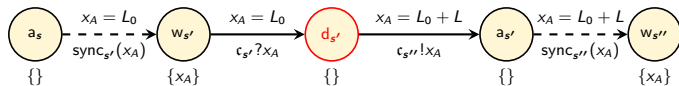


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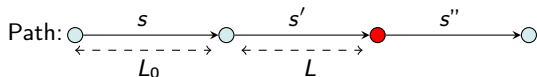


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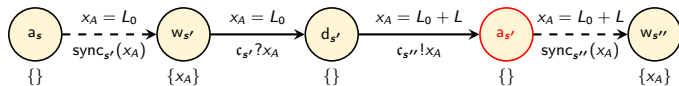


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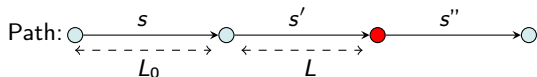


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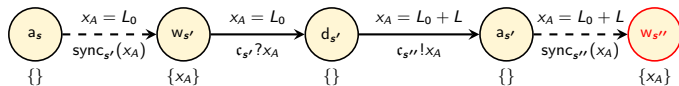


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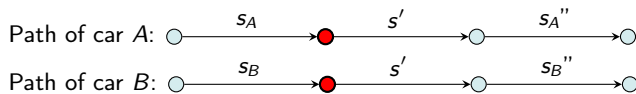


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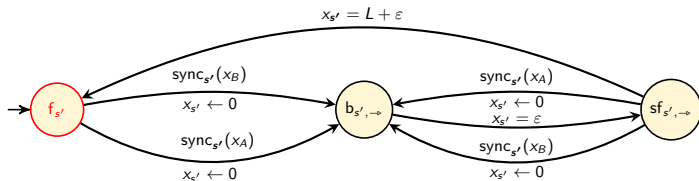


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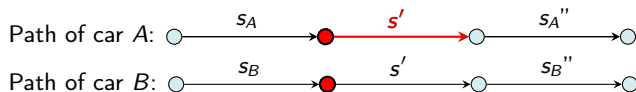
Model distance between cars: intersection



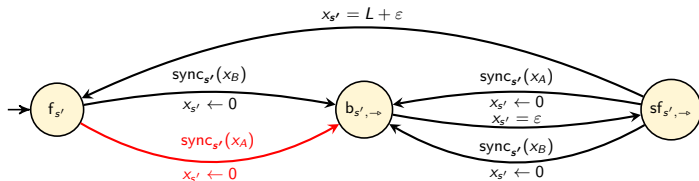
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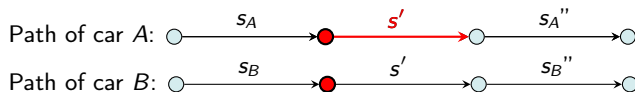
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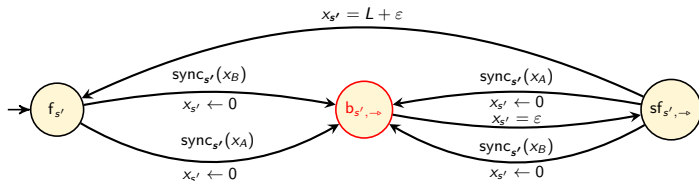
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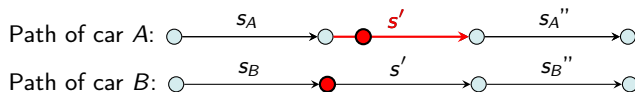
Model distance between cars: intersection



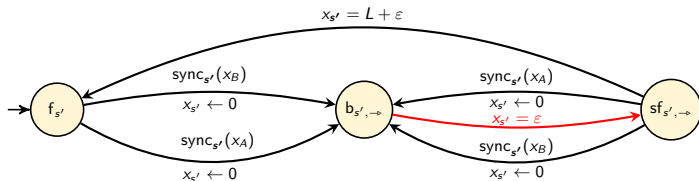
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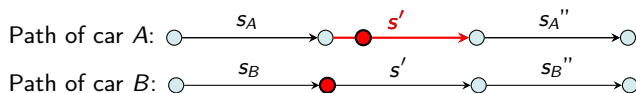
Model distance between cars: intersection



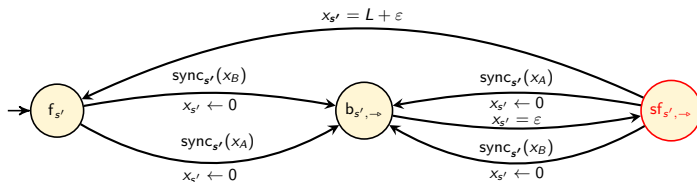
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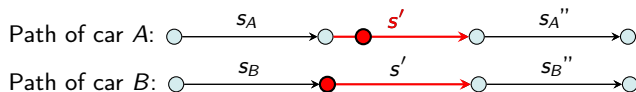
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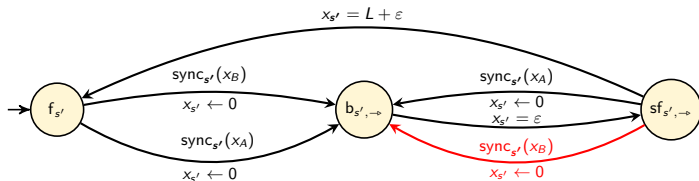
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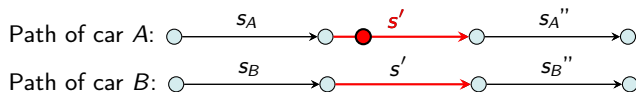
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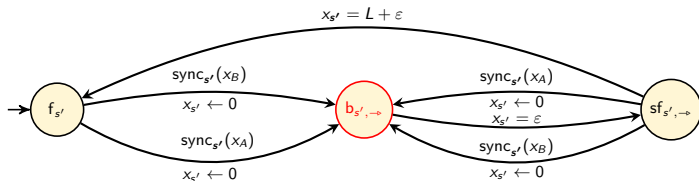
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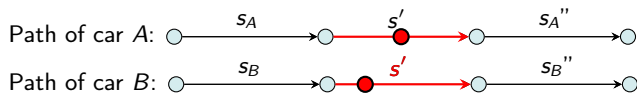
Model distance between cars: intersection



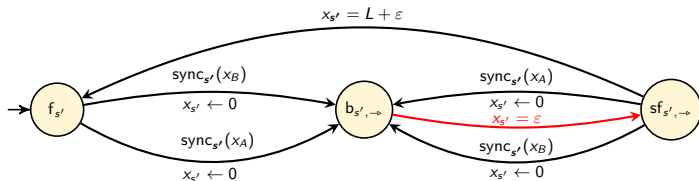
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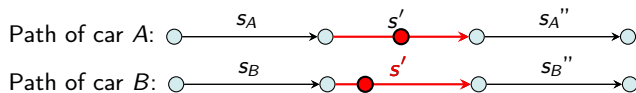
Model distance between cars: intersection



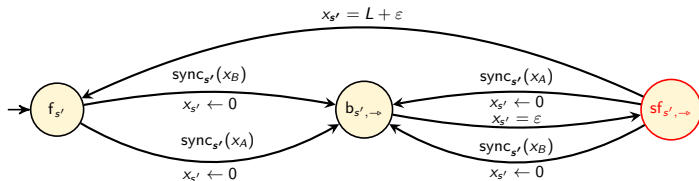
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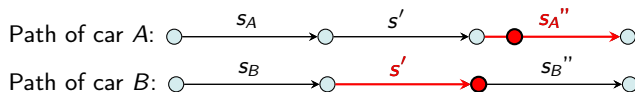
Model distance between cars: intersection



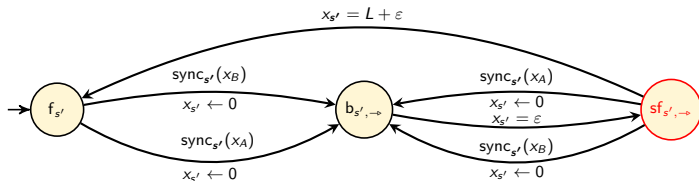
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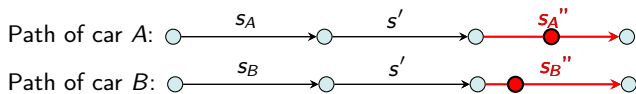
Model distance between cars: intersection



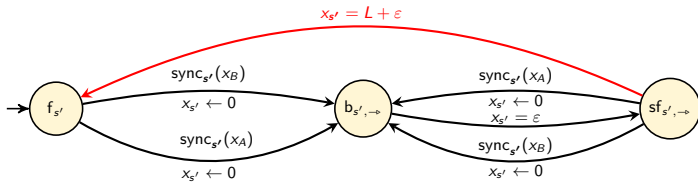
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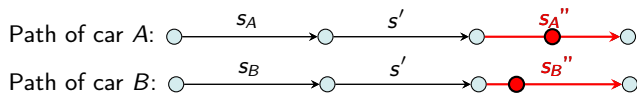
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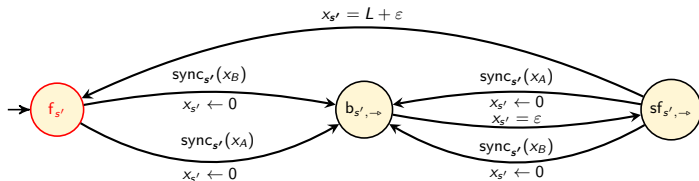
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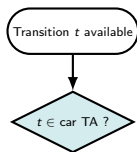
Model distance between cars: intersection



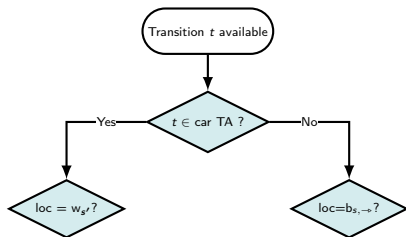
- Intersection automaton



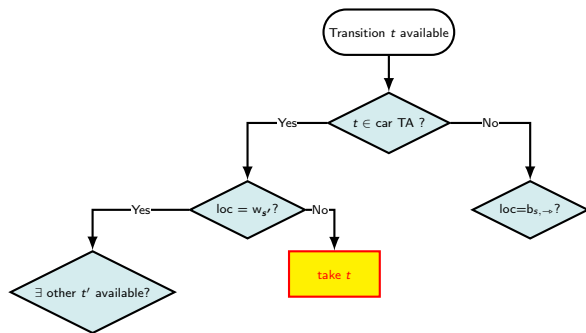
Our Algorithm: a DFS with an optimised succ function



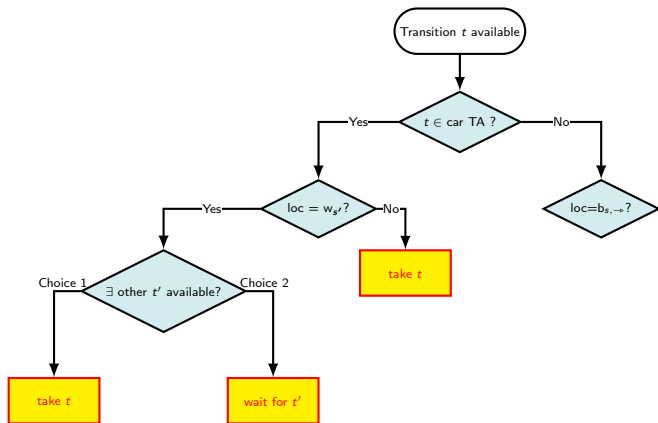
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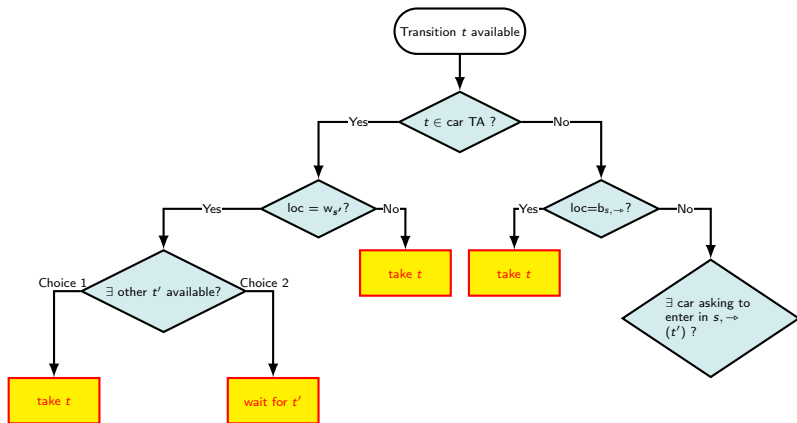
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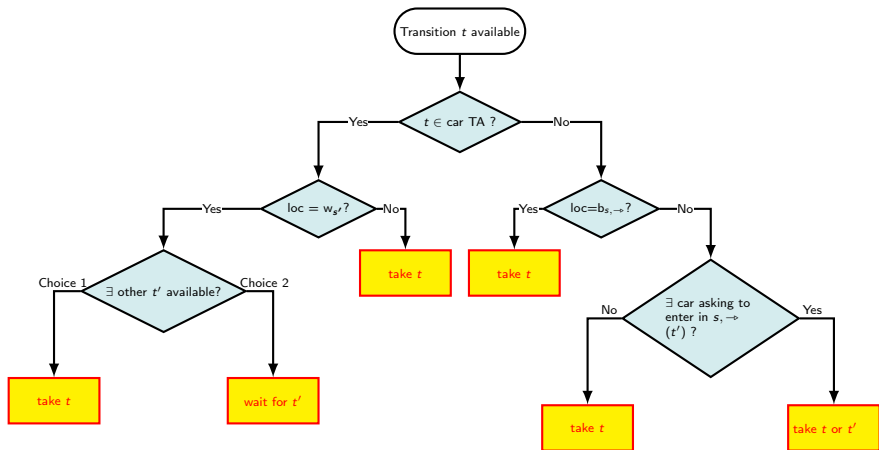
Our Algorithm: a DFS with an optimised succ function



Our Algorithm: a DFS with an optimised succ function



Our Algorithm: a DFS with an optimised succ function



SWA-SMT solver

SMT solver

***Stage 1: Reachability
algorithm on system
of ISWA***

SWA

***Stage 2: Model the
acceleration and de-
celeration***

SMT

Why use of SMT solver?

DFS algorithm

Stage 1: Reachability algorithm on a simplified ISWA model

SWA

*Solved: combinatorial aspect of the problem.
Results: Important events and their relative order*

Drawback: A very abstract model of speed

SMT Solver

Stage 2: Refine the model of the speed

SMT

RL training

Generate a dataset for random initial positions

Dataset

Stage 3: Train an RL algorithm with our dataset

RL

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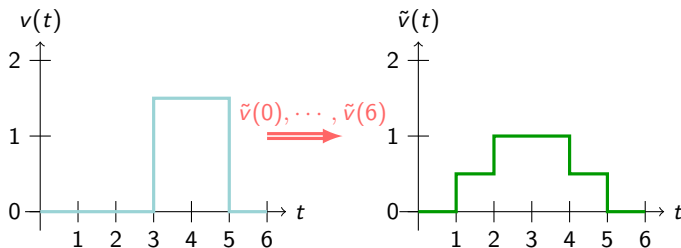
RL

- SMT solver

- ▷ The **continuous** aspect of the problem
- ▷ Introduce a more **realistic** model of speed

- A constant piecewise affine function
 - ▷ A more realistic model that takes into account the **dynamic of the system**
 - ▷ **Different** car speeds
 - ▷ **Bounds** on deceleration and acceleration

$$\begin{aligned}v_i(t) &\Rightarrow \tilde{v}_i(0), \dots, \tilde{v}_i(k-1) \\x(t) &\Rightarrow \tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l)\end{aligned}$$



- New positions/speeds

- ▷ $\tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l)$

- ▷ $\tilde{v}_i(0), \dots, \tilde{v}_i(k-1)$

- New positions/speeds

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- Example of SMT solver's inequalities

For each step k :

- ▷ $\tilde{v}_i(k) - d_{\max} \leq \tilde{v}_i(k+1) \leq \tilde{v}_i(k) + a_{\max}$

- New positions/speeds

- ▷ $\tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l)$

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- Example of SMT solver's inequalities

For each step k :

- ▷ $\tilde{v}_i(k) - d_{\max} \leq \tilde{v}_i(k+1) \leq \tilde{v}_i(k) + a_{\max}$

- ▷ $0 \leq \tilde{v}_i(k) \leq v_{\max}$

RL training

***Generate a dataset
for random initial
positions***

Dataset

***Stage 3: Train an RL
algorithm with our
dataset***

RL

Why use of SMT solver?

DFS algorithm

Stage 1: Reachability algorithm on a simplified ISWA model

SWA

Solved: combinatorial aspect of the problem.
Results: Important events and their relative order

Drawback: A very abstract model of speed

SMT Solver

Stage 2: Refine the model of the speed

SMT

A more realistic model of speed
Results: traces that takes into account the dynamical aspect of the problem
Drawback: runtime execution

RL training

Generate a dataset for random initial positions

Dataset

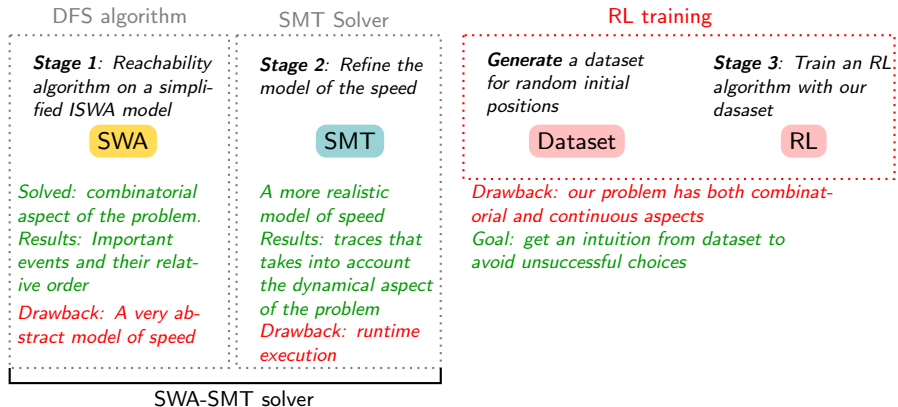
Stage 3: Train an RL algorithm with our dataset

RL

Drawback: our problem has both combinatorial and continuous aspects
Goal: get an intuition from dataset to avoid unsuccessful choices

SWA-SMT solver

Why use of SMT solver?



• RL training dataset

- ▷ Create random initial positions/speeds for cars
- ▷ Generate traces with the SWA-SMT solver

- Markov Decision Process
 - ▶ **Deterministic running example:** deterministic transition function.

- Markov Decision Process

- ▶ **Deterministic running example:** deterministic transition function.

- ▶ **State s_i .** For each section s , if a car c is in s : $v_{i,c}, \text{pos}_{i,c}, \text{id}_c, 1$

- Markov Decision Process

- ▷ **Deterministic running example:** deterministic transition function.

- ▷ **State** s_i . For each section s , if a car c is in s : $v_{i,c}, \text{pos}_{i,c}, \text{id}_c, 1$

- ▷ **Action** act_i : $(\text{acc}_{i,c})_{c \in \text{Cars}}$

$$\underbrace{(\text{pos}_{i,c}, v_{i,c})}_i \text{ ----- } \rightarrow \underbrace{(\text{pos}_{i,c} + v_i, v_{i,c} + \text{acc}_{i,c})}_{i+1}$$

- Markov Decision Process

- ▷ **Deterministic running example:** deterministic transition function.

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- ▷ **Trajectories** $s_i, \text{Obs}_i, \text{act}_i$

- Markov Decision Process

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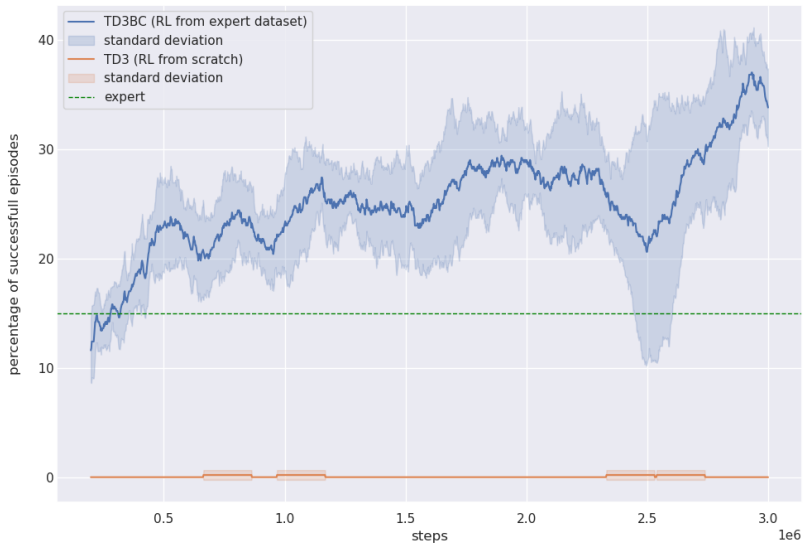
$$\underbrace{(\text{pos}_{i,c}, v_{i,c})}_i \text{ ----- } \rightarrow \underbrace{(\text{pos}_{i,c} + v_i, v_{i,c} + \text{acc}_{i,c})}_{i+1}$$

- ▷ **Trajectories** $s_i, \text{Obs}_i, \text{act}_i$

- ▷ **Reward:**

- +2000 if goals are achieved
 - -100 if distance rules are not respected
 - ↗ with speed
 - ↗ with the increase of distance between cars

Results with SWA-SMT solver, post SWA-SMT solver RL and single RL training



Steps of the layered method

DFS algorithm

Stage 1: Reachability algorithm on a simplified ISWA model

SWA

*Solved: combinatorial aspect of them problem.
Results: Important events and their relative order*

Drawback: A very abstract model of speed

SMT Solver

Stage 2: Refine the model of the speed

SMT

*A more realistic model of speed
Results: traces that takes into account the dynamical aspect of the problem*

Drawback: runtime execution

RL training

Generate a dataset for random initial positions

Dataset

Stage 3: Train an RL algorithm with our dataset

RL

Drawback: our problem has both combinatorial and continuous aspects

*Method: get an intuition from dataset to avoid unsuccessful choices
MDP model to reward short-time episode and distance between cars*

SWA-SMT solver

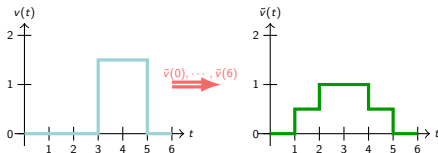
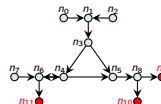
- SWA-SMT Solver

Automata-based model

Efficient algorithm
Abstract model with unrealistic speed model

Piecewise-affine speed graph

Bounded acceleration and deceleration
Different speed
SMT solver to model and solve the distance constraints



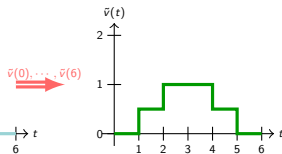
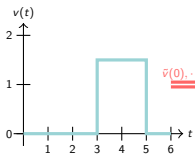
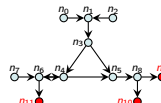
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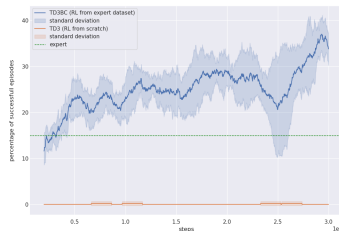
- RL training

Dataset

Trace generated with SWA-SMT solver
Random positions & speeds

Performance of RL (helped with SWA-SMT solver)

Better than single RL
Better than SWA-SMT solver
Runtime: ~ 2 days



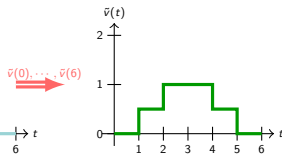
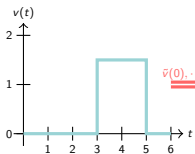
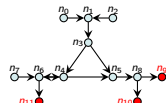
- SWA-SMT Solver

 - Automata-based model

 - Efficient algorithm*
 - Abstract model with unrealistic speed model*

 - Piecewise-affine speed graph

 - Bounded acceleration and deceleration*
 - Different speed*
 - SMT solver to model and solve the distance constraints*



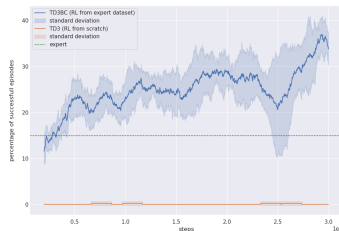
- RL training

 - Dataset

 - Trace generated with SWA-SMT solver*
 - Random positions & speeds*

 - Performance of RL (helped with SWA-SMT solver)

 - Better than single RL*
 - Better than SWA-SMT solver*
 - Runtime: ~ 2 days*



- Future work: Decentralized multi-agent systems