Playing Stochastically in Weighted Timed Games to Emulate Memory

Benjamin Monmege   Julie Parreaux   Pierre-Alain Reynier

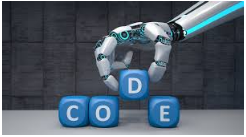
Aix–Marseille Université, France

ICALP 2021

Motivation: game theory for synthesis

Game theory

Interaction between two antagonistic agents: environment and controller



Code synthesis

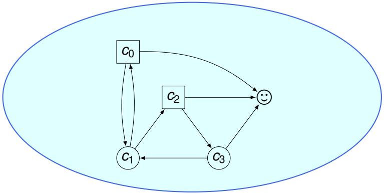
Correct by construction: synthesis of controller

Classical approach

Check the correctness of a system

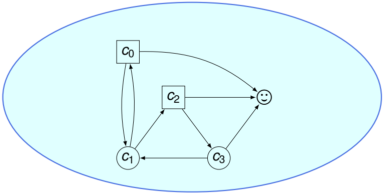
Different classes of games

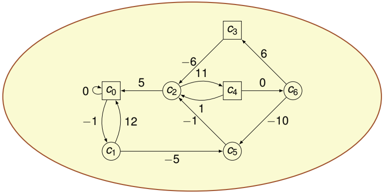
Qualitative games



Different classes of games

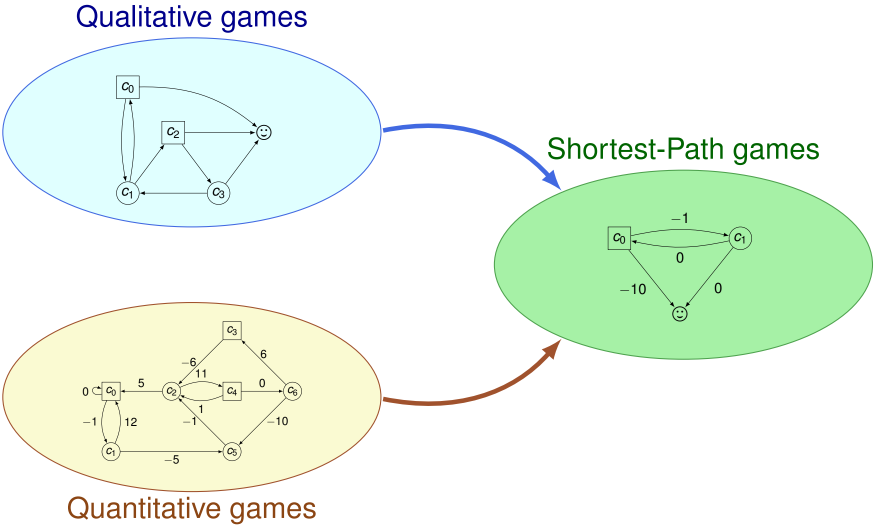
Qualitative games



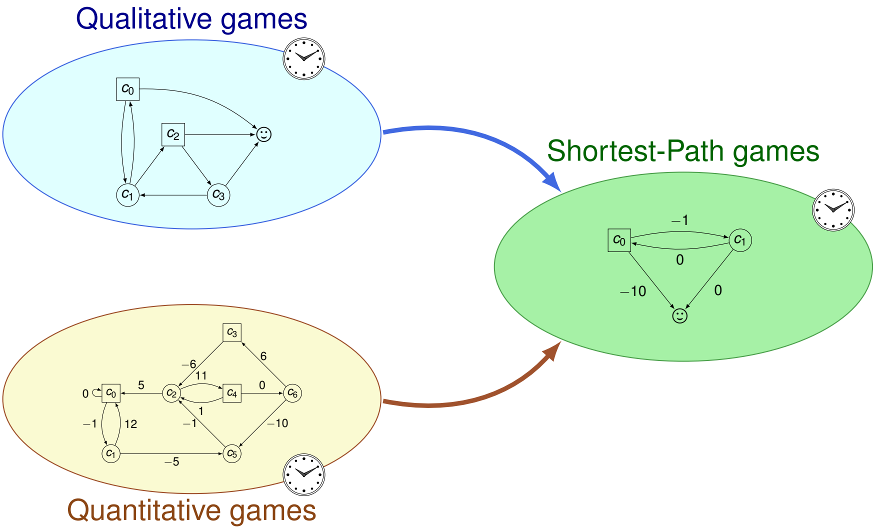


Quantitative games

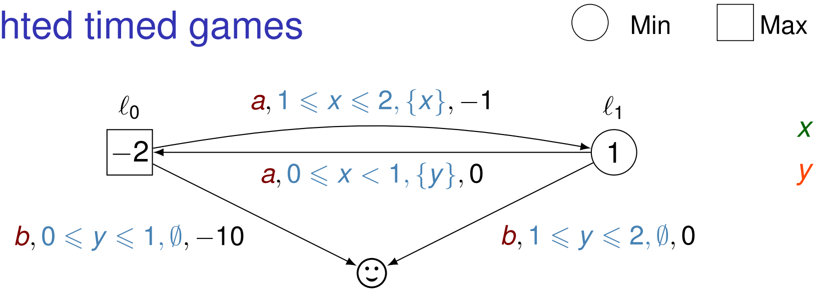
Different classes of games



Different classes of games



Weighted timed games

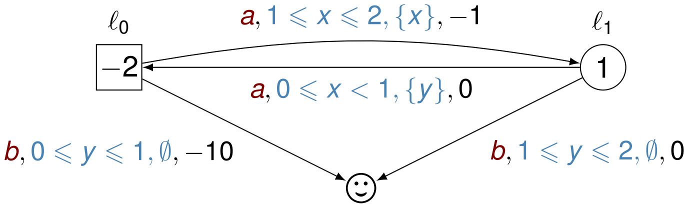


Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow{b,\ 1/3} (☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix})$$

Weighted timed games

○ Min ☐ Max



x

y

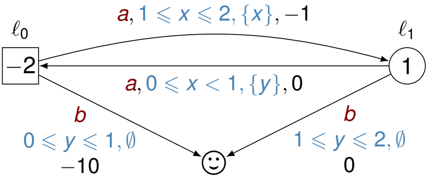
Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow[1 \times 0.5 + 0]{a,\ 0.5} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow[-2 \times 1.25 - 1]{a,\ 1.25} (\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow[1 \times \frac{1}{3} + 0]{b,\ 1/3} (\text{☺}, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}) \rightsquigarrow -\frac{8}{3}$$

Shortest-path payoff

$$\mathbf{SP}(\rho) = \begin{cases} \mathrm{wt}(\rho) & \text{if } \rho \text{ reaches ☺} \\ +\infty & \text{otherwise} \end{cases}$$

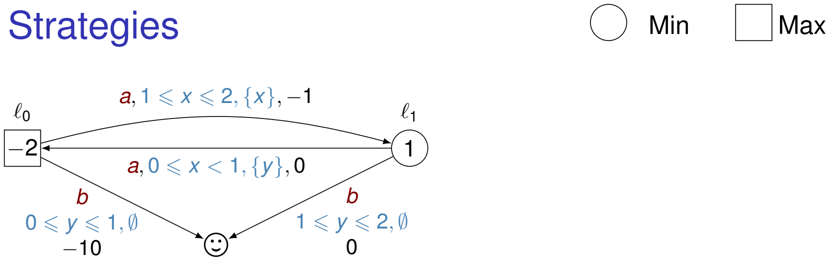
Strategies



Deterministic strategy

Choose an edge and a delay

Strategies



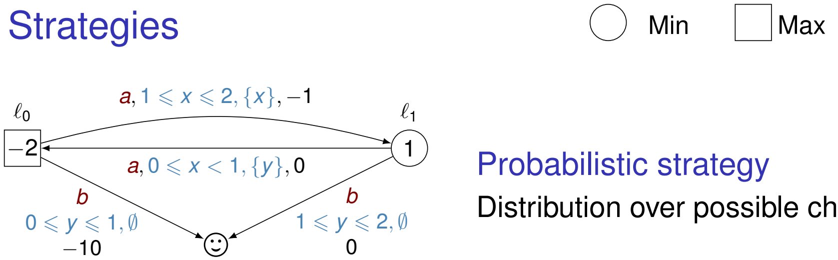
Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0,0))$

Choose a with $t = \frac{1}{3}$

Strategies



Probabilistic strategy

Distribution over possible choices

Deterministic strategy

Choose an edge and a delay

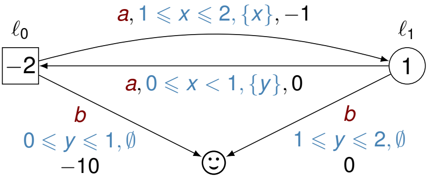
In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinski, 2014, Logical Methods in Computer Science

Strategies

Min Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution

Deterministic strategy

Choose an edge and a delay

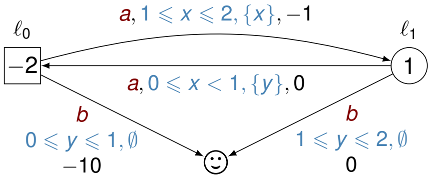
In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

Min Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Deterministic strategy

Choose an edge and a delay

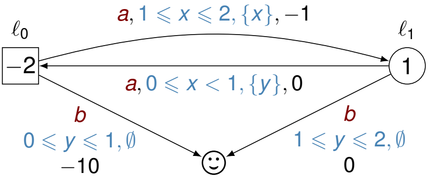
In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

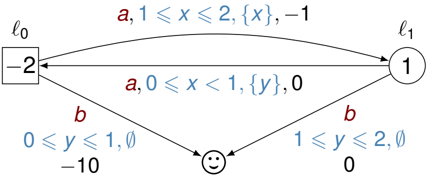
In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

In $(\ell_1, (0, 0))$

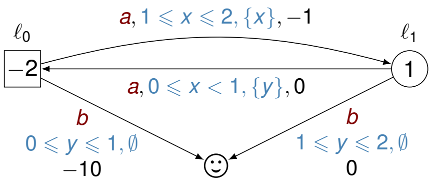
Choose between a or b with $\mathcal{B}(p)$

- a: choose t with $\mathcal{U}([0, 1[)$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose *a* with $t = \frac{1}{3}$

In $(\ell_1, (0, 0))$

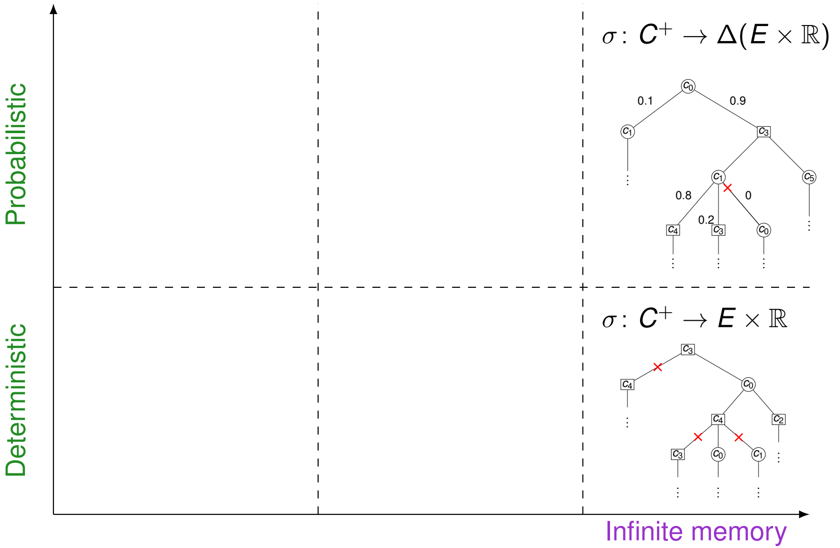
Choose between *a* or *b* with $\mathcal{B}(p)$

- *a*: choose *t* with $\mathcal{U}([0, 1[)$
- *b*: choose *t* with $\delta_{1.5}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

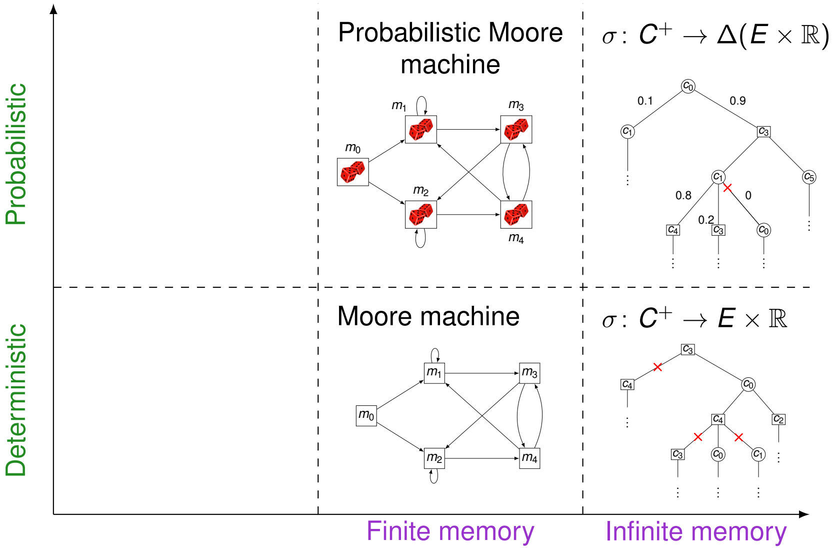
Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



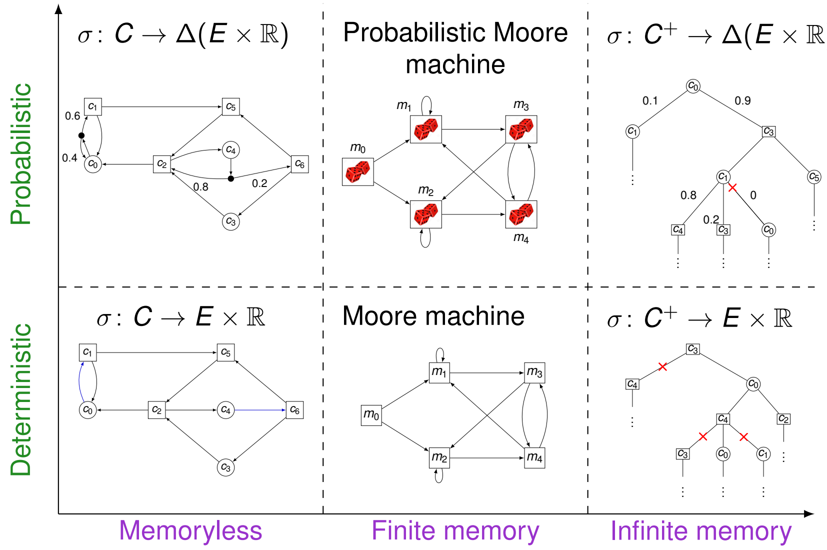
Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



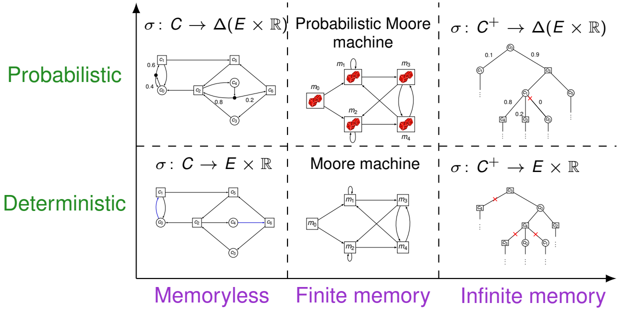
Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Contributions

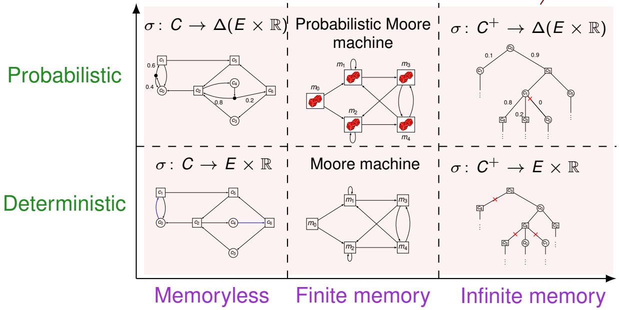
$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$



Contributions

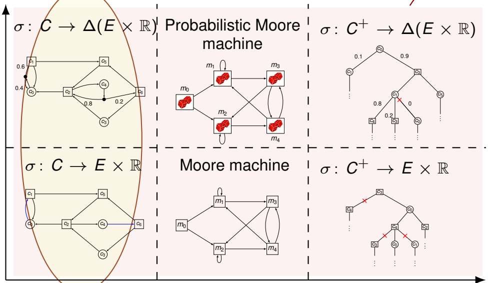
$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\mathrm{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$

mVal

Probabilistic

Deterministic

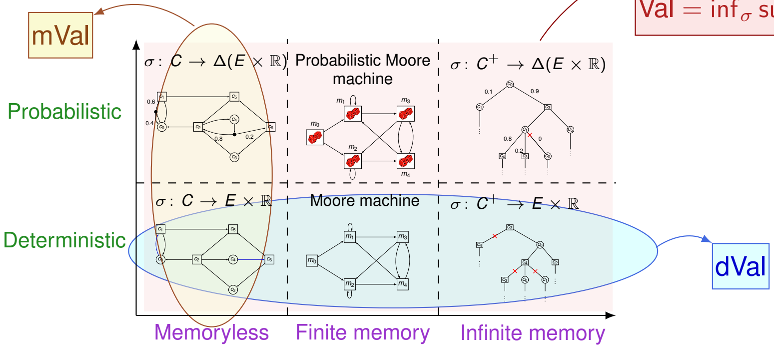


Memoryless Finite memory Infinite memory

Contributions

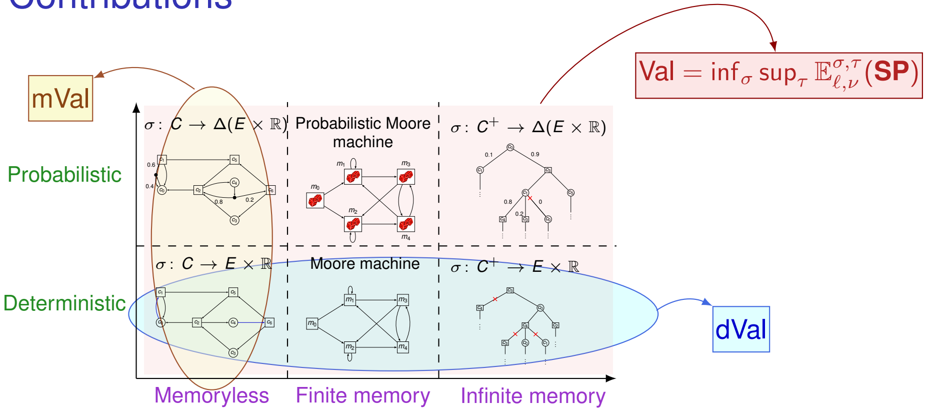
$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}_{\ell,\nu}^{\sigma,\tau}(\textbf{SP})$



Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Theorem

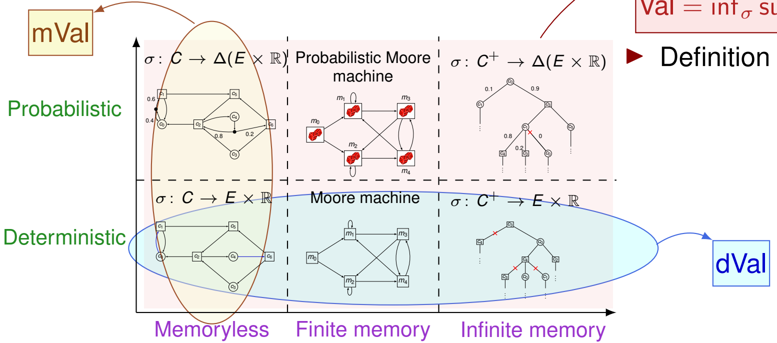
$\mathsf{dVal} = \mathsf{Val} = \mathsf{mVal}$

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$



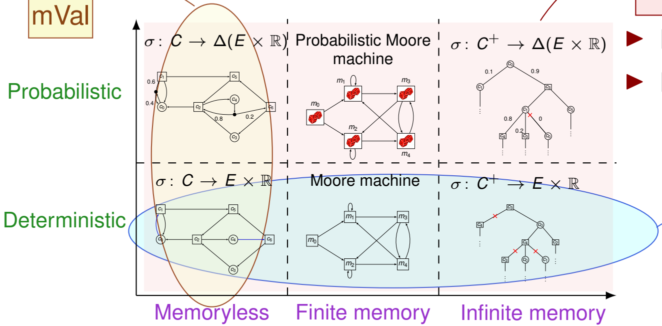
Theorem

$\text{dVal} = \text{Val} = \text{mVal}$

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

mVal



dVal

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$

Theorem

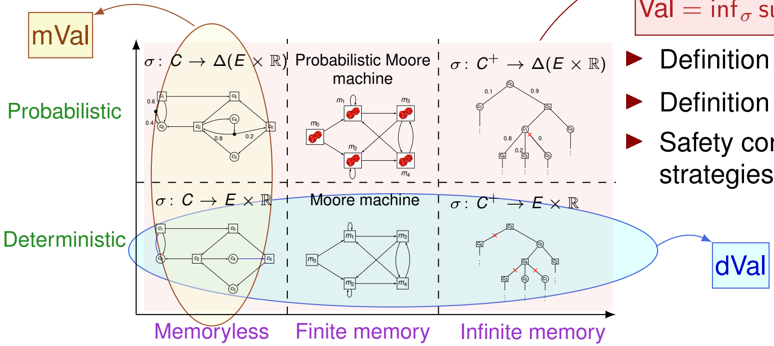
dVal = Val = mVal

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies



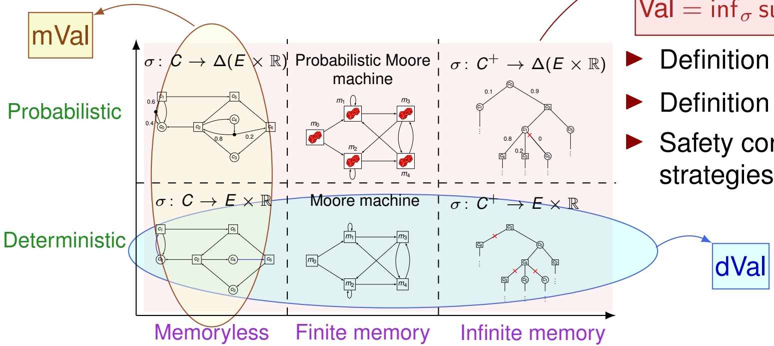
Theorem

dVal = Val = mVal

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\mathsf{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$



- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies

Theorem

$\mathsf{dVal} = \mathsf{Val} = \mathsf{mVal}$

Thank you! Any questions?