Weighted Timed Games: Decidability, Randomness and Robustness

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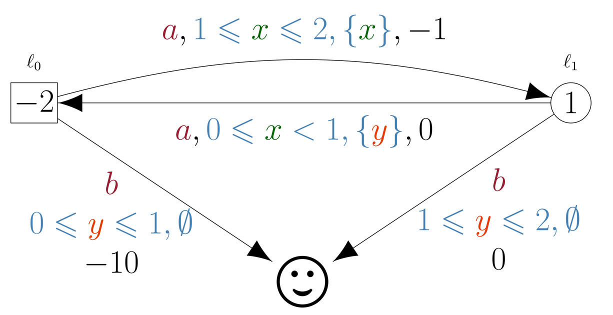
Weighted Timed Games



When it is its turn, □ or ○ chooses an edge and a delay such that clocks satisfy the edge's guard.

$$\rho : \left(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}\right) \xrightarrow{a,\,0.5} \left(\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}\right) \xrightarrow{a,\,1.25} \left(\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}\right) \xrightarrow{b,\,1/3} \left(☺, \begin{pmatrix}1/3\\19/12\end{pmatrix}\right)$$

Objective of players

For all plays ρ, we define

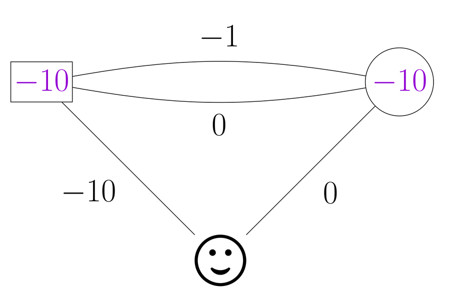
$$\mathbf{SP}(\rho) = \begin{cases} \sum_{i=0}^{n-1}(w(e_i) + t_i\,w(\ell_i)) & \text{if } n \text{ is the smallest index s.t. } \ell_n = ☺ \\ +\infty & \text{if } \rho \text{ does not reach } ☺ \end{cases}$$

Example: $\mathbf{SP}(\rho) = 1 \times 0.5 + 0 + (-2) \times 1.25 - 1 + 1 \times 1/3 + 0 = -8/3$

The objective of each player is

□ Maximizes **SP** ○ Minimizes **SP**

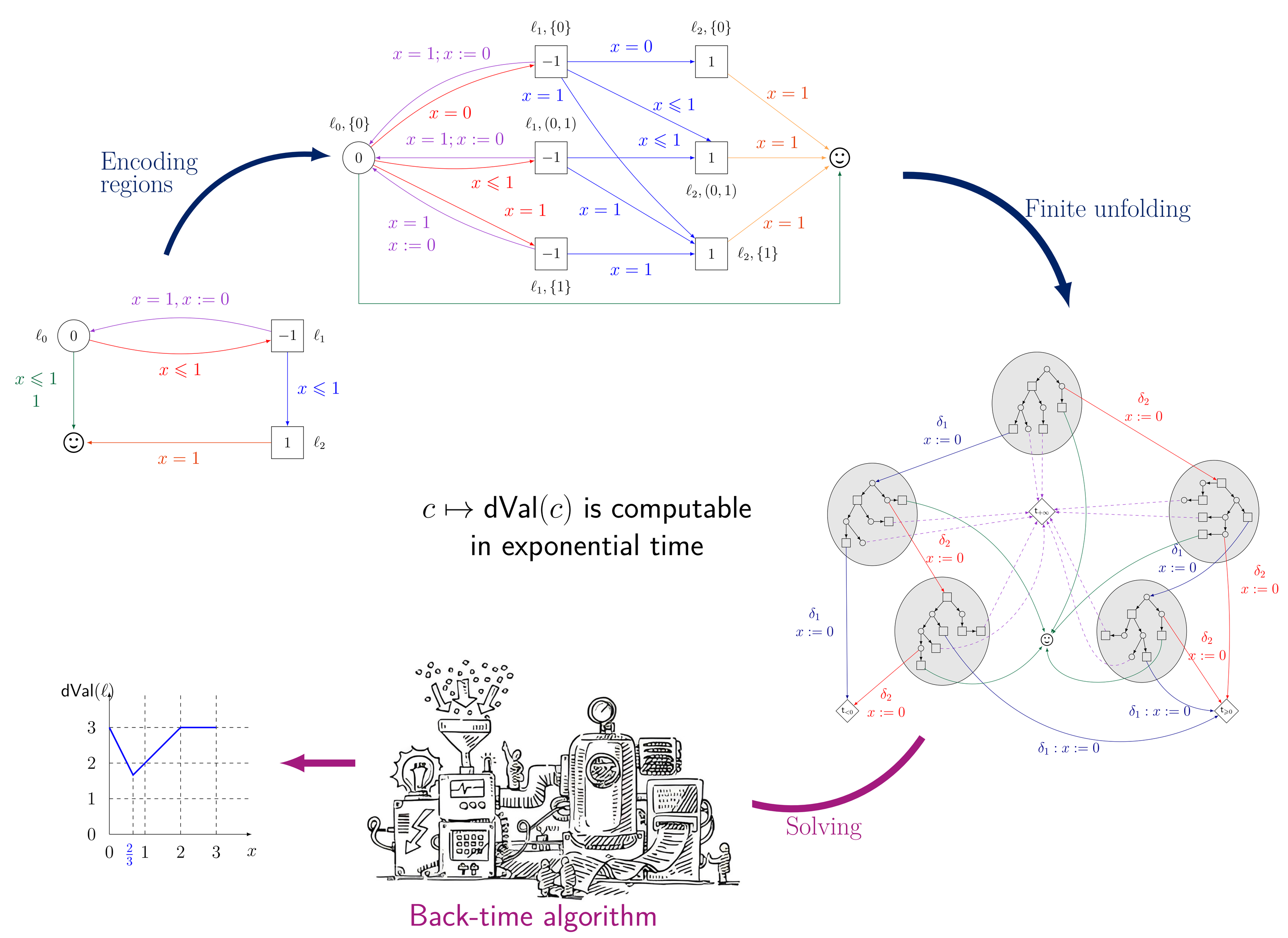
Deterministic value



$$\mathsf{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathsf{Play}(c, \sigma, \tau))$$

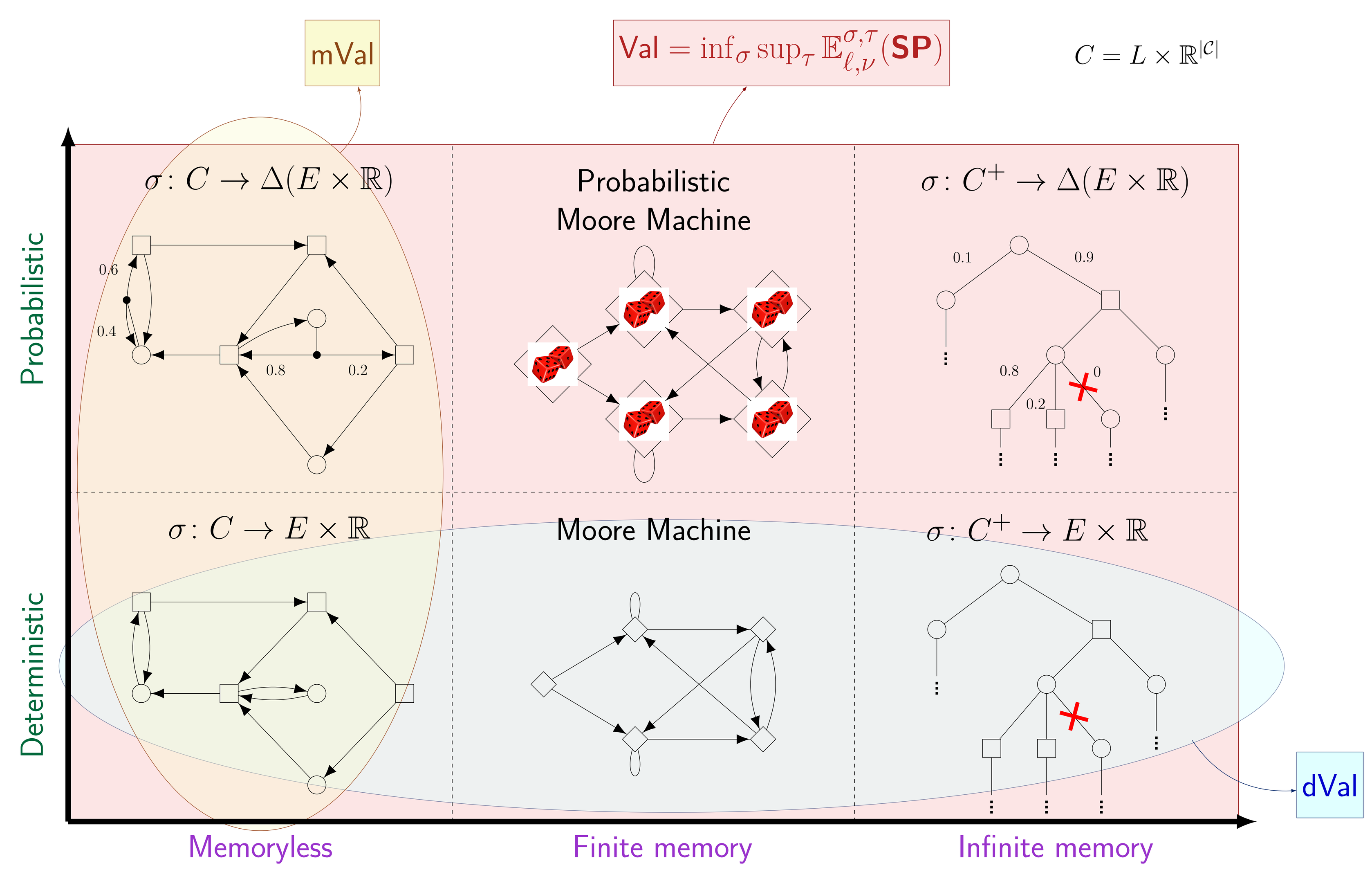
○ needs memory to play (ε-)optimally.

Decidability of deterministic value problem in one-clock WTGs

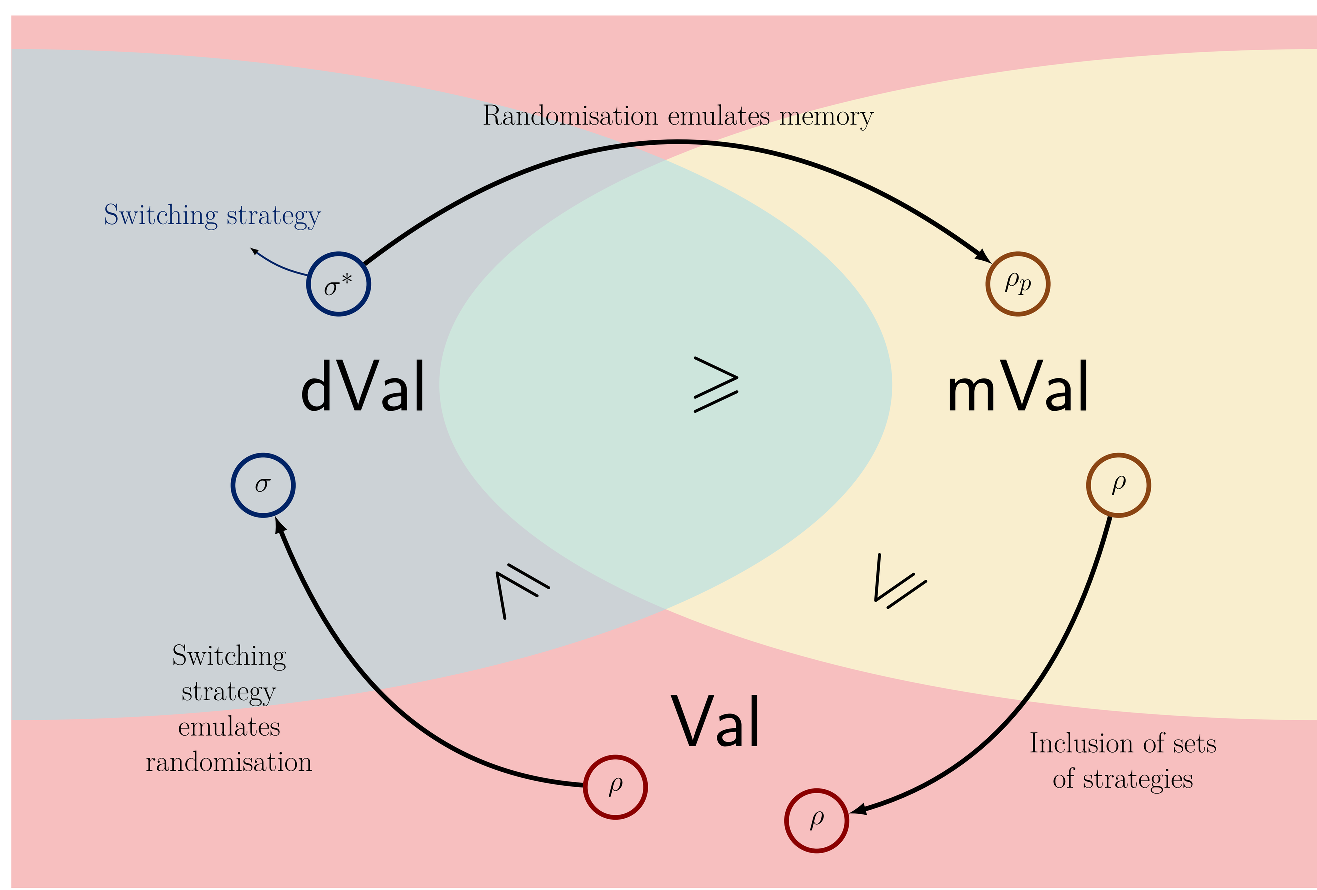


Stochastic strategies

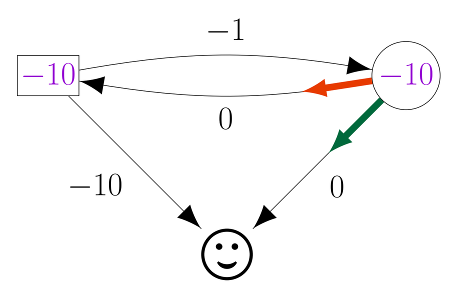
Requires measurability conditions on strategies



Trading memory with probabilities



Switching strategy



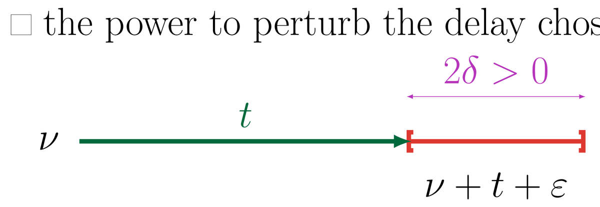
A memory combination between two memoryless strategies
σ_1: reach negative cycle
σ_2: reach ☺

Existence of a switching strategy for ○ ?

Requires divergent WTGs : a restriction over weight of cycles

Robust semantics : a more realistic model

Give to □ the power to perturb the delay chosen by ○



Excessive semantics

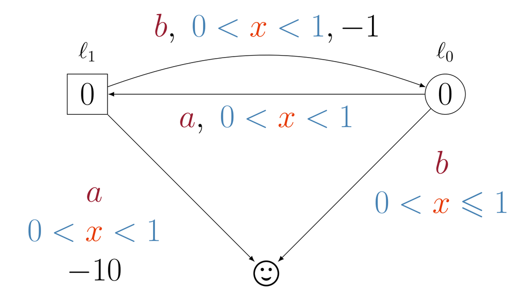
Check the guard before the perturbation : $\nu + t$ satisfies the guard

Conservative semantics

Check the guard after the perturbation : $\nu + t + \varepsilon$ satisfies the guard for all $\varepsilon \in [0, 2\delta]$

Robust value

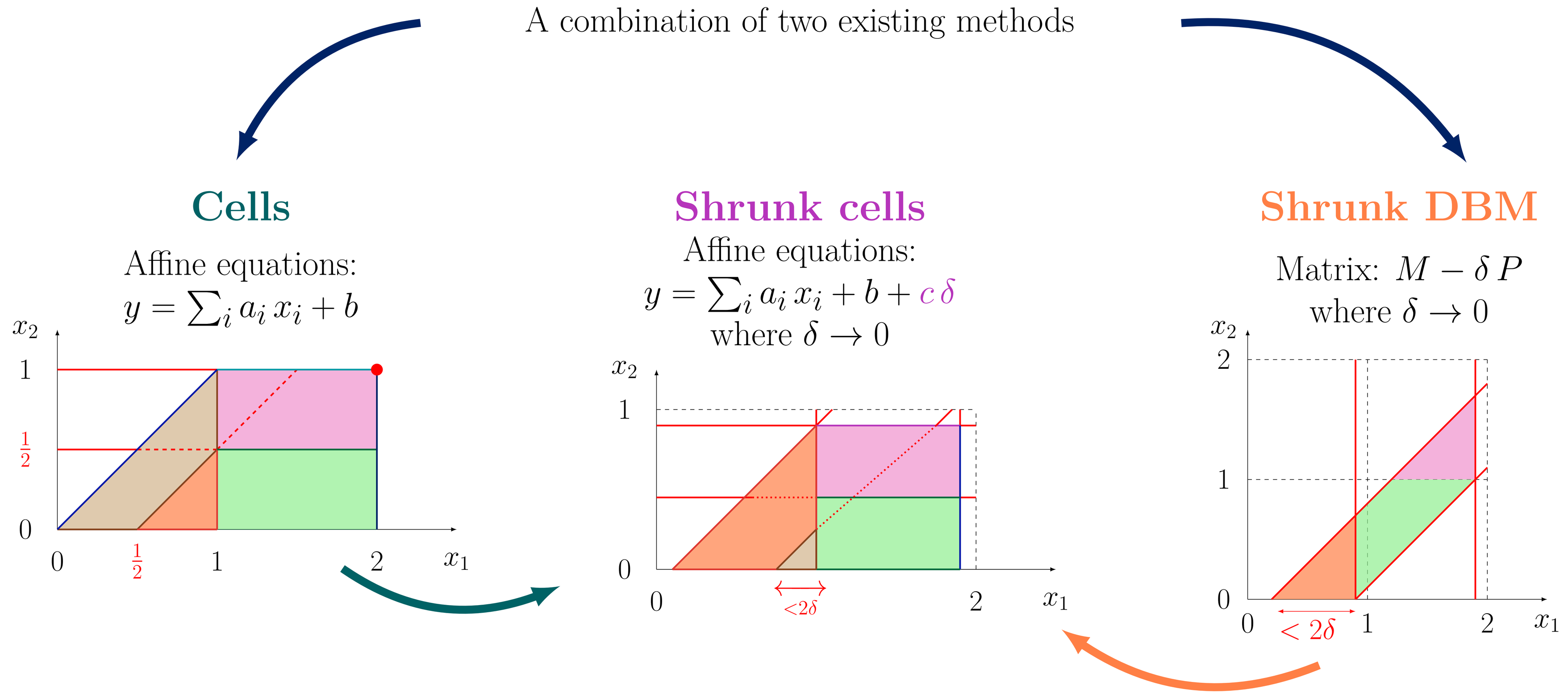
$$\mathsf{rVal}(c) = \lim_{\substack{\delta \to 0\\ \delta > 0}} \inf_{\substack{\chi\\ \delta\text{-robust}}} \sup_{\substack{\zeta\\ \delta\text{-robust}}} \mathbf{SP}(\mathsf{Play}(c, \chi, \zeta))$$



- Under exact semantics :
  ○ can take 10 times a before b, thus
  $\mathsf{dVal}(\ell_1, 0) = -10$
- Under excessive semantics :
  ○ can take only 1 time a before b, thus
  $\mathsf{rVal}(\ell_1, 0) = -1$
- Under conservative semantics :
  ○ cannot take 1 time a before b, thus
  $\mathsf{rVal}(\ell_1, 0) = 0$

Symbolic computation of robust value in acyclic WTGs

A combination of two existing methods



References

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