

# Selective Monitoring Without Delay for Probabilistic Systems

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**Challenge:** fault detection and control of large complex systems in runtime

Formal verification methods to proof the correctness in runtime

- Test: feasible but not proof
- Model Checking: proof for all runs but not feasible
- Monitoring: proof for the current run

**Challenge:** fault detection and control of large complex systems in runtime

Formal verification methods to proof the correctness in runtime

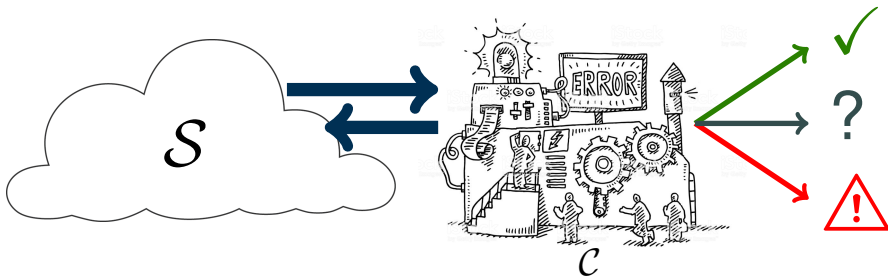
- Test: feasible but not proof
- Model Checking: proof for all runs but not feasible
- **Monitoring: proof for the current run**

# Monitoring and probabilities

## Monitoring problem

*Input:* a system  $S$ , a monitor  $\mathcal{C}$  and a run  $r$  of  $S$

*Output:*  $r$  is a correct run on  $S$



# Monitoring and probabilities

Monitoring introduces overhead

**Problem:** Minimize overhead

## Monitoring with probability

- Runtime Verification with State Estimation problem<sup>1</sup> (RVSE): model with probabilistic system

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<sup>1</sup>Runtime Verification with State Estimation, Stoller et al., 2012

# Monitoring and probabilities

Monitoring introduces overhead

Minimize overhead : use probability

**Problem:** Uncertainty vs Overhead

## Monitoring with probability

- Runtime Verification with State Estimation problem (RVSE): model with probabilistic system
- Runtime Verification with Particle Filtering problem<sup>2</sup>(RVPF): trade-off between uncertainty and overhead

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<sup>2</sup>Runtime Verification with Particle Filtering, Kalajdzic et al., 2013

# Monitoring and probabilities

Monitoring introduces overhead

Minimize overhead : use probability

Uncertainty vs Overhead : trade-off

## Our approach

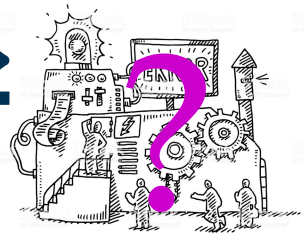
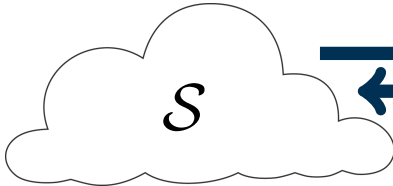
The smallest overhead achievable without compromising precision at all.

# Diagnosability

## Diagnosability problem

*Input:* a system  $S$

*Output:* Exists a monitor for  $S$





# Diagnosability

## Diagnosability problem

*Input:* a system  $\mathcal{S}$

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**Discrete event system**<sup>3</sup>: P to check and EXPTIME to build the monitor<sup>4</sup>

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<sup>3</sup>Diagnosability of discrete event systems, Sampath et al., 1995

<sup>4</sup>A polynomial algorithm for testing diagnosability of discrete-event systems, Jiang et al., 2001

# Diagnosability

## Diagnosability problem

*Input:* a system  $S$

*Output:* Exists a monitor for  $S$

Discrete event system : P to check and EXPTIME to build the monitor

**Probabilistic system**<sup>3</sup>: PSPACE problem<sup>45</sup>

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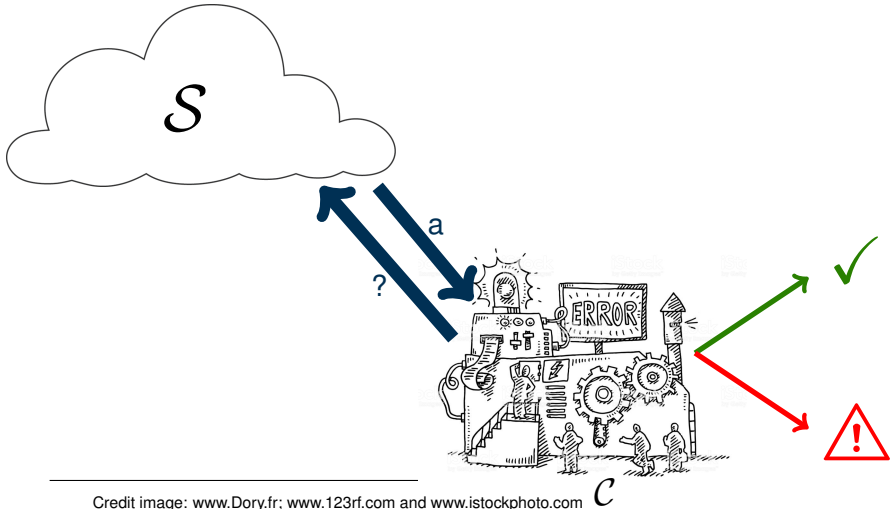
<sup>3</sup>Diagnosability of stochastic discrete-event systems, Thorsley et al., 2005

<sup>4</sup>Foundation of Diagnosis and Predictability in Probabilistic Systems, Bertrand et al., 2014

<sup>5</sup>Accurate Approximate Diagnosability of Stochastic Systems, Bertrand et al., 2016

# Selective monitoring

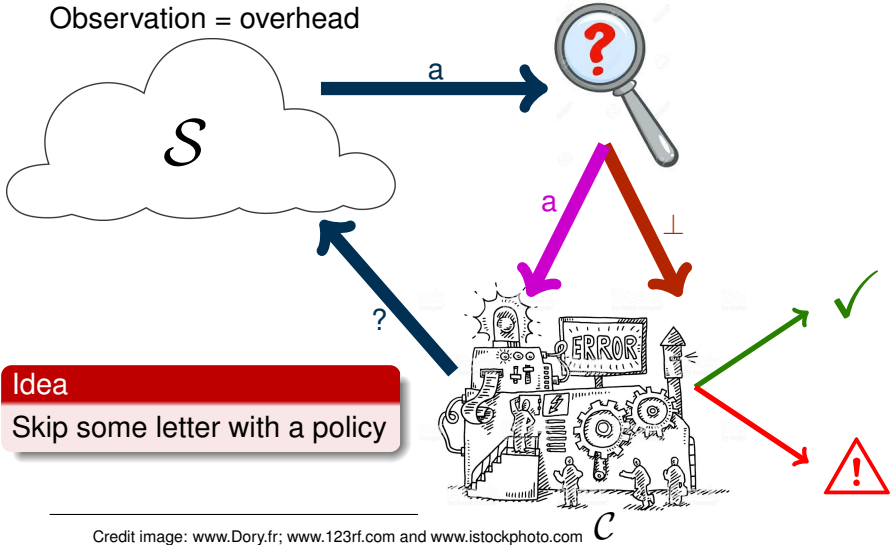
Observation = overhead



Credit image: [www.Dory.fr](http://www.Dory.fr); [www.123rf.com](http://www.123rf.com) and [www.istockphoto.com](http://www.istockphoto.com)

# Selective monitoring

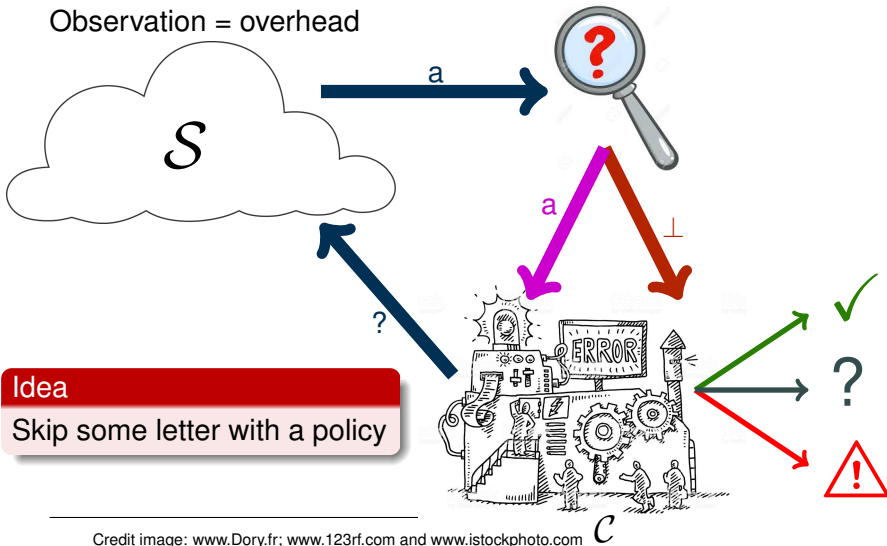
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# Selective monitoring

Observation = overhead



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# Selective monitoring without delay

## Our goal

Design a feasible optimal policy without delay

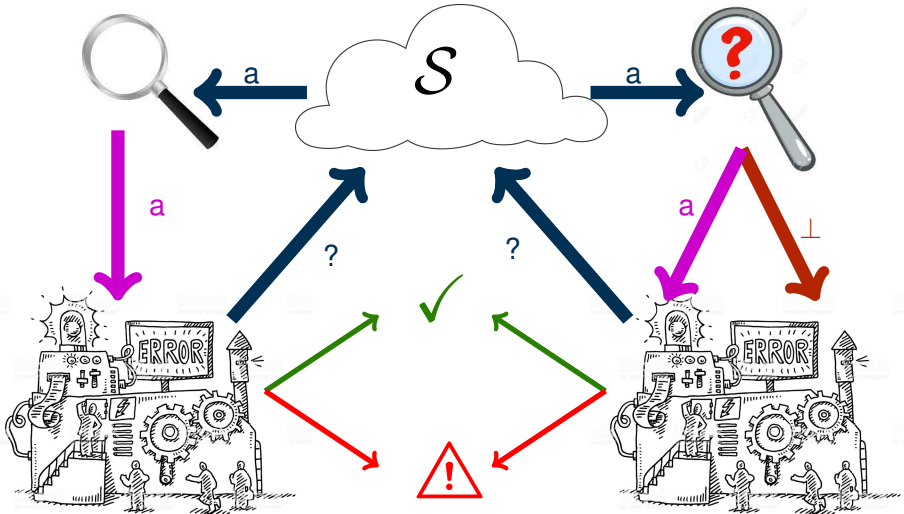
# Selective monitoring without delay

## Our goal

Design a **feasible** optimal policy without delay

**Decide when it is possible**

# Selective monitoring without delay



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# Selective monitoring without delay

## Our goal

Design a **feasible** **optimal** policy without delay

Decide when it is possible

Minimize the number of observations

# Selective monitoring without delay

## Our goal

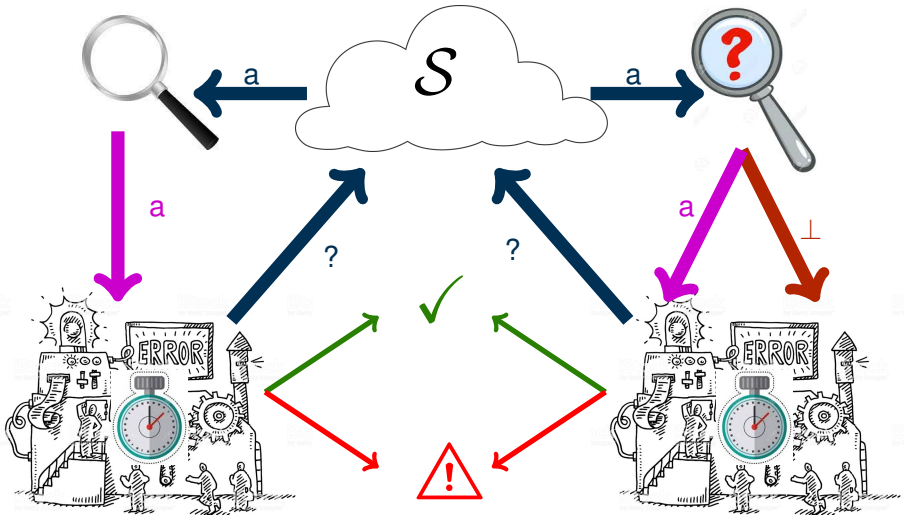
Design a **feasible** **optimal** policy without delay

Decide when it is possible

Minimize the number of observations

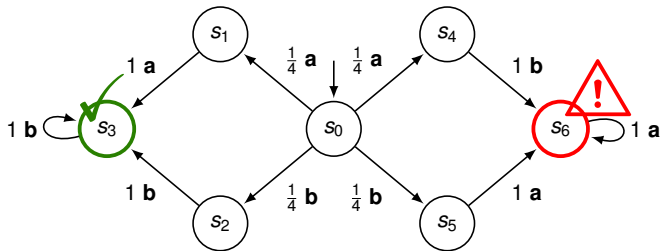
Decide as soon as possible



# Selective monitoring without delay

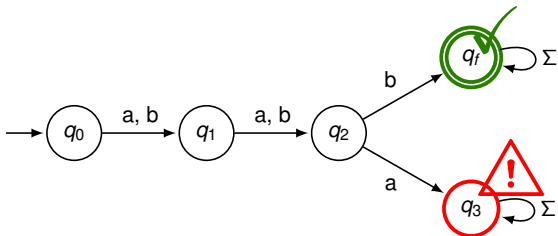


Credit image: [www.Dory.fr](http://www.Dory.fr); [www.123rf.com](http://www.123rf.com); [www.pinterest.fr](http://www.pinterest.fr) and [www.istockphoto.com](http://www.istockphoto.com)

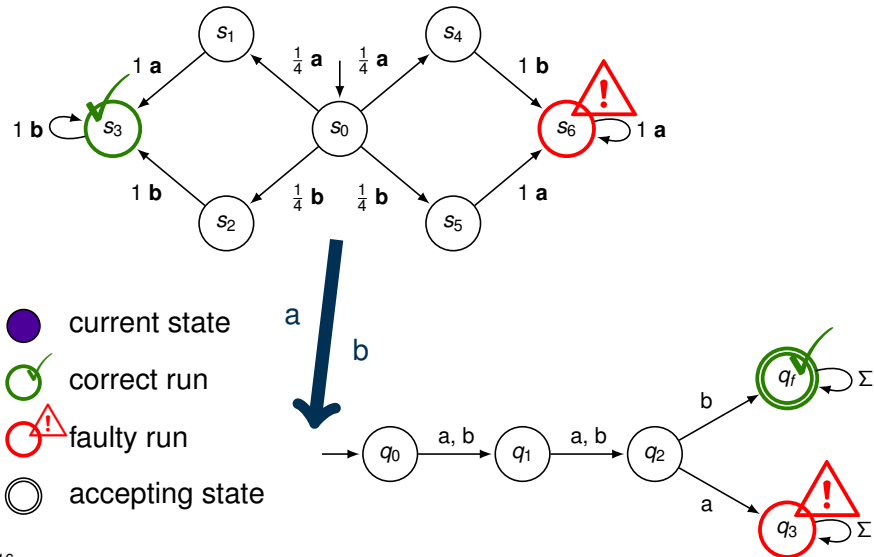
# Markov Chain and Deterministic Finite Automaton



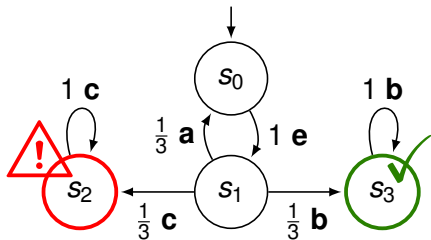
-  current state
-  correct run
-  faulty run
-  accepting state



# Markov Chain and Deterministic Finite Automaton

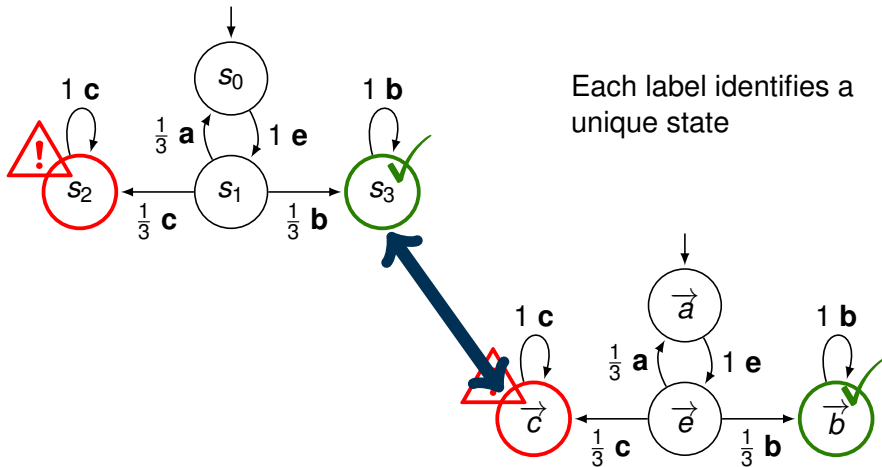


# Non-Hidden Markov Chain



Each label identifies a unique state

# Non-Hidden Markov Chain



# Observation policy

**Observation policy:** choice between observing and skipping

Observation policy

*aaaaaa*  $\implies$  *a*⊥⊥*aaa*⊥

**Assumption:** the policy decides with a finite prefix



## A feasible optimal policy

*Input:* a Markov Chain  $\mathcal{M}$ , a Deterministic Finite Automaton  $\mathcal{A}$  and a threshold  $r$

*Output:* feasible policy whose expected number of observations is smaller than  $r$

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*Input:* a Markov Chain  $\mathcal{M}$ , a Deterministic Finite Automaton  $\mathcal{A}$  and a threshold  $r$

*Output:* feasible policy whose expected number of observations is smaller than  $r$

Result in the general case <sup>6</sup>

Problem is undecidable.

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<sup>6</sup>Selective Monitoring, Grigore et al., 2018

# A feasible optimal policy

*Input:* a Markov Chain  $\mathcal{M}$ , a Deterministic Finite Automaton  $\mathcal{A}$  and a threshold  $r$

*Output:* feasible policy whose expected number of observations is smaller than  $r$

## Result in the general case

Problem is undecidable.

## Result in the non-Hidden Markov Chain case <sup>6</sup>

Problem is decidable in P

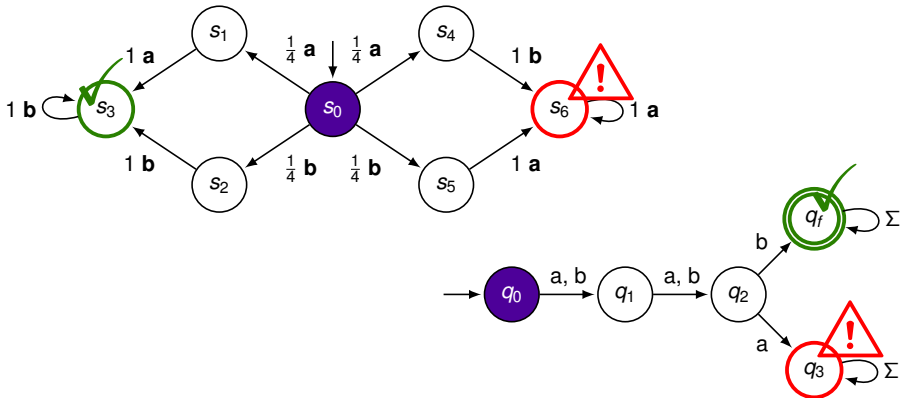
There exists a feasible optimal policy can be compute in P.

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<sup>6</sup>Selective Monitoring, Grigore et al., 2018

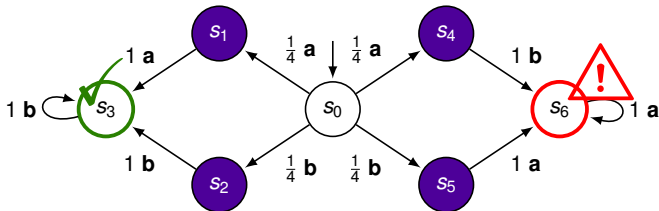
# A feasible optimal policy with delay

The feasible policy decides with delay.

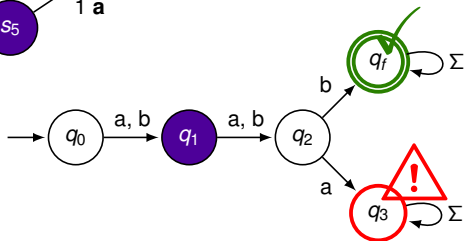


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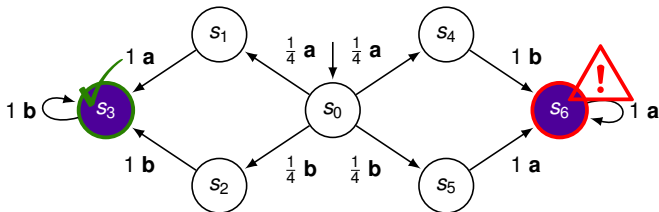


Policy:  $\perp$

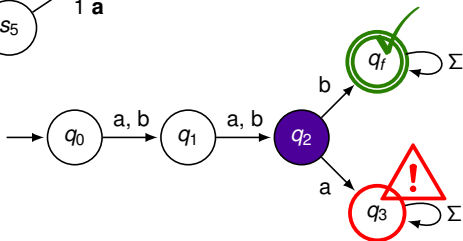


# A feasible optimal policy with delay

The feasible policy decides with delay.

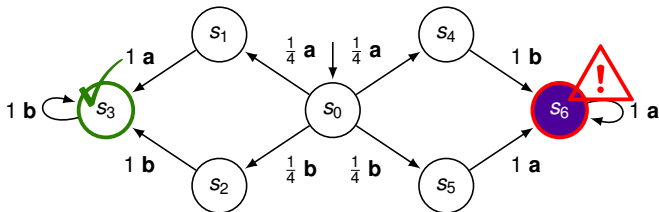


Policy:  $\perp\perp$

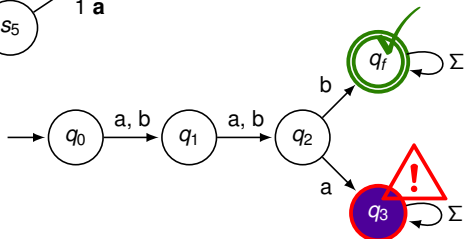


# A feasible optimal policy with delay

The feasible policy decides with delay.

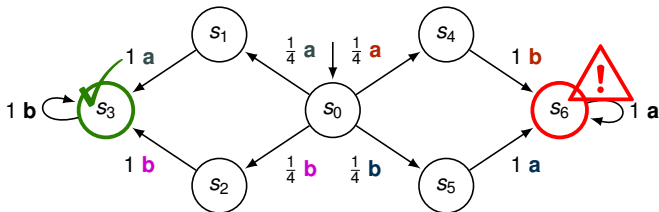


Policy:  $\perp \perp a$



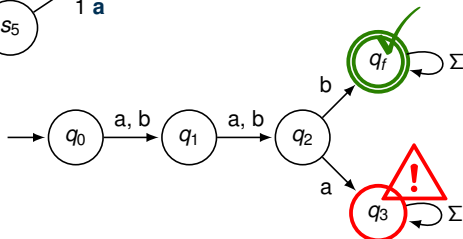
# A feasible optimal policy with delay

The feasible policy decides with delay.



Policy:  $\perp \perp a$

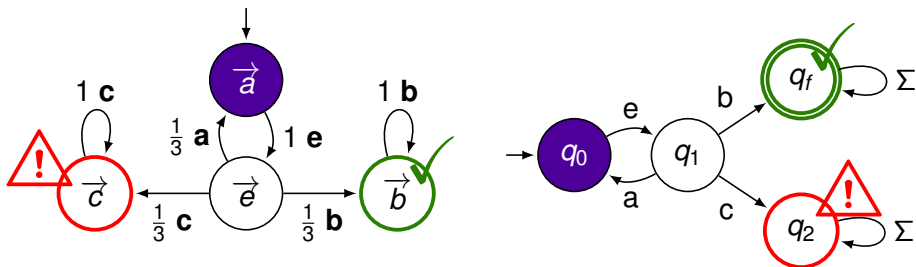
aa positively deciding  
bb  
ab negatively deciding  
ba





# A feasible policy without delay $\rho_{ZeroDelay}$

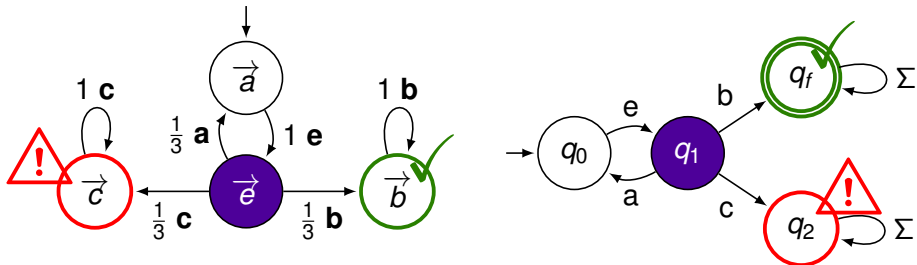
Apply the policy on  $w = eb^\omega$



The policy skips the next letter

# A feasible policy without delay $\rho_{ZeroDelay}$

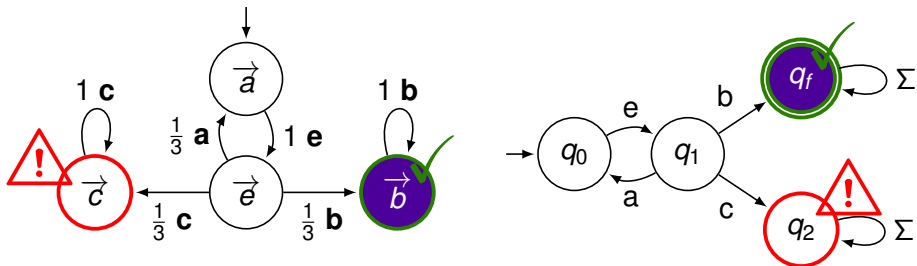
Apply the policy on  $w = ebb^\omega$



The policy observes the next letter

# A feasible policy without delay $\rho_{ZeroDelay}$

Apply the policy on  $w = eb^\omega$



The policy return yes

## Results in general case

### Feasible policy without delay

If  $v$  is a deciding prefix then the observation of  $v$  by  $\rho_{ZeroDelay}$  is also deciding.

### Non-optimal policy

The policy  $\rho_{ZeroDelay}$  is not optimal.

### Complexity

We can compute  $\rho_{ZeroDelay}$  in PSPACE.

## Results in non-Hidden Markov Chain case

### Optimal policy

The policy  $\rho_{ZeroDelay}$  is optimal.

### Complexity

We can compute  $\rho_{ZeroDelay}$  in P.

## Conclusion: our policy

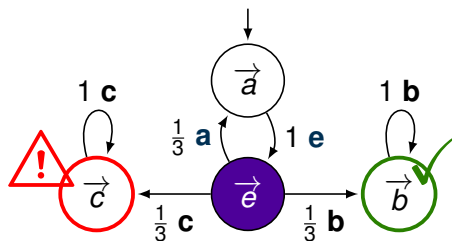
	General case	Non-Hidden Markov Chain case
Feasible	✓	✓
Without delay	✓	✓
Optimality	×	✓
Complexity	PSPACE	P

- Complexity hardness result on the optimal policy choice
- Undecidability result
- Feasible optimal policy without delay in other classes of Markov Chain
- Policy implementation and study in practice

# Prefix

## Class of prefix

- *enabled*: emit by the MC



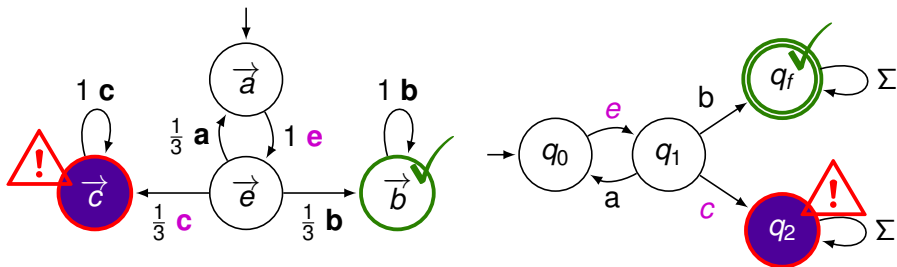
ea**e** is enabled but eaa is not enabled



# Prefix

## Class of prefix

- *enabled*: emit by the MC
- *negatively deciding*: all infinite word describe a faulty run

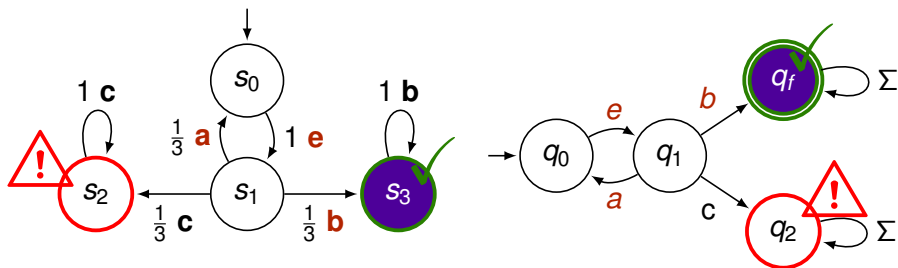


ec is negatively deciding

# Prefix

## Class of prefix

- *enabled*: emit by the MC
- *negatively deciding*: all infinite word describe a faulty run
- *positively deciding*: all infinite word describe a correct run



**eaeb** is positively deciding

# Observation policy

## Deciding policy

A policy *decides*  $w$  when the observation of  $w$  has a deciding prefix.

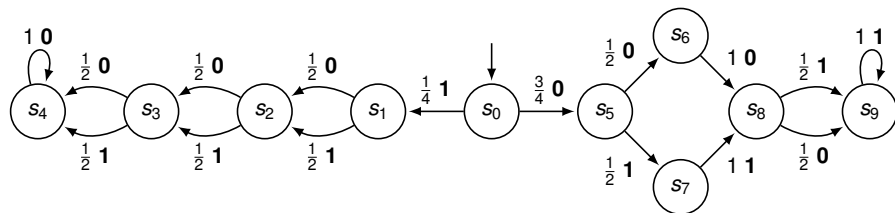
## Feasible policy

A *feasible* policy decides  $w$  if and only if  $w$  has a deciding prefix.

# Non-optimality

## Non-optimal policy

The policy  $\rho_{ZeroDelay}$  is not optimal.



$\rho$  observes 4 letters  
 $\rho_{ZeroDelay}$  observes 3 letters

$\rho$  observes 2 letters  
 $\rho_{ZeroDelay}$  observes 3 letters

$$3 = Ex(\rho_{ZeroDelay}) > Ex(\rho) = 2.5$$