

RÈGLES DE LA DÉDUCTION NATURELLE

axiome	$\frac{}{\Sigma, A \vdash A} \text{ax}$
affaiblissement	$\frac{\Sigma \vdash A}{\Sigma, B \vdash A} \text{aff}$
conjonction	$\frac{\Sigma \vdash A \quad \Sigma \vdash B}{\Sigma \vdash A \wedge B} \wedge_i \quad \frac{\Sigma \vdash A \wedge B}{\Sigma \vdash A} \wedge_e \quad \frac{\Sigma \vdash A \wedge B}{\Sigma \vdash B} \wedge_d$
disjonction	$\frac{\Sigma \vdash A}{\Sigma \vdash A \vee B} \vee_i \quad \frac{\Sigma \vdash B}{\Sigma \vdash A \vee B} \vee_d \quad \frac{\Sigma \vdash A \vee B \quad \Sigma, A \vdash C \quad \Sigma, B \vdash C}{\Sigma \vdash C} \vee_e$
implication	$\frac{\Sigma, A \vdash B}{\Sigma \vdash A \rightarrow B} \rightarrow_i \quad \frac{\Sigma \vdash A \rightarrow B \quad \Sigma \vdash A}{\Sigma \vdash B} \rightarrow_e$
absurdités (classique / intuitioniste)	$\frac{\Sigma, \neg A \vdash \perp}{\Sigma \vdash A} \perp_c \quad \frac{\Sigma \vdash \perp}{\Sigma \vdash A} \perp_i$
négation	$\frac{\Sigma, A \vdash \perp}{\Sigma \vdash \neg A} \neg_i \quad \frac{\Sigma \vdash A \quad \Sigma \vdash \neg A}{\Sigma \vdash \perp} \neg_e$
travaux-exclue	$\frac{\Sigma, A \vdash B \quad \Sigma, \neg A \vdash B}{\Sigma \vdash B} \text{t.e.}$
loi de Peirce	$\frac{\Sigma, \neg A \vdash A}{\Sigma \vdash A} \text{l.p.}$
quantification universelle	$\frac{\Sigma \vdash A[x:=t] \text{ et } x \notin \text{varlib}(\Sigma)}{\Sigma \vdash \forall x A} \forall_i \quad \frac{\Sigma \vdash \forall x A}{\Sigma \vdash A[x:=t]} \forall_e$
quantification existentielle	$\frac{\Sigma \vdash A[x:=t]}{\Sigma \vdash \exists x A} \exists_i \quad \frac{\Sigma \vdash \exists x A, \Sigma, A \vdash C \text{ et } x \notin \text{varlib}(\Sigma, C)}{\Sigma \vdash C} \exists_e$
égalité	$\frac{}{\Sigma \vdash t=t} =_i \quad \frac{\Sigma \vdash u=t \quad \Sigma \vdash A[x:=t]}{\Sigma \vdash A[x:=u]} =_e$
contraposition	$\frac{\Sigma \vdash \neg B \rightarrow \neg A}{\Sigma \vdash A \rightarrow B} \text{cp.}$

LC = logique classique = LM + \perp_c

LI = logique intuitioniste = LM + \perp_i

LM = logique minimale (peut se passer de \perp)