Inégalités linéaires de dominance pour l'ordonnancement juste-à-temps avec date d'échéance commune non restrictive

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ROADEF, Montpellier, février 2020

Outline

1. Introduction
2. Dominance properties
3. Dominance inequalities
4. Conclusion

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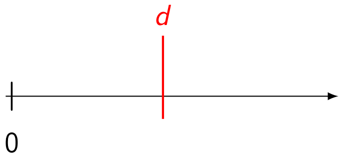

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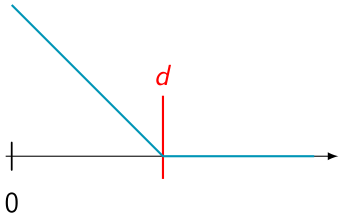

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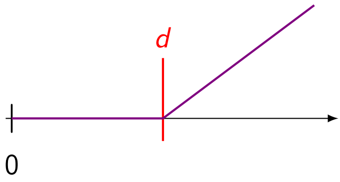

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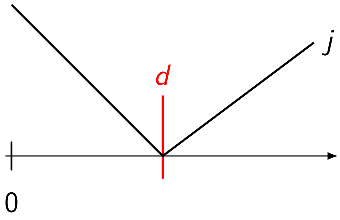

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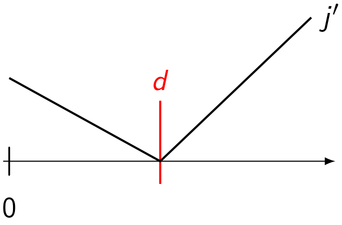

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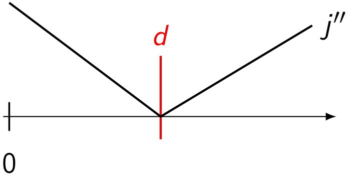

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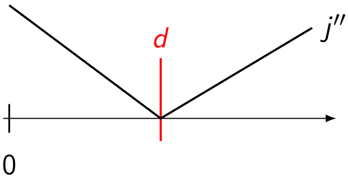

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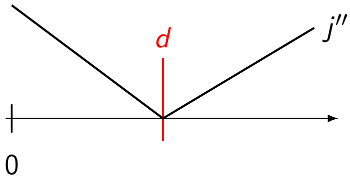

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The objective = $\min \sum_{j \in J} \alpha_j E_j + \beta_j T_j$

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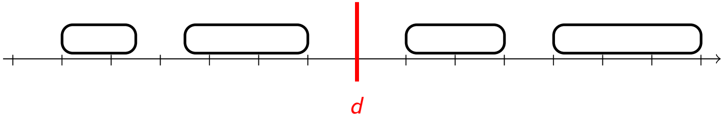
In both cases, the searching space can be reduced to T, other solutions can be discarded.

Dominance properties for the UCDDP



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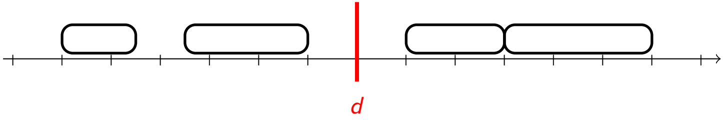

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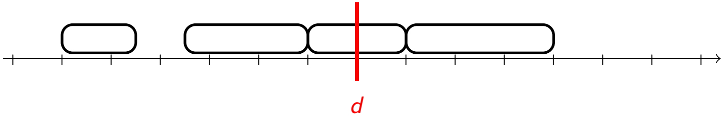

Hall and Posner, 1991, Operations research

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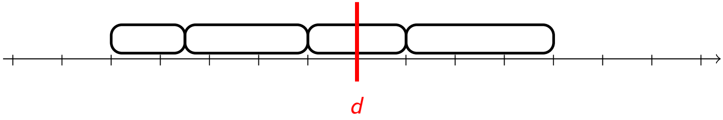

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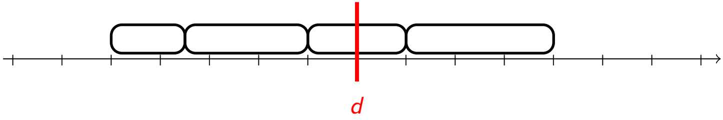

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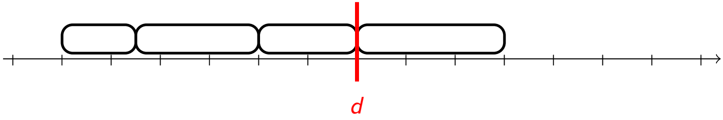

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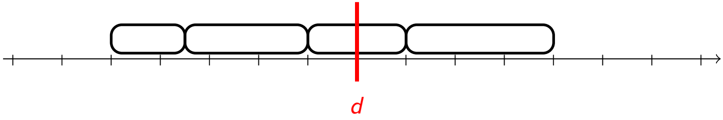

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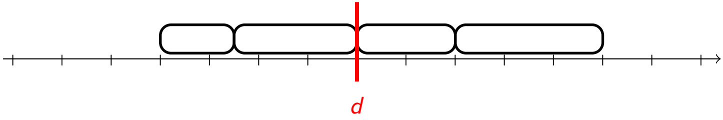

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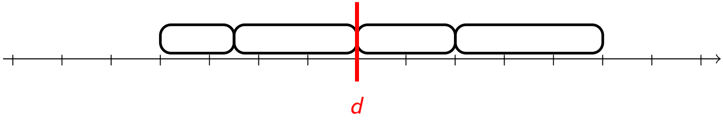

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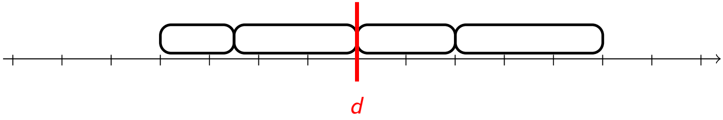

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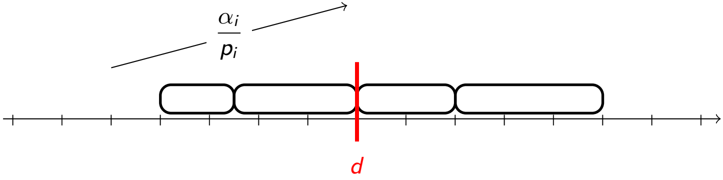

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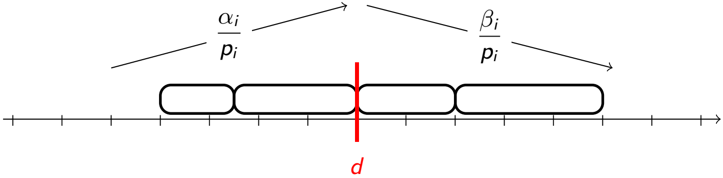

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- This work: translate dominance properties in a MIP context

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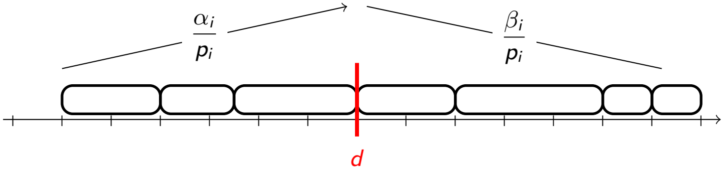
Structural dominance properties



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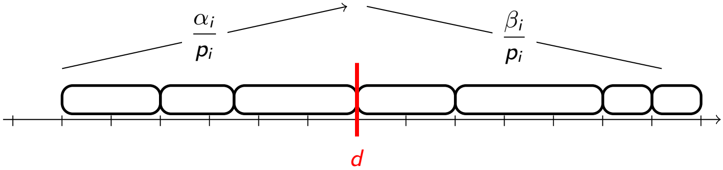

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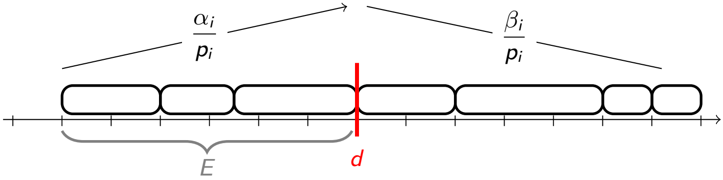

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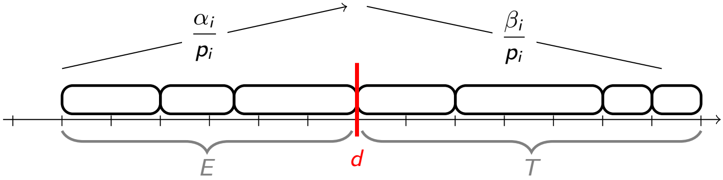

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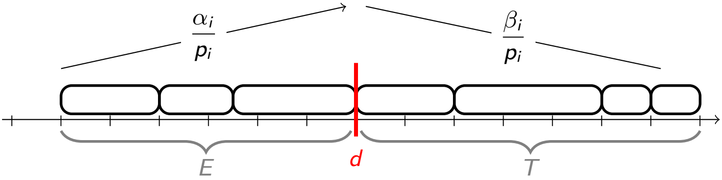

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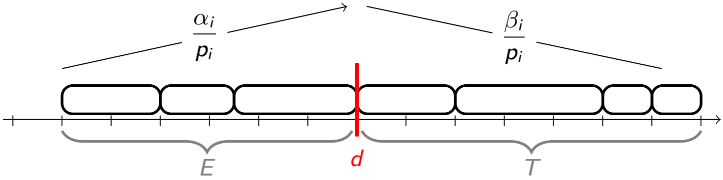

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- V-shaped d-blocks having the same early/tardy partition (E, T) have the same penalty $f(E, T)$
- formulate UCDDP as a partition problem $\min_{(E,T) \in \vec{\mathcal{P}}_2^*(J)} f(E, T)$

where $\vec{\mathcal{P}}_2^*(J) = \{(E, T) \mid \{E, T\} \text{ is a partition of } J \text{ and } E \neq \emptyset\}$

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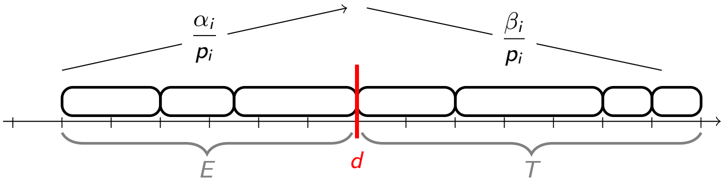

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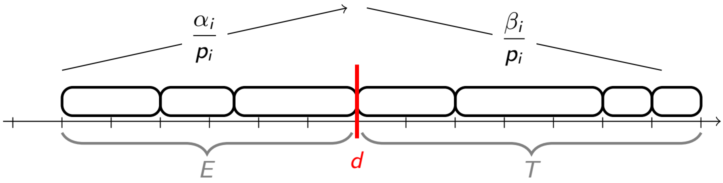

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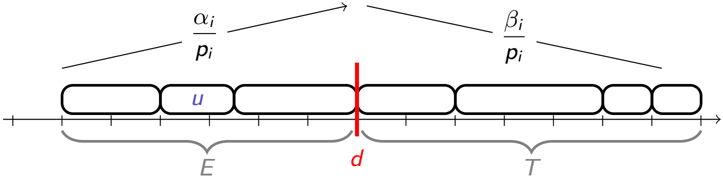

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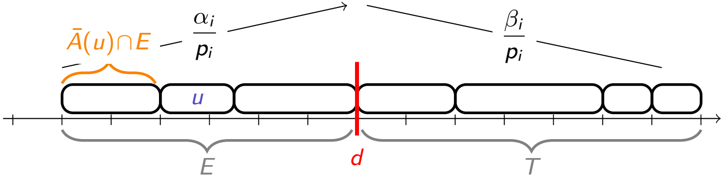

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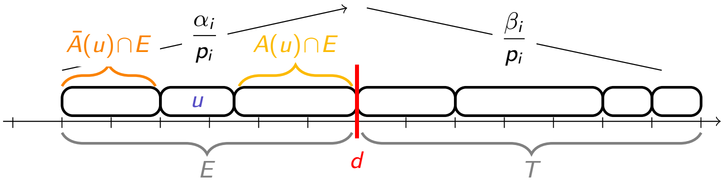

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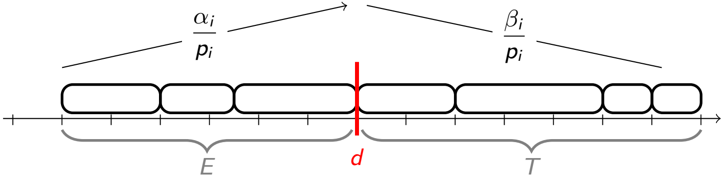

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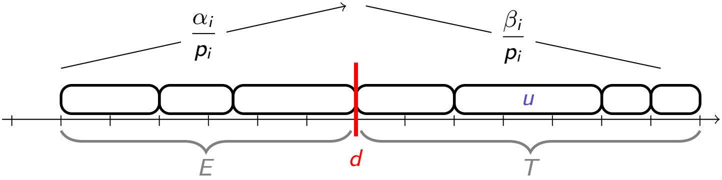

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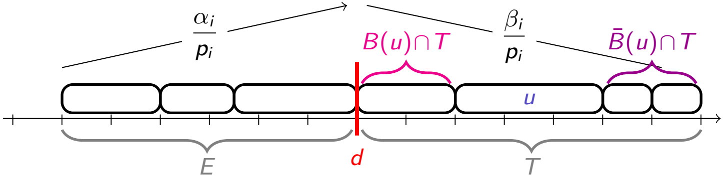

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$$B(u) = \left\{ i \in J \mid \frac{\beta_i}{p_i} > \frac{\beta_u}{p_u} \right\} \quad \text{and} \quad \bar{B}(u) = \left\{ i \in J \mid i \neq u, \frac{\beta_i}{p_i} \leqslant \frac{\beta_u}{p_u} \right\}$$

Neighborhood based dominance properties : generic idea

Remark: If a solution is dominated by one of its neighbors, then it is not an optimal solution.

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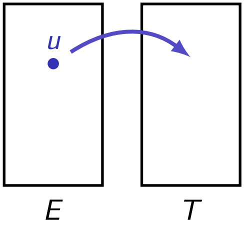
Our approach:

- define a neighborhood based on operations
- translate the associate dominance property by constraints

Insert and swap operations on partitions



insert *of an early task*

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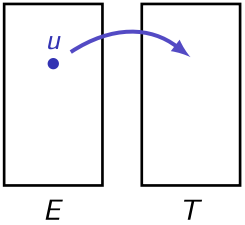

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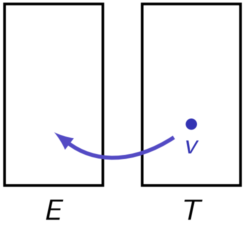
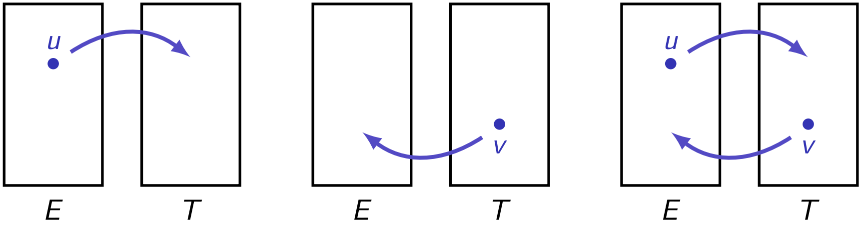


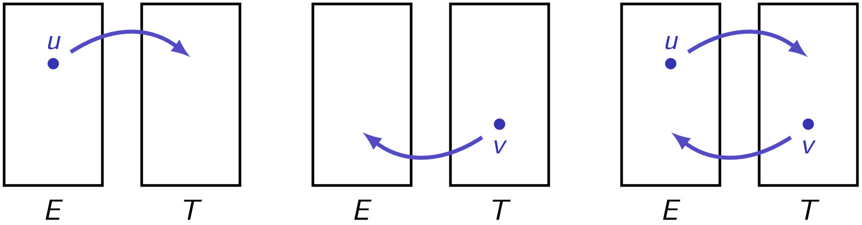
insert *of a tardy task*

Insert and swap operations on partitions



insert** of an early task* | ***insert** of a tardy task* | ***swap

Insert and swap operations on partitions



insert of an early task *insert of a tardy task* *swap*

A partition (E, T) is said:

- insert non-dominated if $\begin{cases} \forall v \in T,\ f(E,T) \leqslant f(E \cup \{v\},\ T_{\setminus\{v\}}) \\ \forall u \in E,\ f(E,T) \leqslant f(E_{\setminus\{u\}},\ T \cup \{u\}) \end{cases}$
- swap non-dominated if $\forall (u,v) \in E \times T,\ f(E,T) \leqslant f(E_{\setminus\{u\}} \cup \{v\},\ T_{\setminus\{v\}} \cup \{u\})$

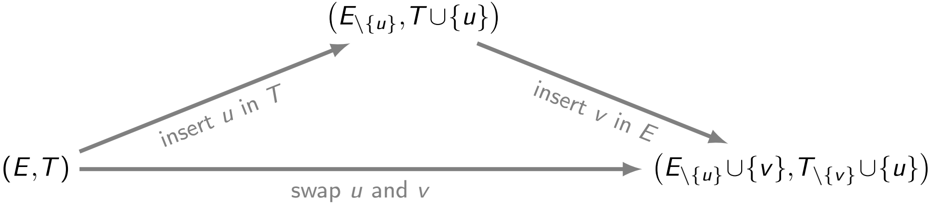
Compare insert dominance and swap dominance

(E, T)

Compare insert dominance and swap dominance

$$(E, T) \xrightarrow{\text{swap } u \text{ and } v} \left(E_{\setminus\{u\}} \cup \{v\}, T_{\setminus\{v\}} \cup \{u\}\right)$$

Compare insert dominance and swap dominance



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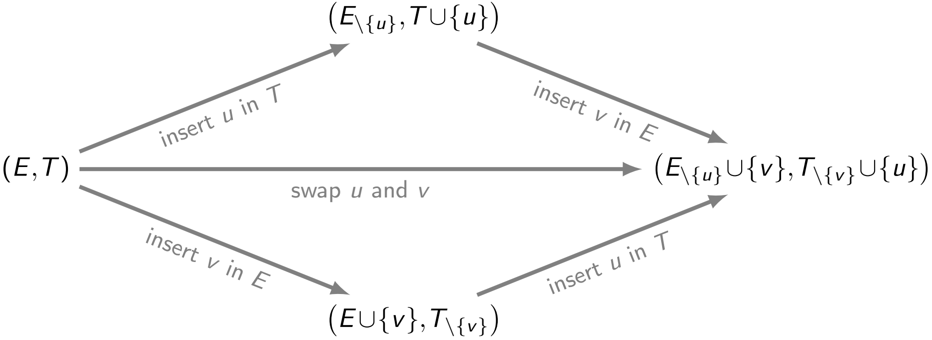

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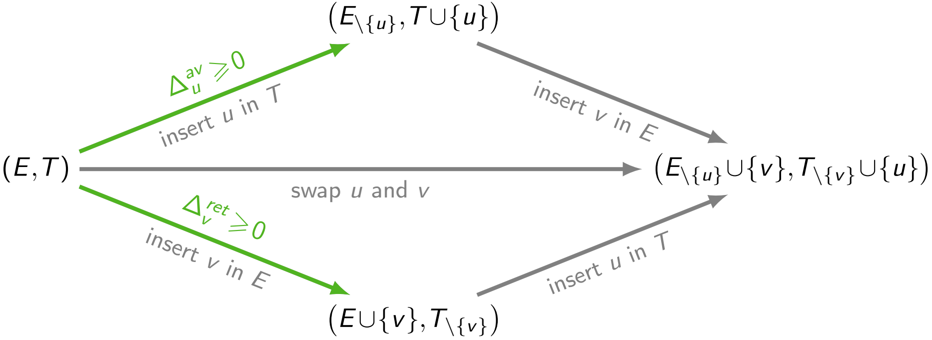

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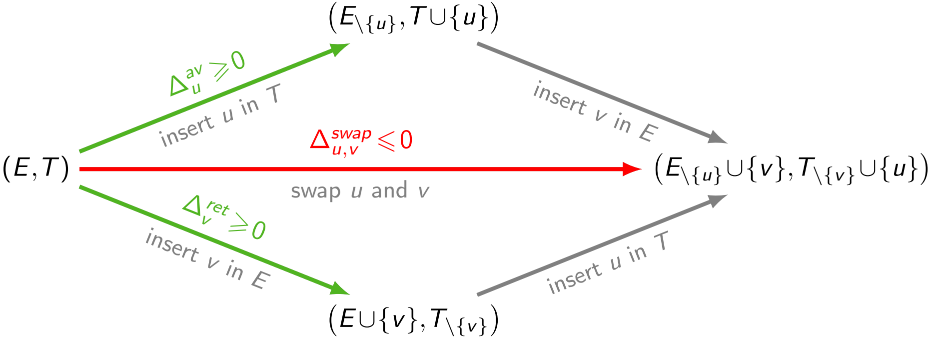

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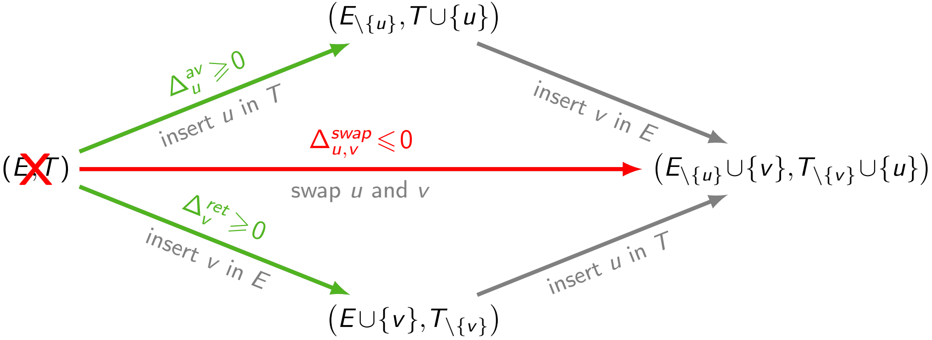

An example on schedules

Instance :

α_j	4	3	3	-	5	-
β_j	-	3	-	6	3	1

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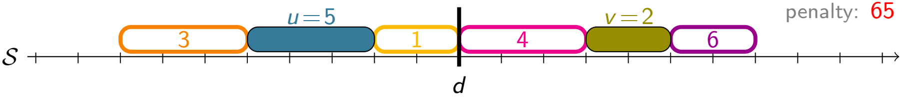

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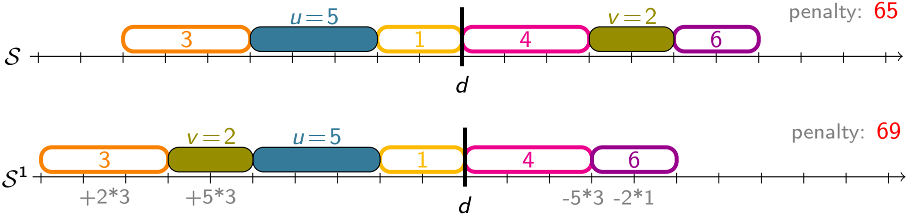

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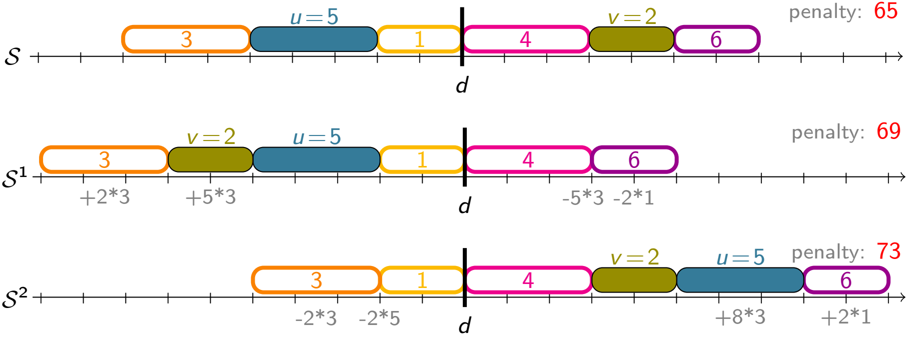

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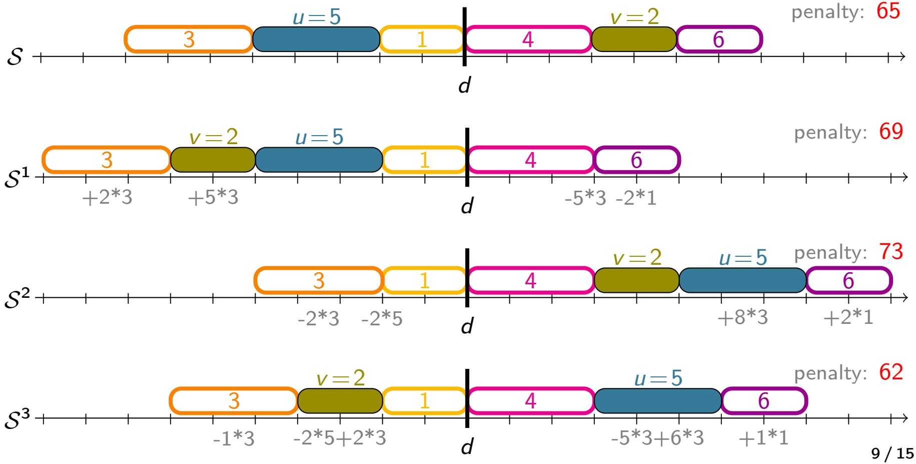
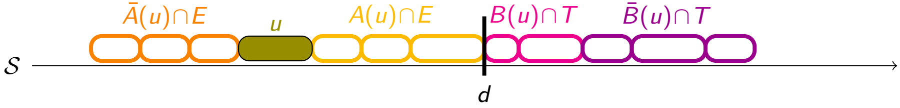


Cost variation induced by the insertion of an early task u



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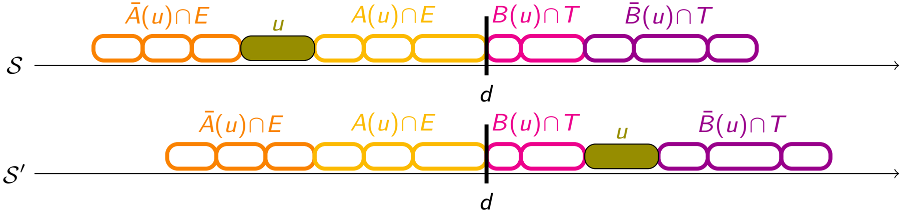

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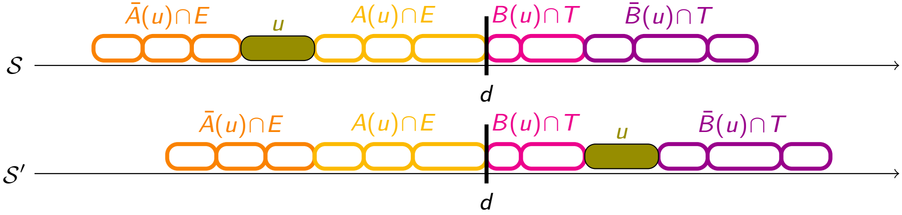

$\text{cost}(\mathcal{S}') = \text{cost}(\mathcal{S})$

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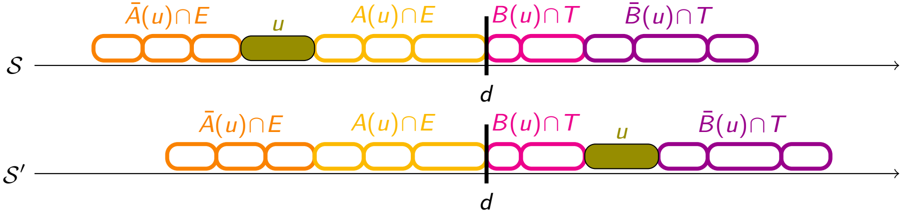

$$\text{cost}(\mathcal{S}') = \text{cost}(\mathcal{S}) - \alpha_u \, p\big(\underline{A(u) \cap E}\big)$$

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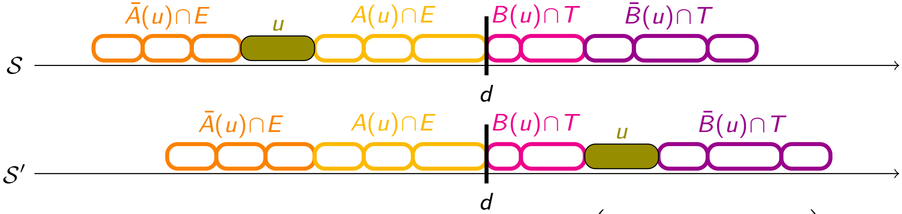

$$\text{cost}(\mathcal{S}') = \text{cost}(\mathcal{S}) - \alpha_u\, p\big(\underline{A(u)\cap E}\big) + \beta_u \Big(p\big(\underline{B(v)\cap T}\big) + p_u\Big)$$

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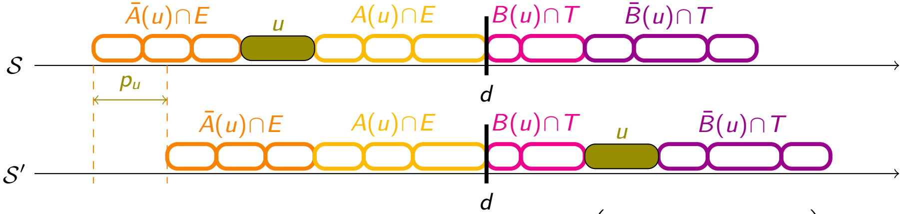

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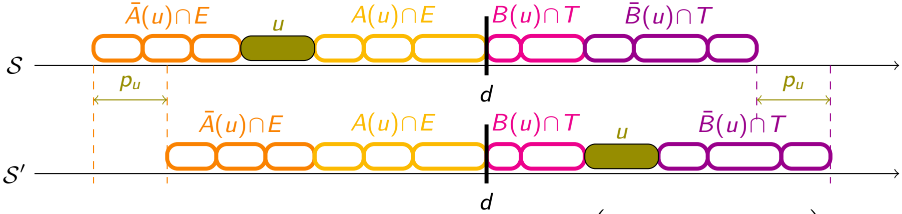

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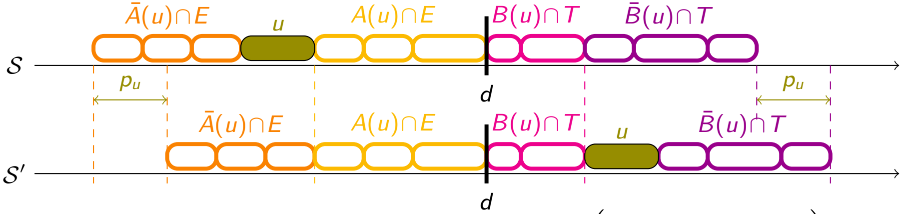

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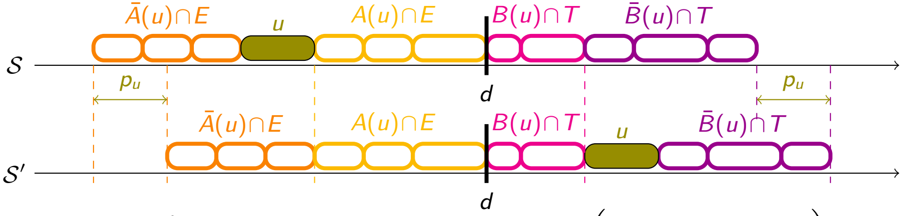

$$\Delta_u^{early}(E,T) = -\alpha_u\, p\big(\underline{A(u)\cap E}\big) + \beta_u \Big(p\big(\underline{B(v)\cap T}\big) + p_u\Big)$$
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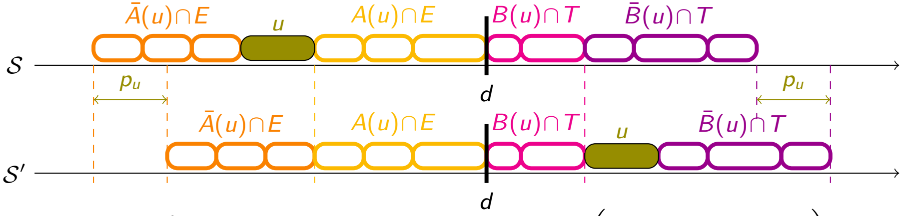

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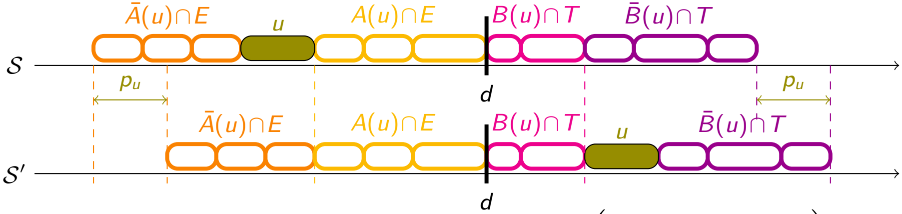
dominance constraint
for the early-insert of $u \in J$

$$\boxed{\Delta_u^{early}(E,T) \geqslant 0 \text{ if } u \in E}$$

Cost variation induced by the insertion of an early task u



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Similarly: dominance constraint for the tardy-insert of $v \in J$: $\boxed{\Delta_v^{tardy}(E,T) \geqslant 0 \text{ if } v \in T}$

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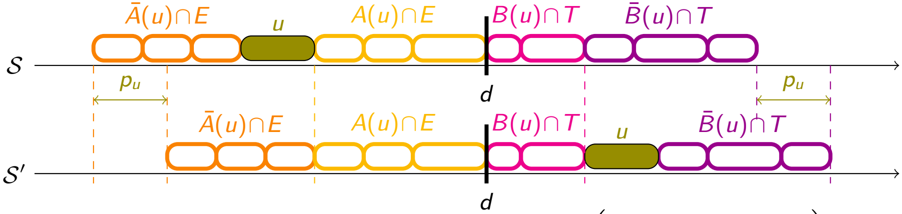

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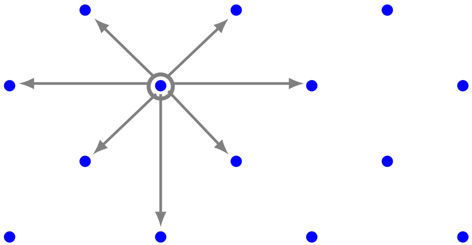
dominance constraint for the early-insert of $u\in J$ — $\Delta_u^{early}(E,T)\geqslant 0$ if $u\in E$

Similarly: dominance constraint for the tardy-insert of $v\in J$ — $\Delta_v^{tardy}(E,T)\geqslant 0$ if $v\in T$

dominance constraint for the swap of $u\in J$ and $v\in J$ — $\Delta_{u,v}^{swap}(E,T)\geqslant 0$ if $(u,v)\in E\times T$

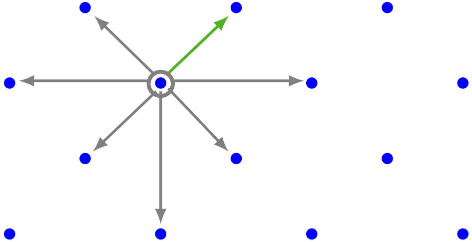
Neighborhood: solution-centered vs operation-centered point of view



Solution-centered

= *consider* ***all*** *the neighbors of* ***one*** *given solution*

Neighborhood: solution-centered vs operation-centered point of view



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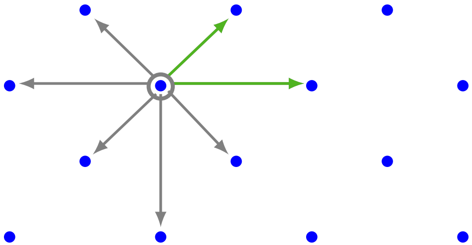

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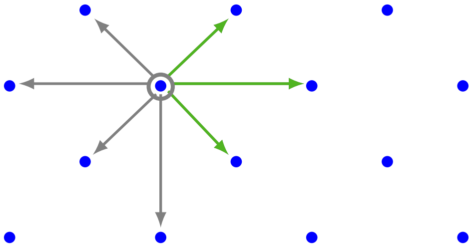

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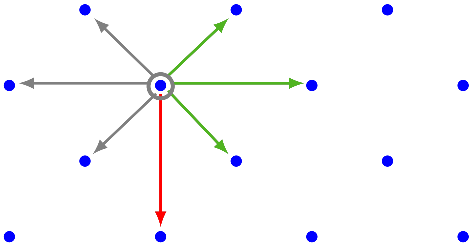

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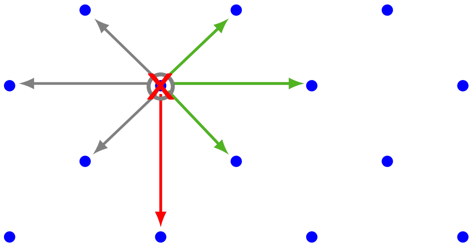

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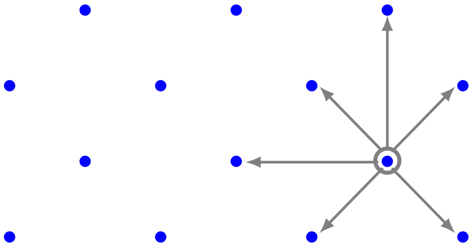

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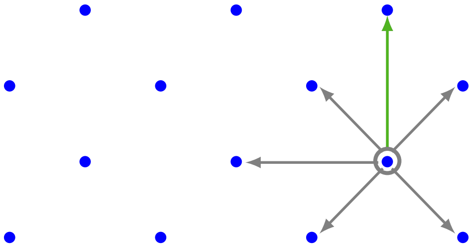

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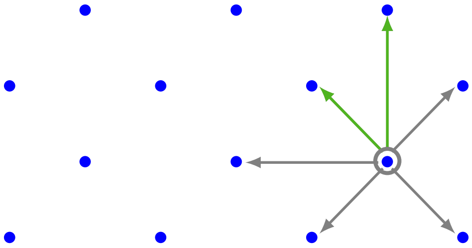

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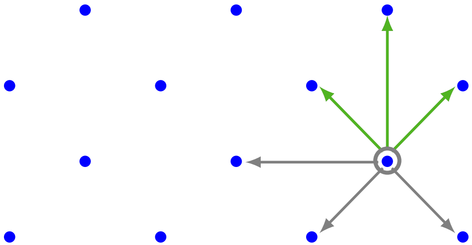

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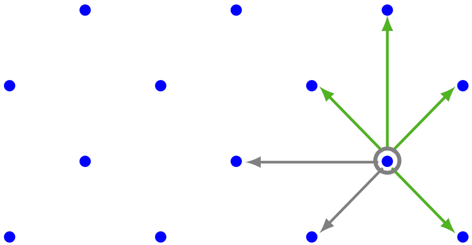

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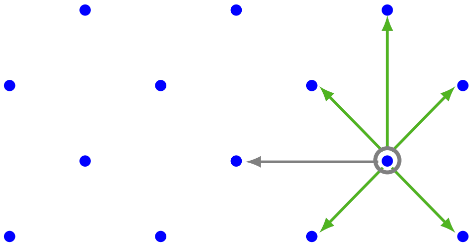

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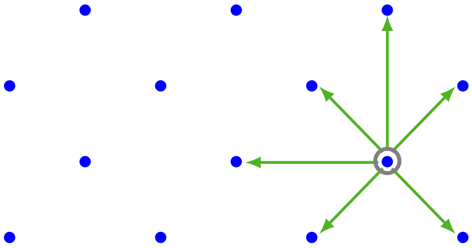

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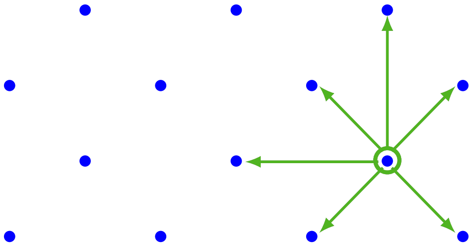

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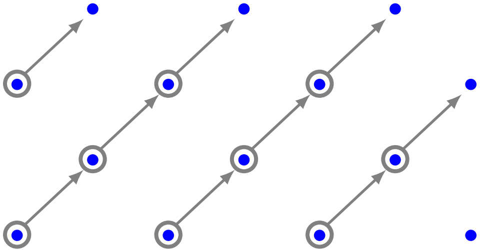

Operation-centered

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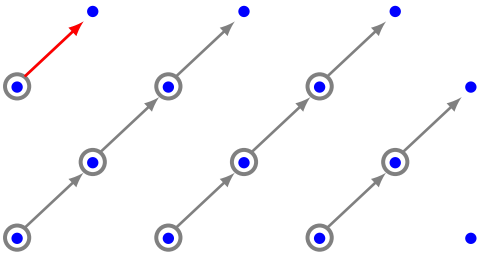

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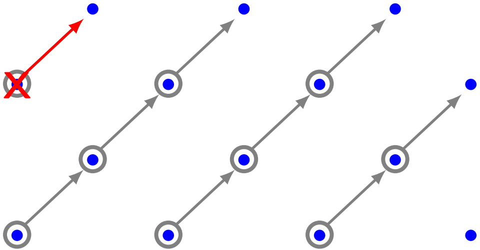

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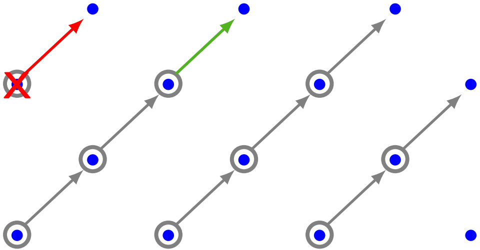

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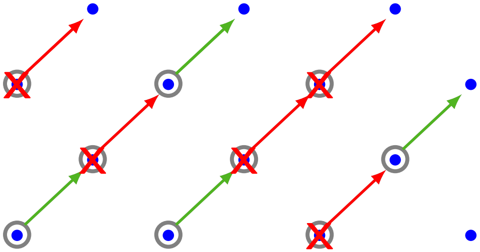

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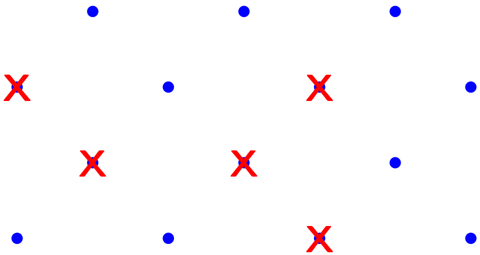

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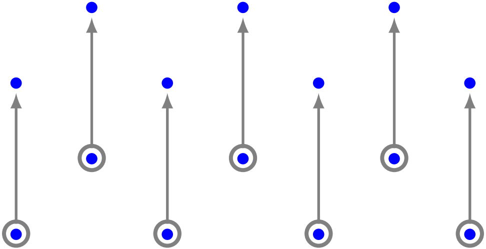

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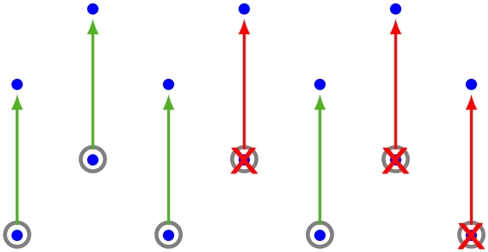

Operation-centered

= *consider* ***one*** *given type of neighbor for* ***all*** *the solutions*

Outline

Linear formulation

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$$\boxed{\Delta_u^{early}(E, T) \geqslant 0 \text{ if } u \in E} \longrightarrow \boxed{\Delta_u^{early}(\delta) \geqslant -M(1 - \delta_u)}$$

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Experimental results

20	6
50	1835
60	1/10

Framework:

- machine
  RAM 144 Go
  1 core at 3.47 Ghz
- PL Solver
  Cplex 12.6.3
- time limit
  3600s

Benchmark:

- by Biskup&Feldmann
- $n \in \{10, 20, 30, 40, 50, 60\}$
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- p, α and β integers
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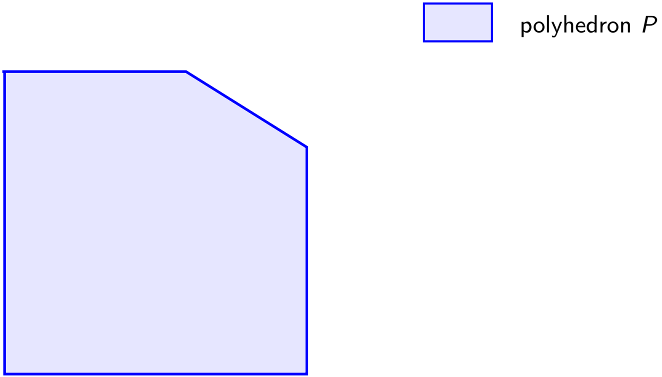

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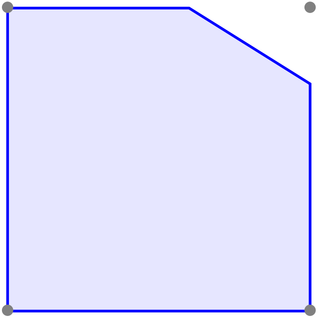

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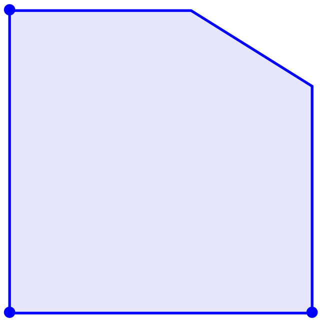

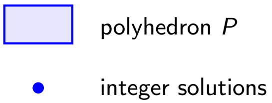

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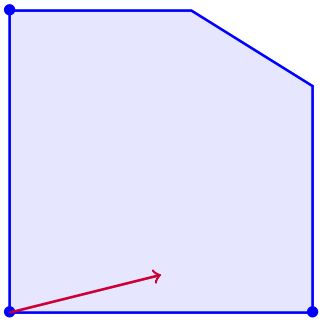

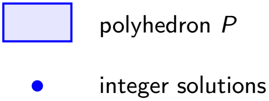

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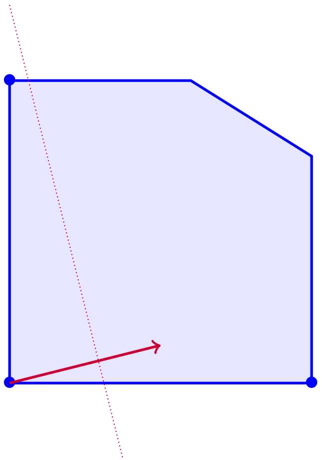
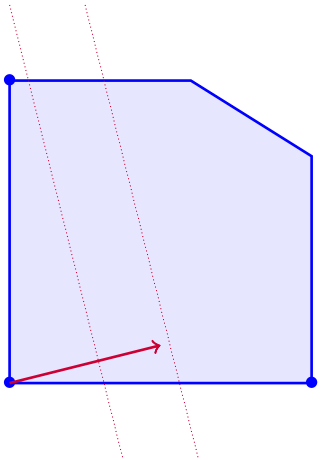
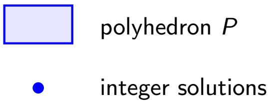


polyhedron P

integer solutions

Unusual inequalities





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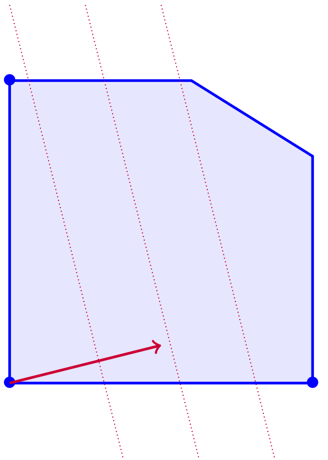

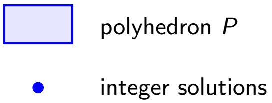

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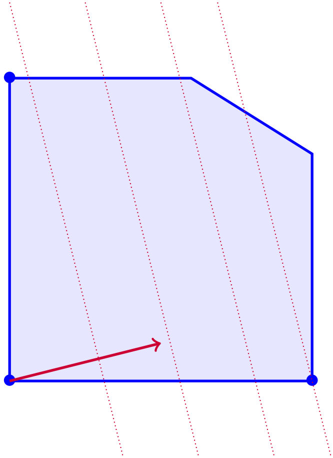

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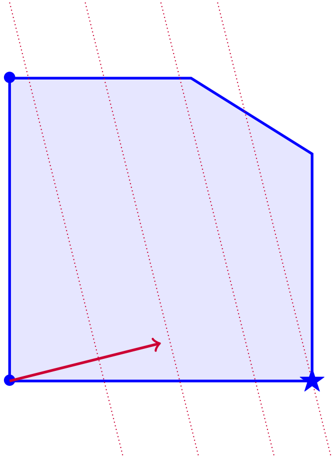

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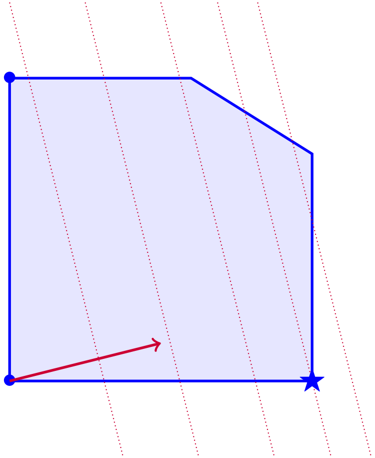

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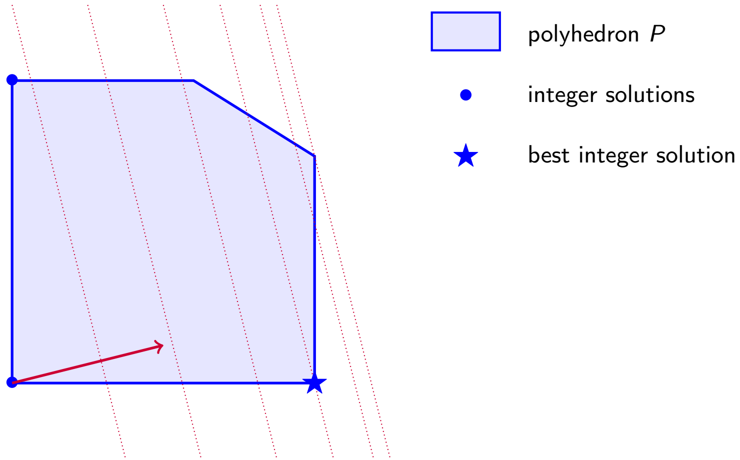

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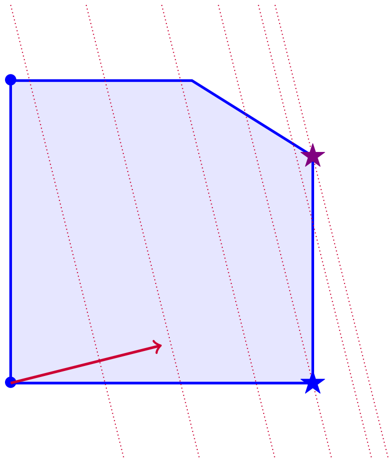

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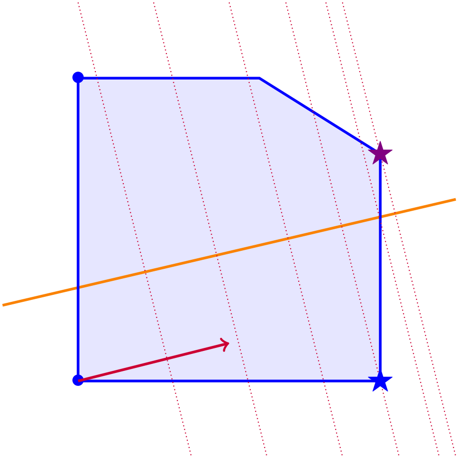

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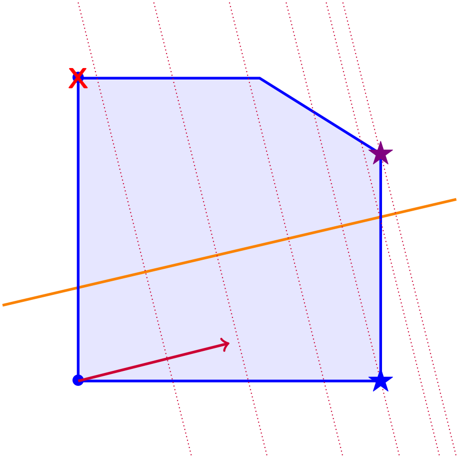

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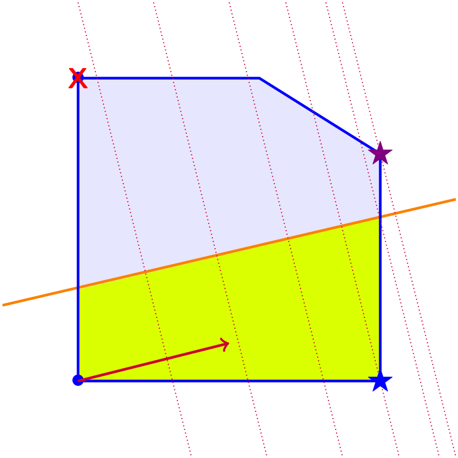

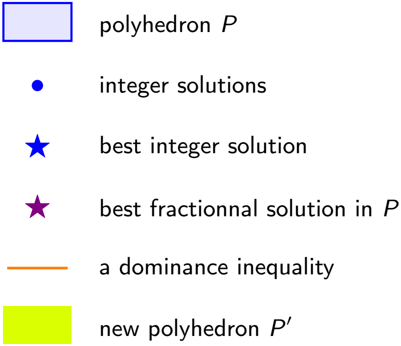

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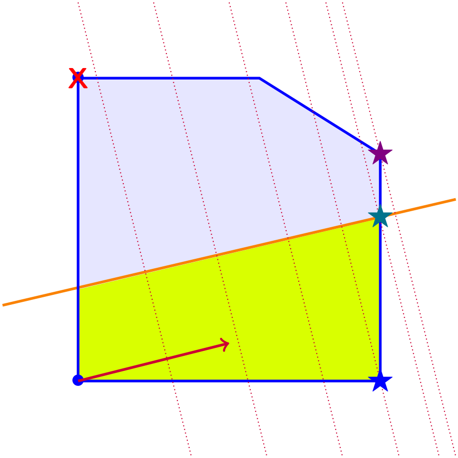

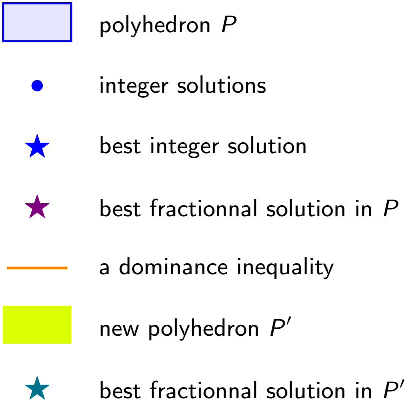

Outline

We propose linear inequalities

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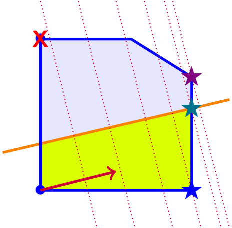

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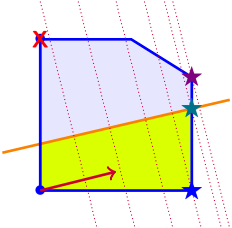
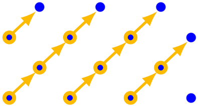


- by translating neigborhood-based dominance properties



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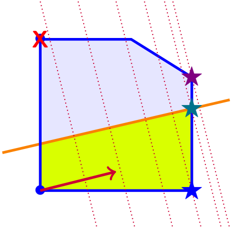

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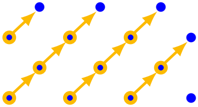

Future work:

- applying the dominance inequalities principle to other combinatorial problems

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