

# Adaptive Multi-Objective Local Search Algorithms

## Permutation Flowshop Scheduling Problem

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## Multi-Objective Optimisation

- ▶ Local search algorithms
- ▶ Bi-objective optimisation
- ▶ Permutation problems (PFSP, TSP, QAP)

## Automatic Algorithm Design

- ▶ Offline design (algorithm selection, parameter tuning)
- ▶ Online design (hyper-heuristics, parameter control)

[Eiben et al., 1999][Hamadi et al., 2012][Karafotias et al., 2015]

# Motivations

## Offline Design

- ▶ Prediction based
- ▶ Instance classes / distributions
- ▶ Computationally expensive

## Online Design

- ▶ Adaptation based
- ▶ Single current instance
- ▶ *Slight* overhead

## Beyond Automatic Offline Design

- ▶ How to get an adaptive version of our MOLS algorithm?
- ▶ What parameter(s) can we control?
- ▶ What control mechanism can we use?

# Obstacles

## Adaptive algorithm

- ▶ No easy recipe

## Search space

- ▶ Too many parameters
- ▶ Categorical main parameters

## Control mechanisms

- ▶ Infrequently generalisable

# Multi-Objective Local Search (MOLS) Algorithms

## MOLS Algorithms

- ▶ Efficient metaheuristics
- ▶ Used on many problems (e.g., scheduling, routing, assignment)
- ▶ Many combinations of parameters/strategies

## MOLS Principles

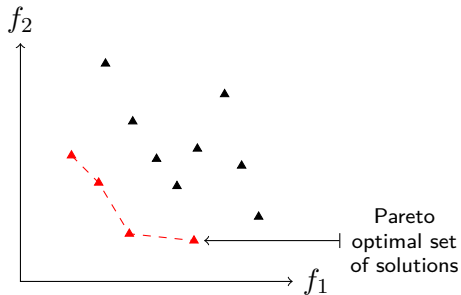
- ▶ Iteratively improve a **set of solutions**
- ▶ Using a neighbouring relation

[Blot et al., Journal of Heuristics, 2018]

# Multi-Objective Optimisation

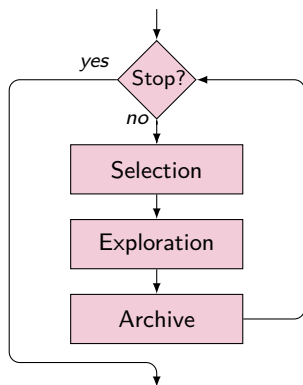
## Performance Criteria

- ▶ Convergence
- ▶ Distribution
- ▶ Diversity
- ▶ Size



# Static MOLS Algorithm

## Core MOLS

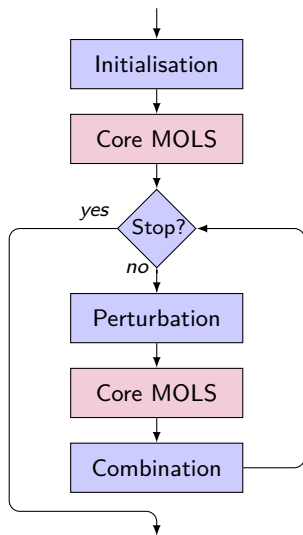


## Core MOLS

- ▶ Termination
- ▶ Selection
- ▶ Exploration
- ▶ Archive

# Static MOLS Algorithm

## Iterated MOLS



## Iterated MOLS

- ▶ Initialisation
- ▶ Perturbation
- ▶ Combination



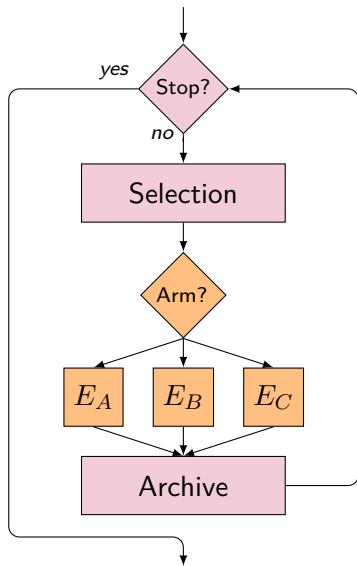
# MOLS Configuration Space

Phase	Parameter	Parameter values
Selection	select-strat	{all, rand, newest, oldest}
Selection	select-size	$\mathbb{N}^+$
Exploration	explor-strat	{all, all-imp, imp, ndom, imp-ndom}
Exploration	explor-ref	{sol, arch}
Exploration	explor-size	$\mathbb{N}^+$
Archive	bound-strat	{unbounded, rand, replace}
Archive	bound-size	$\mathbb{N}^+$
Perturbation	perturb-strat	{kick, kick-all, restart}
Perturbation	perturb-size	$\mathbb{N}^+$
Perturbation	perturb-strength	$\mathbb{N}^+$

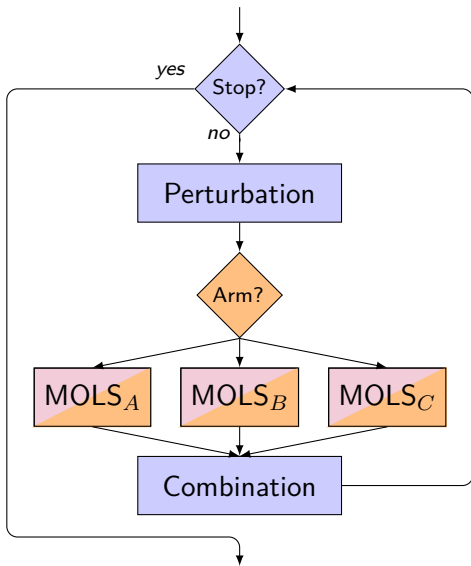
# MOLS Configuration Space

Phase	Parameter	Parameter values
Selection	select-strat	{all, rand, newest, oldest}
Selection	select-size	$\mathbb{N}^+$
Exploration	explor-strat	{all, all-imp, imp, ndom, imp-ndom}
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Perturbation	perturb-size	$\mathbb{N}^+$
Perturbation	perturb-strength	$\mathbb{N}^+$

# Adaptive MOLS Algorithm

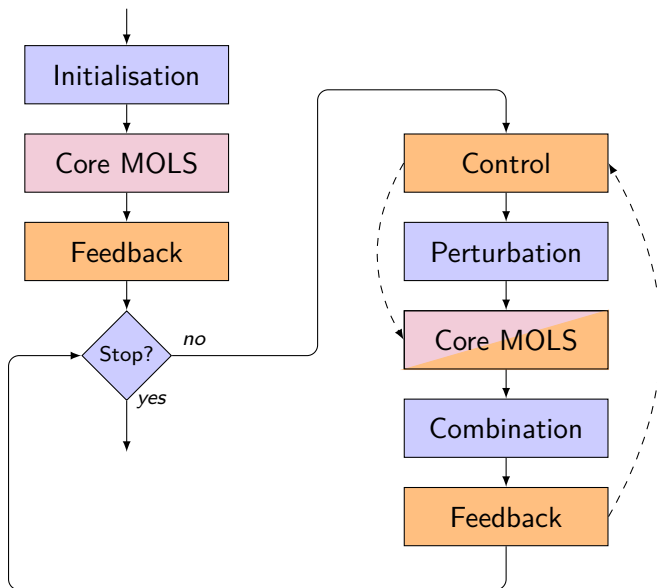


**Core MOLS**



**Iterated MOLS**

# Adaptive MOLS Algorithm



# Control Mechanisms

## Constraints

- ▶ Single categorical parameter
- ▶ Few possible values (up to 9)

## Generic Parameter Control

- ▶ Random mechanisms
- ▶ Probability based mechanisms
- ▶ Multi-armed bandits mechanisms
- ▶ Reinforcement learning

[Karafotias et al., 2015]

# Control Mechanisms

## Constraints

- ▶ Single categorical parameter
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- ▶ Random mechanisms
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[Karafotias et al., 2015]

## Feedback

- ▶ Hypervolume (convergence performance criteria)

# Selected Generic Parameter Control

## Uniform Random Control

$$p_i(t+1) = \frac{1}{N}$$

## $\epsilon$ -greedy

$$p_i(t+1) = \begin{cases} (1 - \epsilon) + \epsilon/N, & \text{if } i = \arg \max_j q_j(t) \\ \epsilon/N, & \text{otherwise} \end{cases}$$

## Why no UCB1?

- ▶ Very few possible arms (2 or 3)
- ▶ Very few possible control behaviours (2?)
- ▶ **First results**

# Experimental Setup I

## 3-arm Control

- ▶ `ndom`
- ▶ `imp-ndom`
- ▶ `imp`
- ▶ `random { ndom, imp-ndom, imp }`
- ▶ `0.1-greedy { ndom, imp-ndom, imp }`

## 2-arm Control

- ▶ `ndom`
- ▶ `imp-ndom`
- ▶ `random { ndom, imp-ndom }`
- ▶ `0.1-greedy { ndom, imp-ndom }`



# Experimental Setup II

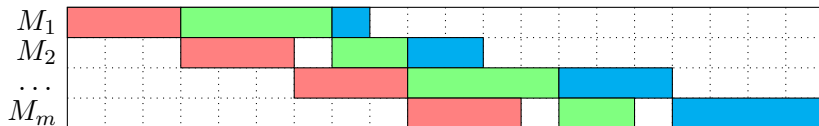
## Long Term Learning (random)

- ▶ random { ndom, imp-ndom, imp }
- ▶ random { ndom, imp-ndom, imp }  $\rightarrow$  (50%) { ndom, imp-ndom }
- ▶ random { ndom, imp-ndom, imp }  $\rightarrow$  (20%) { ndom, imp-ndom }
- ▶ random { ndom, imp-ndom }

## Long Term Learning (0.1-greedy)

- ▶ 0.1-greedy { ndom, imp-ndom, imp }
- ▶ 0.1-greedy { ndom, imp-ndom, imp }  $\rightarrow$  (50%) { ndom, imp-ndom }
- ▶ 0.1-greedy { ndom, imp-ndom, imp }  $\rightarrow$  (20%) { ndom, imp-ndom }
- ▶ 0.1-greedy { ndom, imp-ndom }

# Permutation Flowshop Scheduling Problem



## PFSP Instances

- ▶ Classical Taillard instances
  - ▶  $n \in \{20, 50, 100, 200, 500\}$  jobs
  - ▶  $m \in \{5, 10, 20\}$  machines
  - ▶ 12 valid combinations
  - ▶ 120 instances
- ▶ 2 classical objectives
  - ▶ Makespan (max of completion times)
  - ▶ Flowtime (sum of completion times)

# Experimental Results

## 3-arm Ranking

Approach	Instance ( $N, M$ )												Avg.
	20			50			100			200		500	
	5	10	20	5	10	20	5	10	20	10	20	20	
<b>imp</b>	5	5	5	5	5	5	5	5	5	5	5	5	5
<b>imp-ndom</b>	4	4	3	4	4	4	4	1	2	1	2	1	2.8
<b>ndom</b>	1	1	3	1	1	1	1	1	1	1	1	1	1.2
<b>rand_3</b>	1	1	1	1	1	1	1	1	2	3	3	3	1.6
<b>greedy_3</b>	1	1	1	1	1	1	1	1	2	3	3	3	1.6

Wilcoxon signed ranked tests, Friedman post-hoc analysis  
200 runs/dataset

# Experimental Results

## 2-arm Ranking

Approach	Instance ( $N, M$ )											Avg.	
	20			50			100			200			500
	5	10	20	5	10	20	5	10	20	10	20		20
<i>imp-ndom</i>	4	4	3	4	4	4	4	4	4	4	4	1	3.7
<i>ndom</i>	1	1	3	1	1	1	1	1	1	1	1	1	1.2
<i>rand_2</i>	1	1	1	1	1	1	1	1	2	1	1	1	1.1
<i>greedy_2</i>	1	1	1	1	1	1	1	1	2	1	1	1	1.1

Wilcoxon signed ranked tests, Friedman post-hoc analysis  
200 runs/dataset

# Experimental Results

## Long Term Learning Rankings

Approach	Instance ( $N, M$ )												Avg.
	20			50			100			200		500	
	5	10	20	5	10	20	5	10	20	10	20	20	
rand_3	4	4	2	4	4	4	4	4	4	4	4	3	3.8
rand_lt1_50	3	1	2	1	1	1	3	3	3	2	3	3	2.2
rand_lt1_20	1	1	2	1	1	1	1	1	1	2	2	2	1.3
rand_2	1	1	1	1	1	1	1	1	1	1	1	1	1
greedy_3	1	1	1	1	4	4	4	4	4	4	4	3	2.9
greedy_lt1_50	1	1	1	1	1	1	3	3	3	3	2	3	1.9
greedy_lt1_20	1	1	1	1	3	1	1	1	1	1	2	2	1.3
greedy_2	1	1	1	1	1	1	1	1	1	1	1	1	1

Wilcoxon signed ranked tests, Friedman post-hoc analysis  
200 runs/dataset

# Wrap-up

## Conclusions

- ▶ Very good strategies are essential
- ▶ Good strategies don't hurt control
- ▶ Bad strategies can be identified during the search

## Perspectives

- ▶ New strategies
- ▶ New complex control mechanisms
- ▶ More than a single strategy