Higher Dimensional Timed Automata

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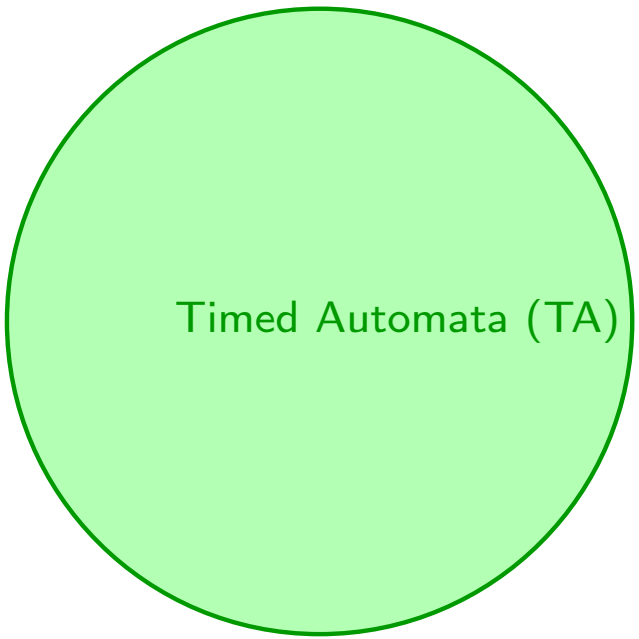
14th of December 2023




Motivation



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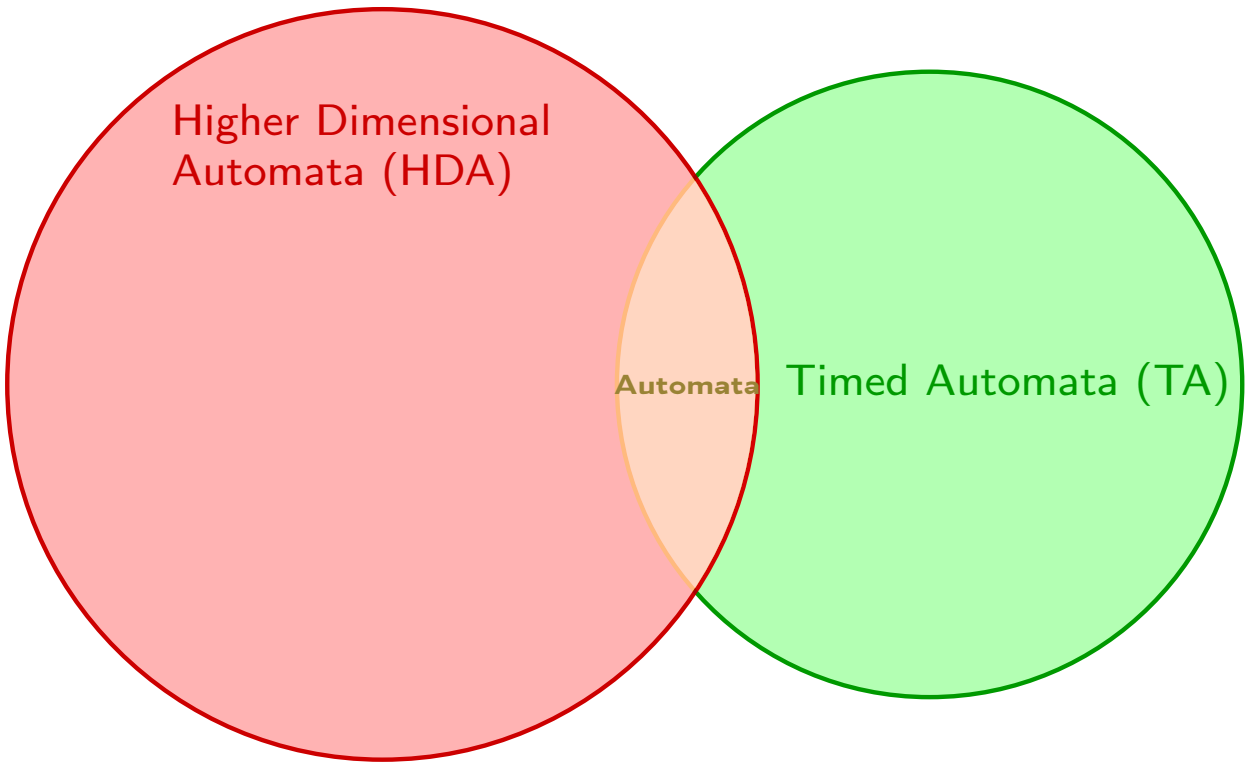
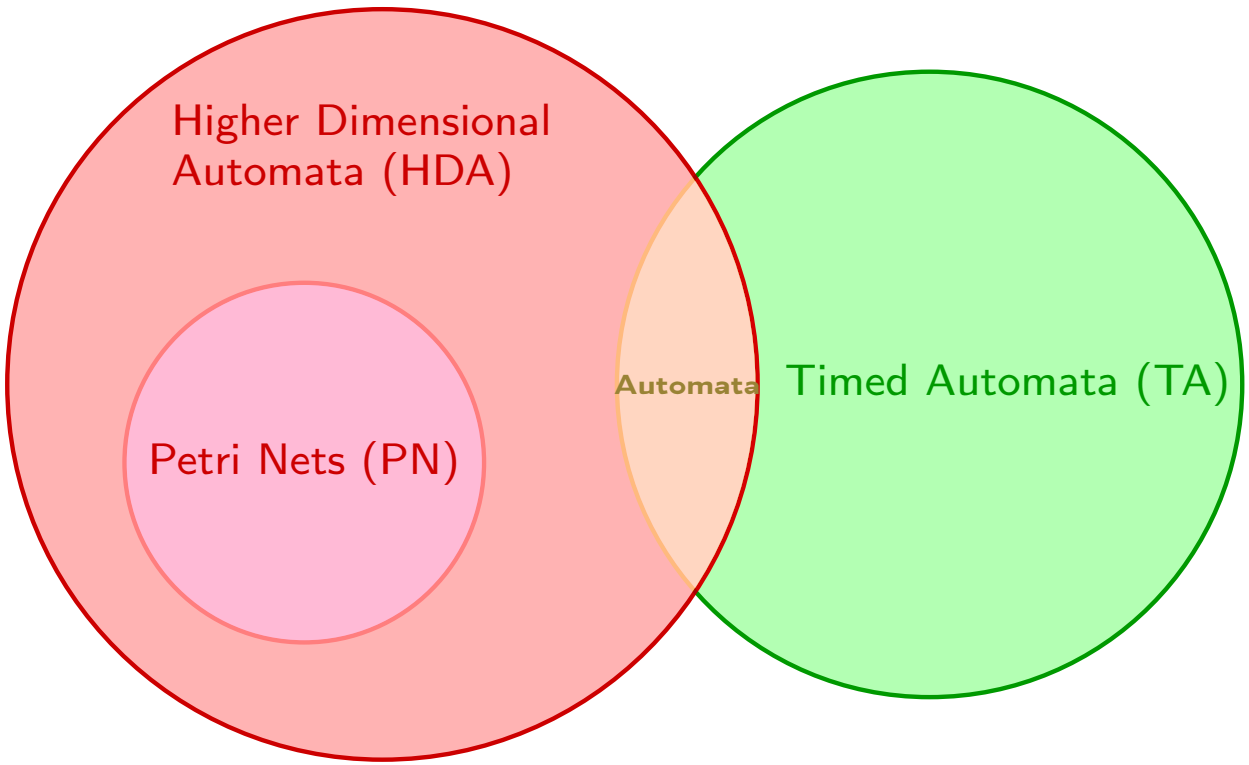

Higher Dimensional
Automata (HDA)
Automata
Timed Automata (TA)

Motivation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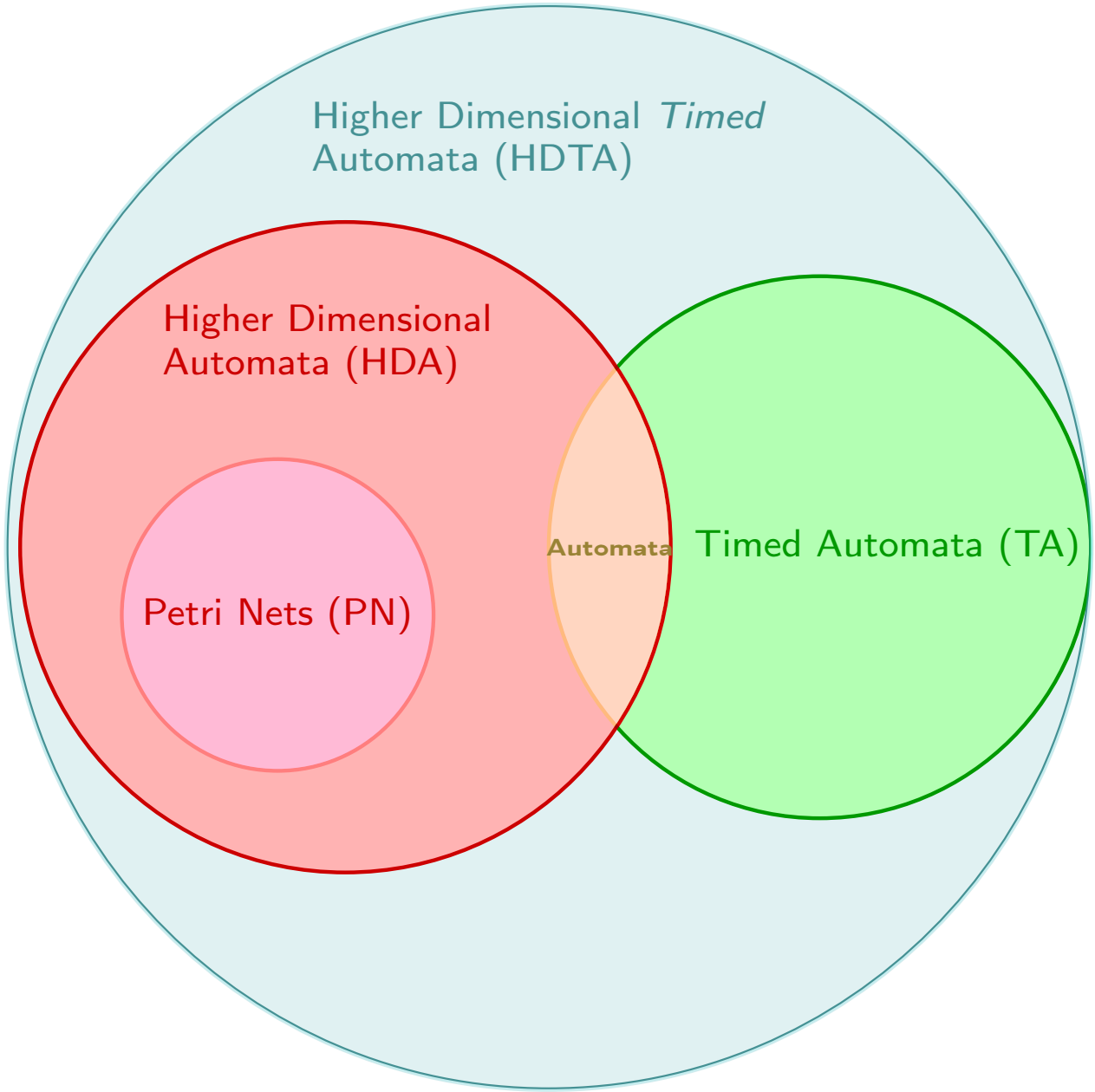
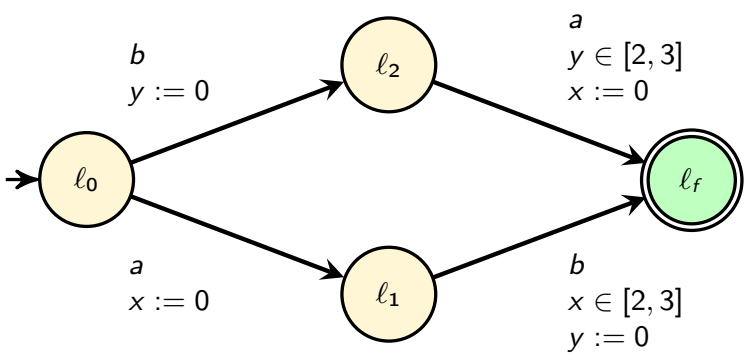

Higher Dimensional *Timed* Automata (HDTA)
Higher Dimensional Automata (HDA)
Petri Nets (PN)
Automata
Timed Automata (TA)

- Timed Automata
  - **Alur & al**: 1994
  - **Application**: synchronous real-time systems
  - **Decidable Problems**: Reachability, Emptriness, LTL MC PSPACE-Complete...
  - **Undecidable Problem**: Universality.



Models and applications

- Timed Automata
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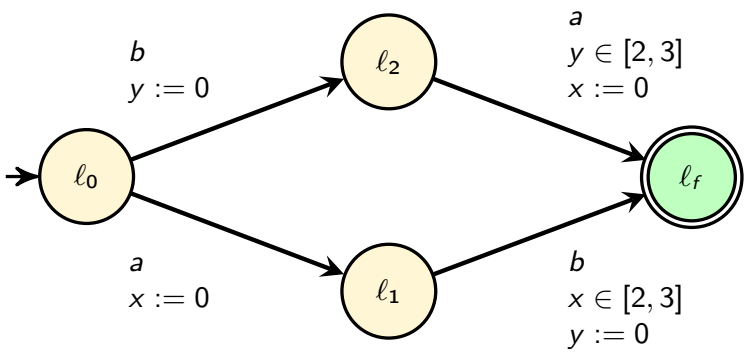
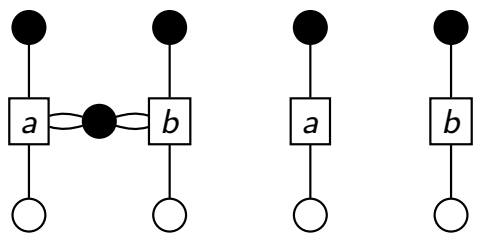
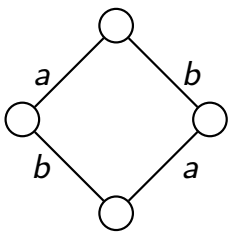


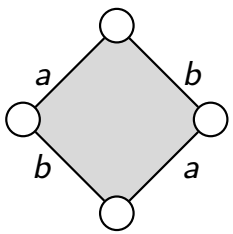
- Petri Nets
  - **Petri**: 1962
  - **Applications**: Discrete events, concurrency.



Our goal/motivation

- Concurrency in Automata and their variants
  - **Timed Automata/Automata**: $b.a + a.b$.
  - **Higher dimensional Automata**: $a||b$.





- Goal: model Time duration of events & concurrency

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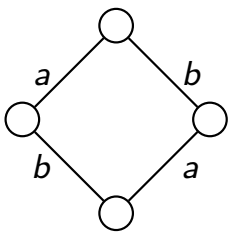

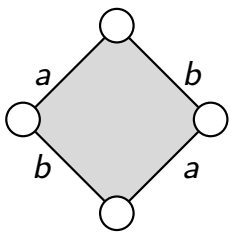

- Goal: model Time duration of events & concurrency

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Ex: distributed cyber-physical systems

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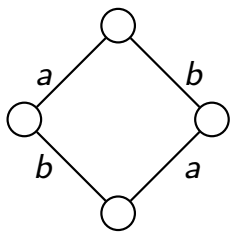
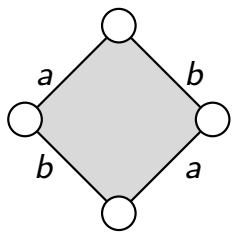




- Goal: model Time duration of events & concurrency

- Application: Real-time concurrent systems

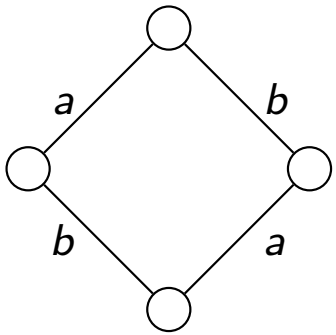
Ex: distributed cyber-physical systems

- Method: Higher Dimensional Timed Automata

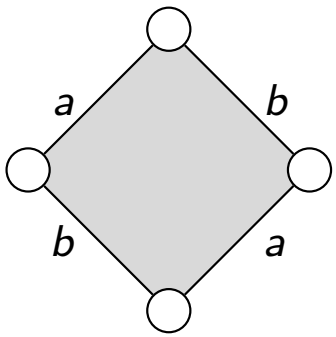
Augment Higher Dimensional Automata with **time duration** of events.

Higher Dimensional Automata: Intuition

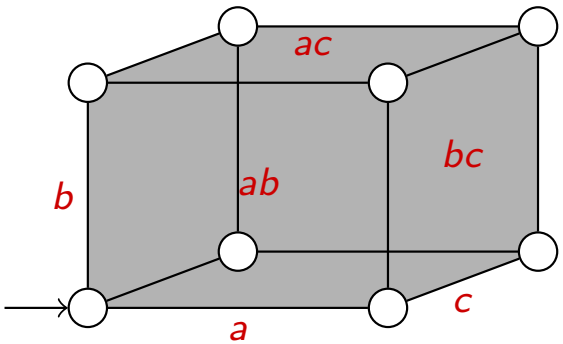
- Examples with two events
  - $a.b + b.a$:



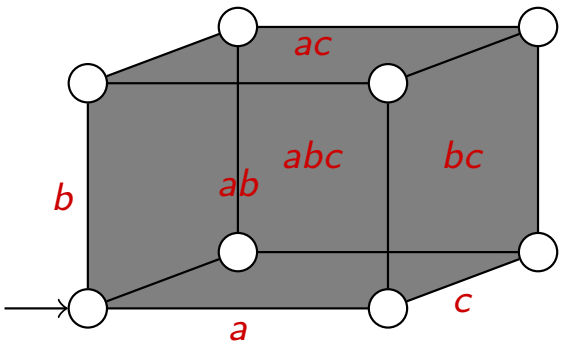
  - $a||b$:



- Examples with three events
  - $a||b + b||c + a||c$:



  - All events independent:

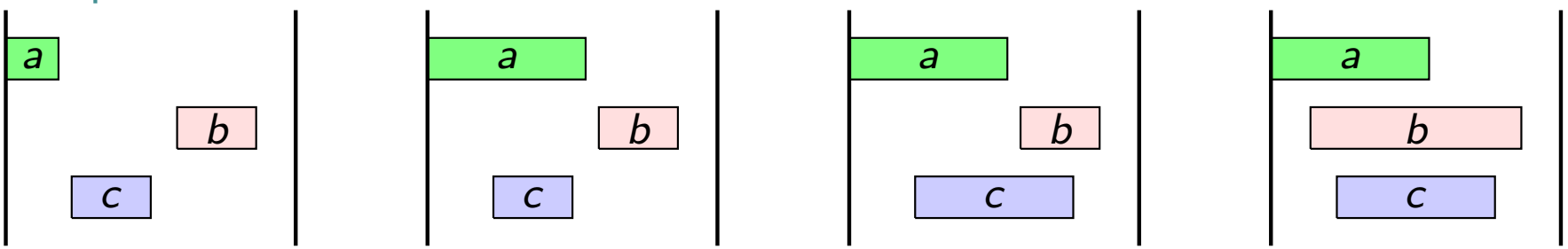


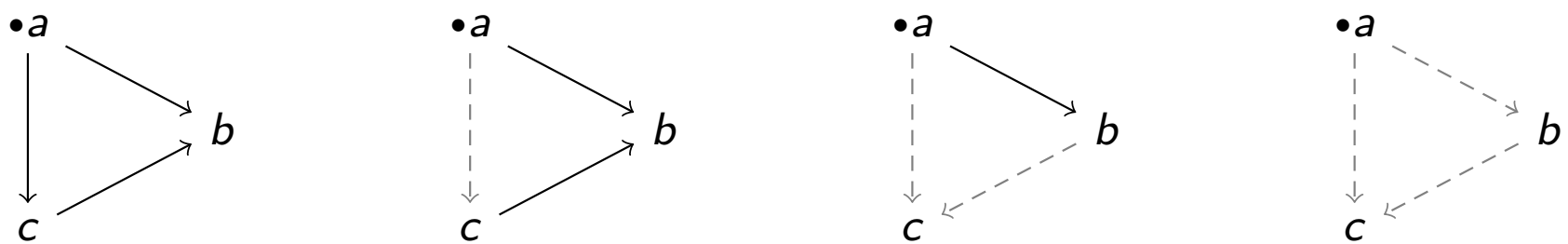
Events representation: pomset with interface

- Two partial order events
  - $<$: precedence order (rep with $\longrightarrow$)
  - $\dashrightarrow$: event order.
  - $< \cup \dashrightarrow$: **total**.
- Interfaces

Source/Target interfaces: S/T: $<$ −minimal/maximal.

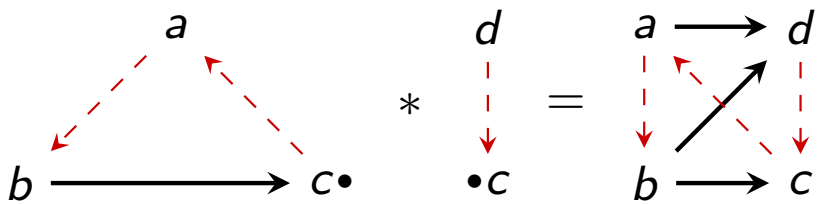
- Representation of events as interval



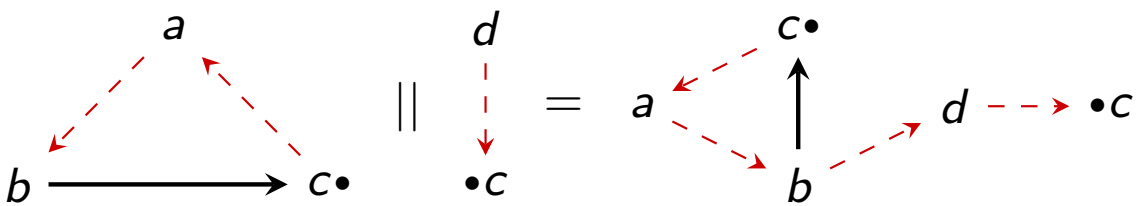


Example of operations on ipomsets

- Gluing composition:



- Parallel composition



Definition of Higher Dimensional Automata

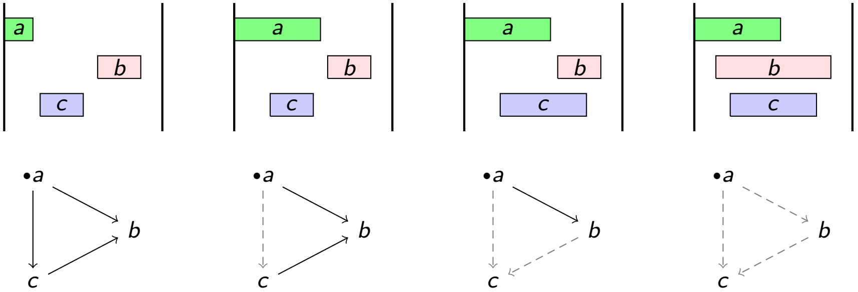
- An Higher Dimensional Automata A:
  - A tuple $(X, X_\perp, X_\top)$ where X is a finite precubical set, $X_\perp$ (*resp.* $X_\top$) $\subseteq X$ a start (*resp.* accept) cell and λ a labelling function.
- List of events
  - a conclist (concurrent list): a finite, totally ordered ($\dashrightarrow$) $\Sigma-$labelled set.
- Precubical set X:
  - a set of cells X, each $x \in X$ associated with a conclist $ev(x)$ (a list of active events in cell x)
  - $X[U] = \{x \in X | ev(x) = U\}$ represents the cell of type U, for a conclist U.
  - For each conclist U and $A \subseteq U$, we can define: lower\upper face map ($\delta^0_A \backslash \delta^1_A$): that represents unstarting\terminating events A:

$$\delta^0_A : X[U] \to X[U - A], \delta^1_A : X[U] \to X[U - A]$$

  - Respects the identity:

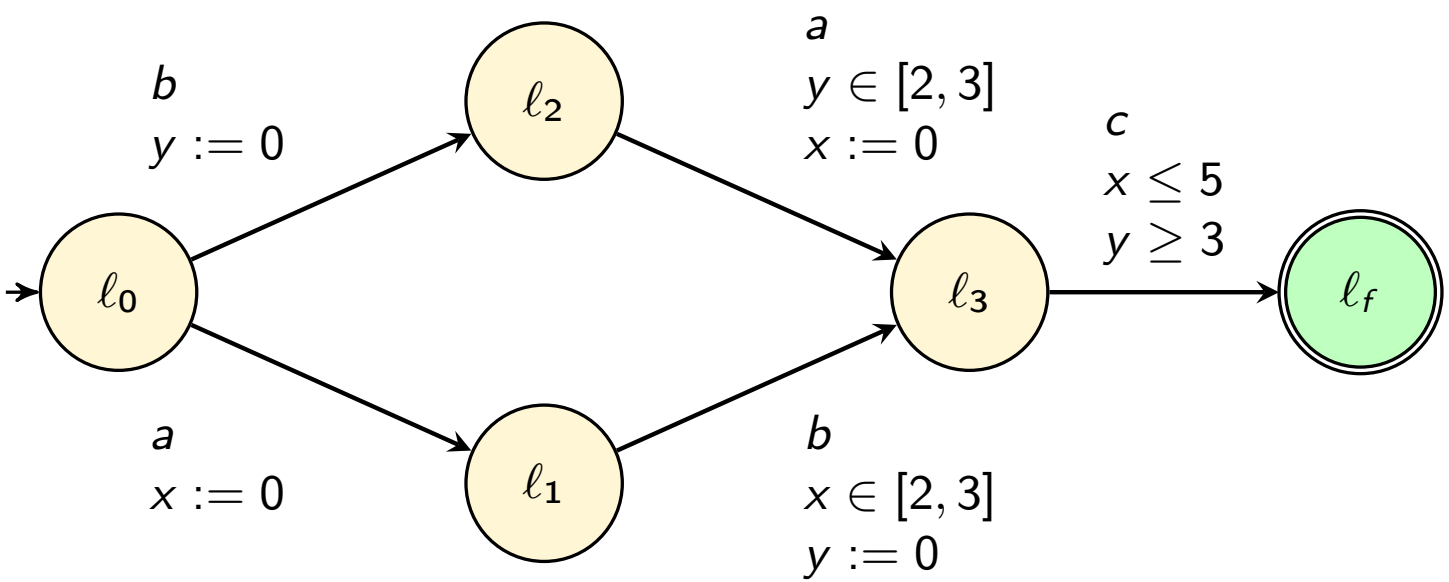
$$\boxed{\delta^\mu_A \delta^\nu_B = \delta^\nu_B \delta^\mu_A \ : \ \forall A, B \ s.t. \ A \cap B = \varnothing, \forall \mu, \nu \in \{0, 1\}}$$

What about time?



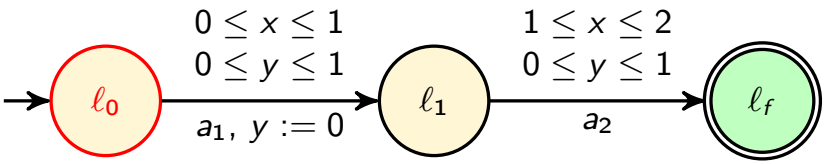
Timed Automata: example of scheduling

- Example of Scheduling of events a, b, c
  - Time constraints impose that between event a and b, at least (*resp.* at most) 2 (*resp.* 3) time units elapses
  - Resets ($x := 0$) of clocks

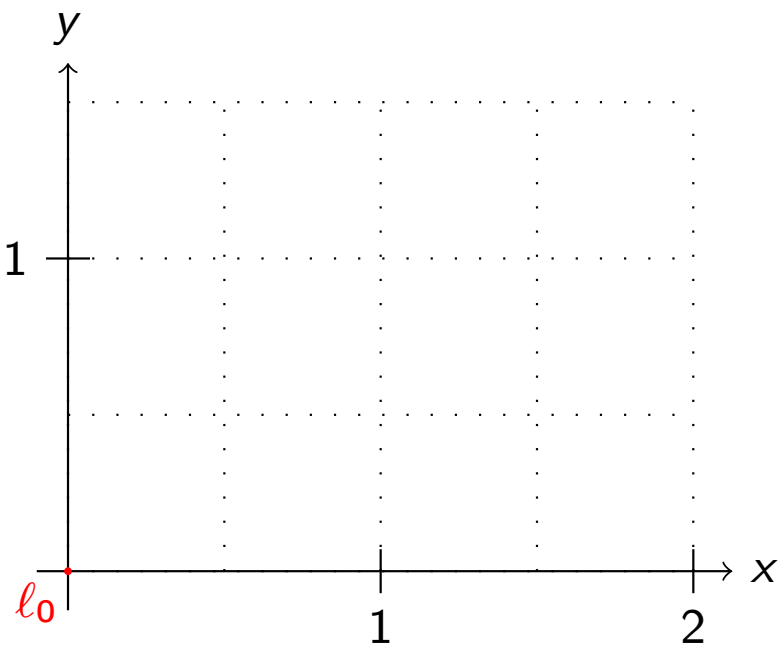


Clocks evolution example

- Timed automaton $\mathcal{A}$:



- Evolution of clocks x and y during the run



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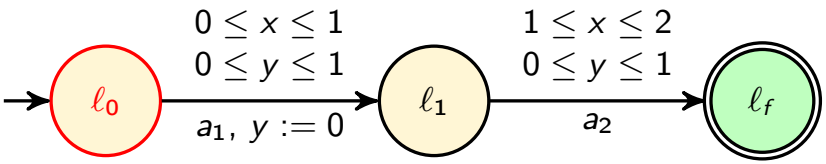

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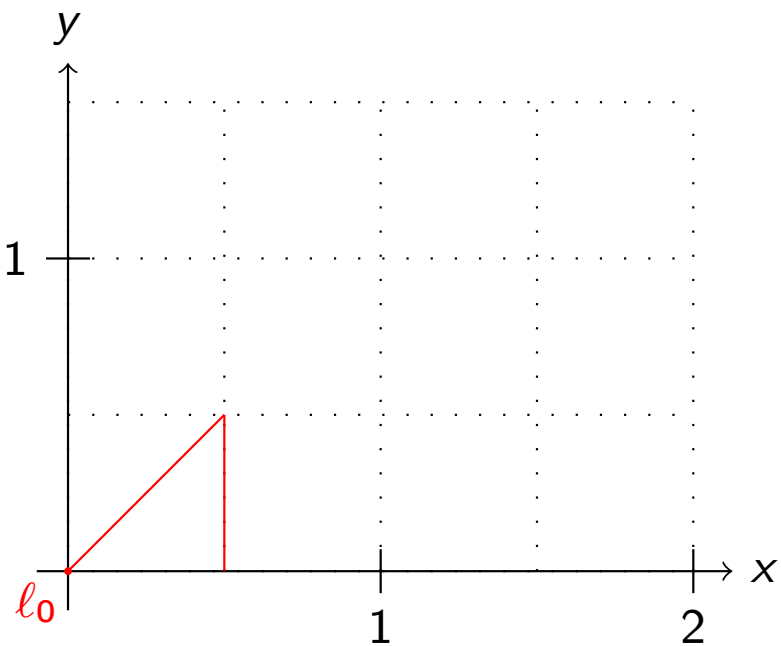

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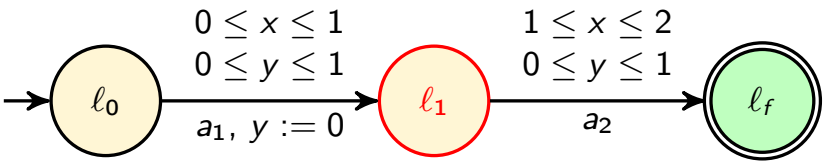

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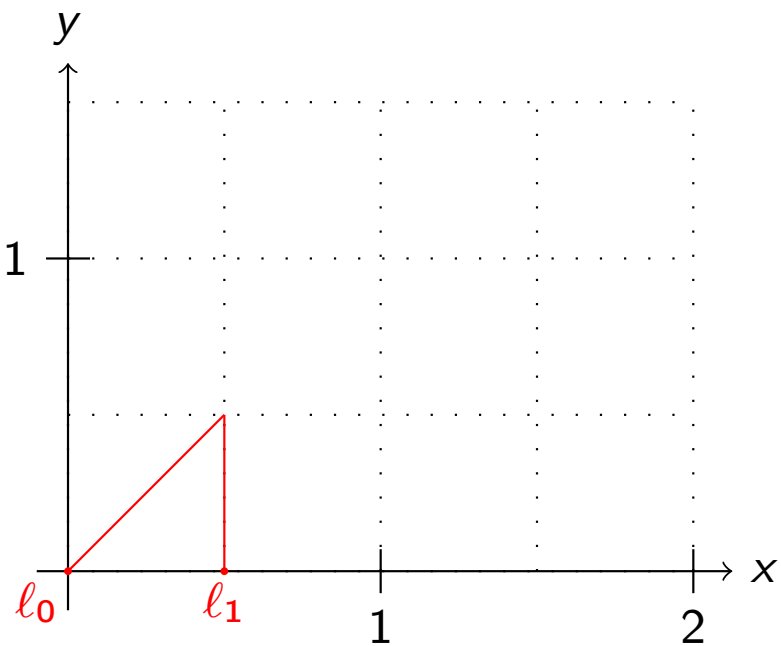

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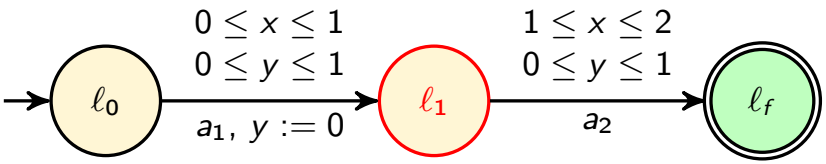

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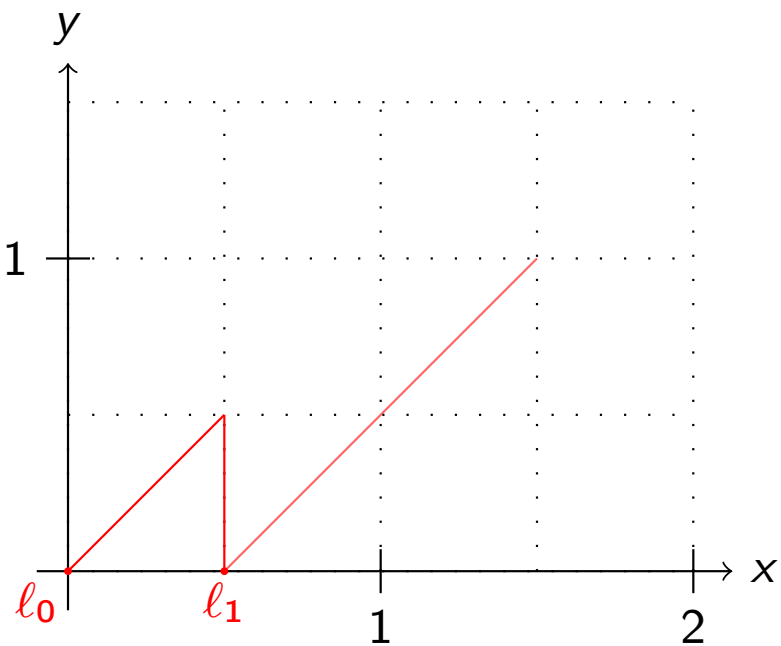

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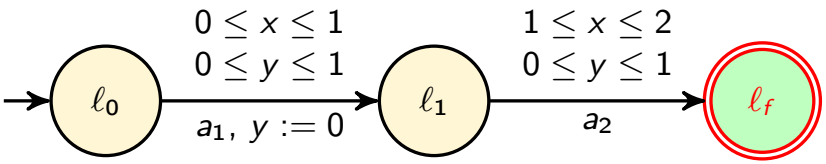

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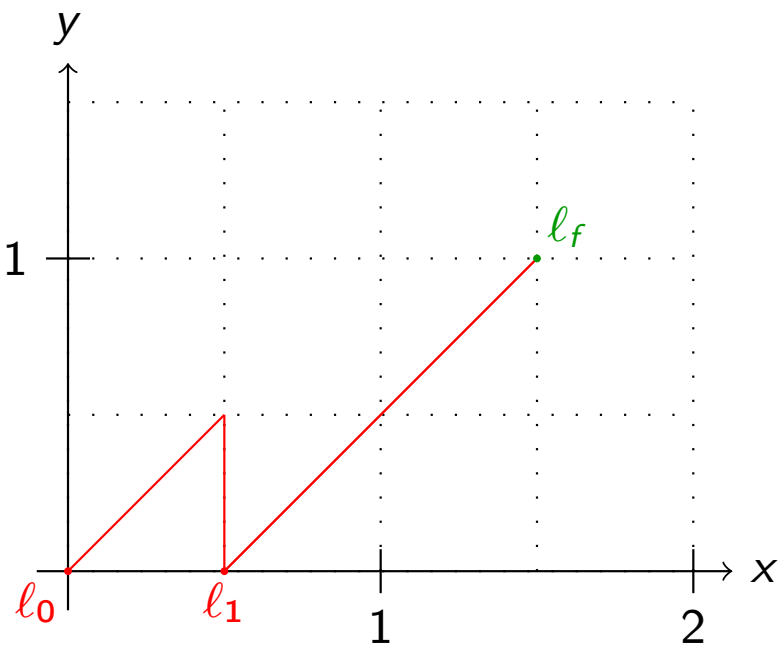
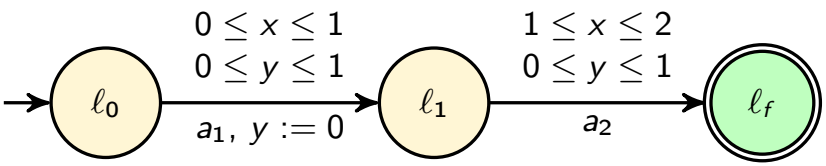


Events: semantics of transitions



- Delay-based transitions

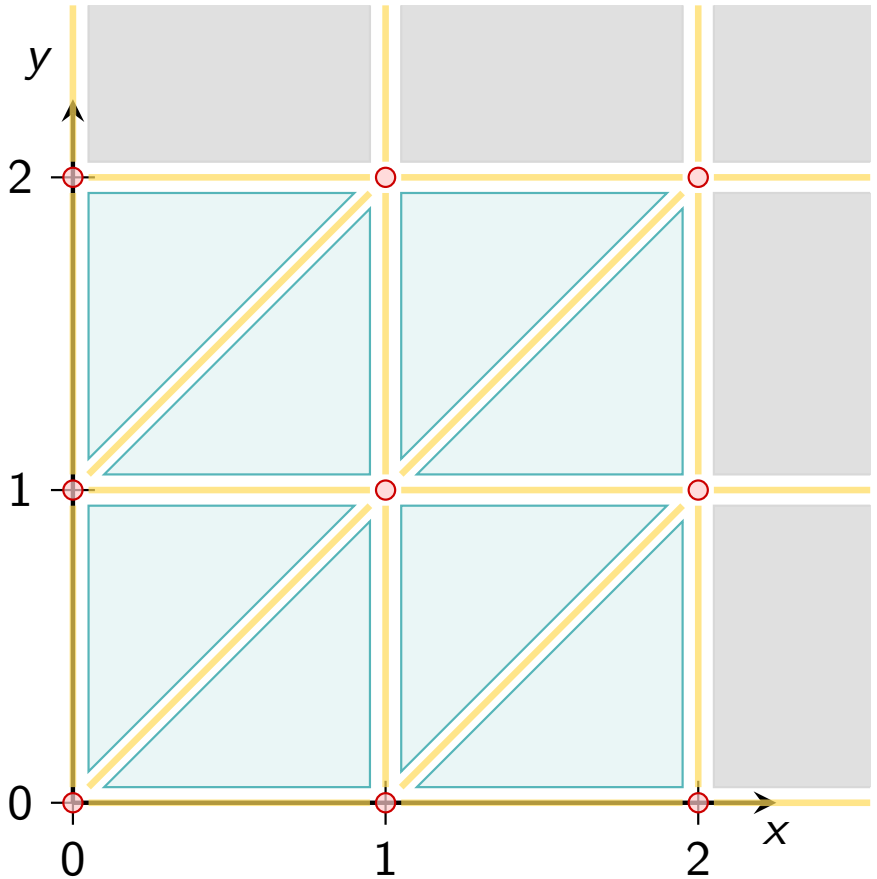
$$(\ell, v) \xrightarrow{\delta} (\ell, v + \delta)$$

- Action-based transitions

$$(\ell, v) \xrightarrow{a_1} (\ell_1, v[y := 0])$$

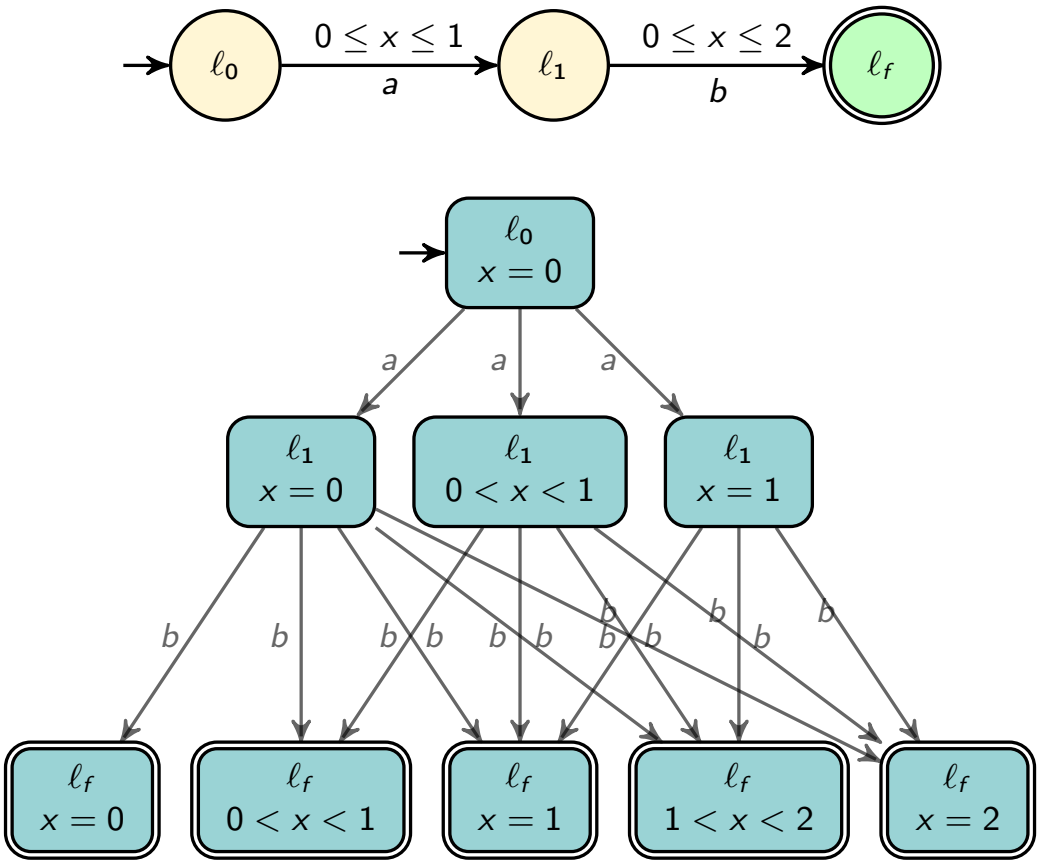
Region

- Ex: Region of the constraint $0 \leq x, y \leq 2$



Region Automaton: example for one-clock TA

- A timed Automaton and its region automaton

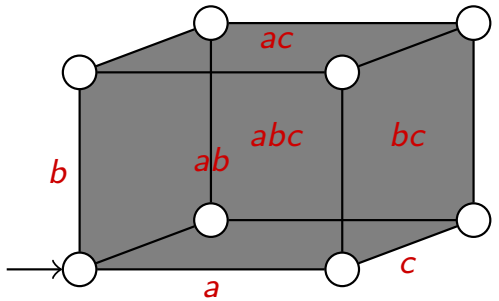


- Reachability problem for TA

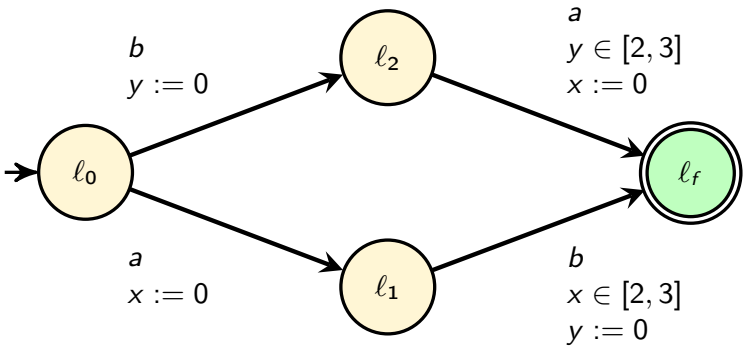
PSPACE (Alur et al, 1994): correspondance between runs of TA and the one of the corresponding region automata.

Motivation

- Higher Dimensional Automata:
  - Events can happen in parallel, but...
  - No timing duration information nor timing constraints



- Timed Automata
  - Timing constraints, but...
  - Instantaneous events
  - interleaving concurrency



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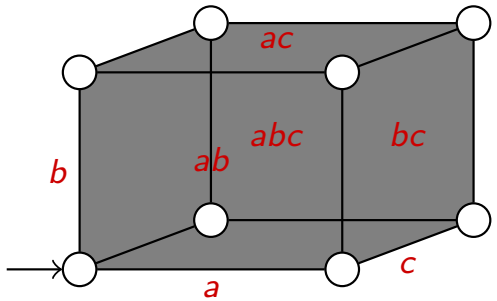

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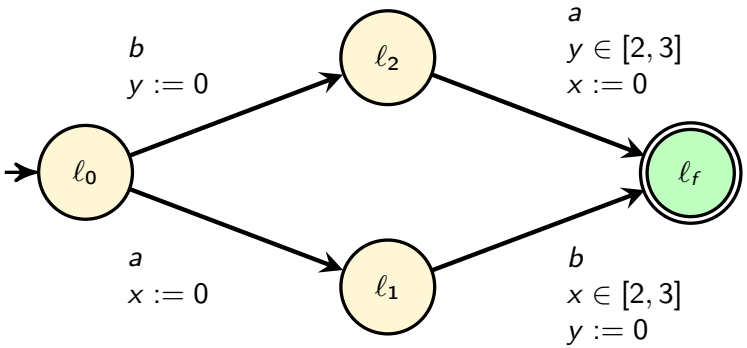
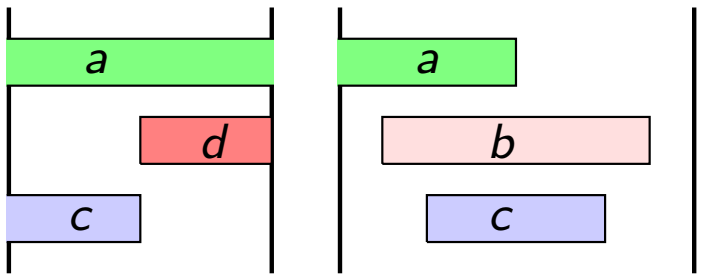


- Our goal: study of Higher Dimensional *Timed* Automata
  - Represent timing constraints
  - Consider timing duration of events

Represent ipomset with timing information

- Timed ipomsets is composed of:
  - An ipomset
  - A duration
  - A map labelling all events to time intervals

- Ipomsets in HDA:



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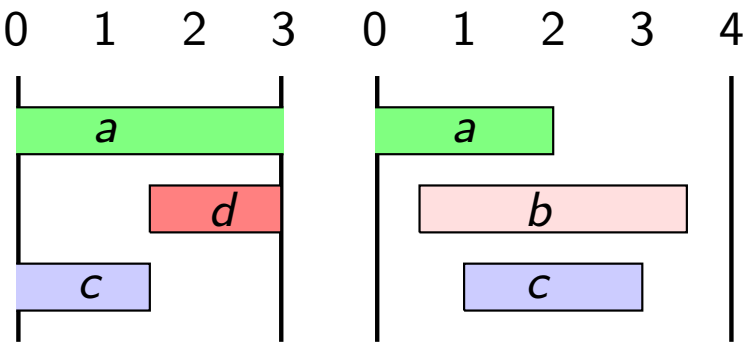
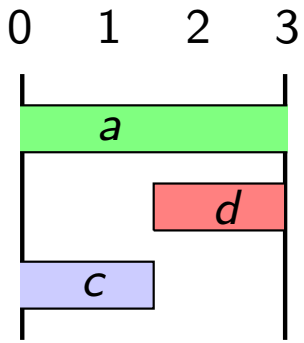
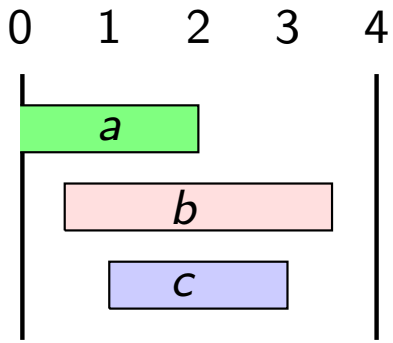


- Example of Timed Ipomset:
  - ipomset: $(\{x_1, x_2, x_3\}, <, x_1 \dashrightarrow x_2 \dashrightarrow x_3, \{x_1, x_3\}, \{x_2\}, \lambda_1)$
  - $\lambda(x_1) = a, \lambda(x_2) = d, \lambda(x_3) = c$
  - $d = 3$
  - $\sigma(a) = (0, 3), \sigma(c) = (0, 1.5), \sigma(d) = (1.5, 3)$

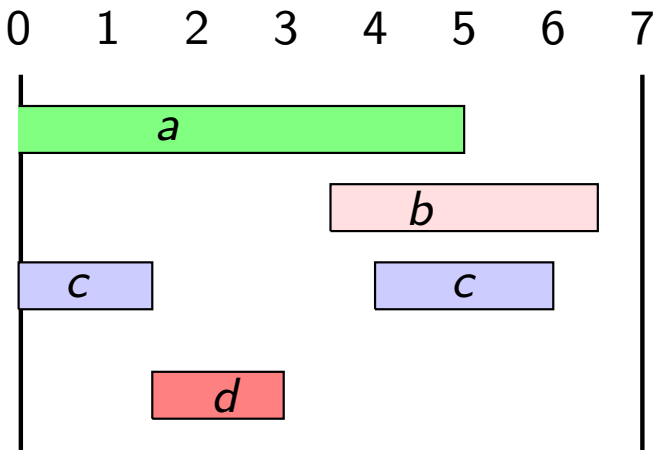
Operation on Timed Ipomsets

- Timed ipomsets T_1, T_2:





- Gluing of T_1 and T_2



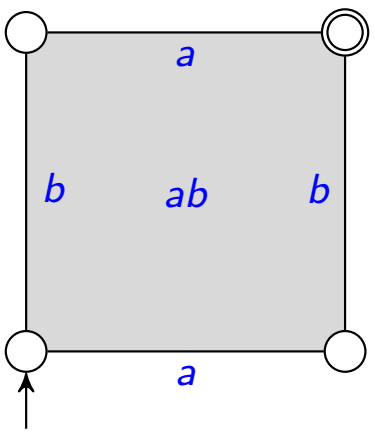
Higher Dimensional Timed Automata: definition

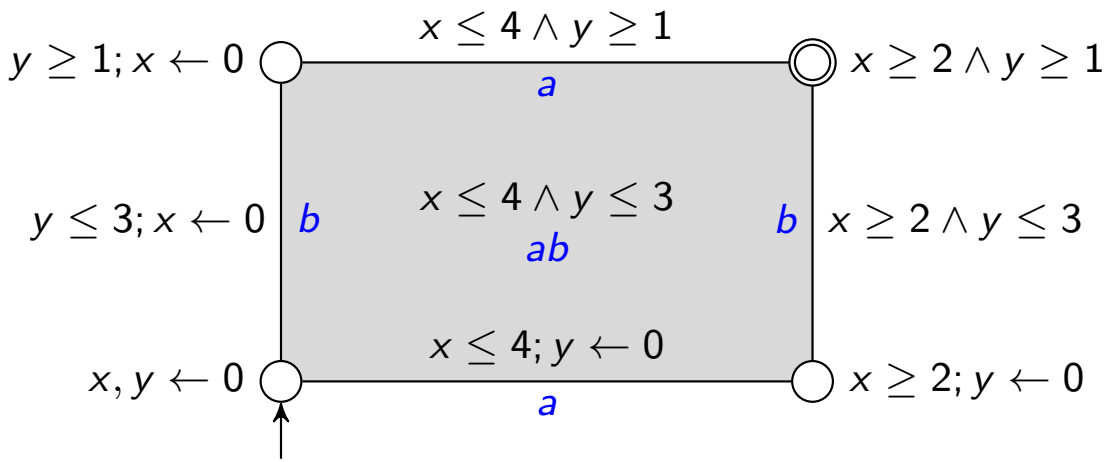
- Definition:

A HDTA is a tuple $(X, X_\perp, X_\top, \lambda, \mathcal{C}, \text{inv}, \text{exit})$ where:

  - $(X, X_\perp, X_\top, \lambda)$ is an HDA
  - $\mathcal{C}$: set of clocks
  - $\text{inv} : X \to \phi(\mathcal{C})$ (*resp.* $\text{exit} : X \to 2^{\mathcal{C}}$) assign invariant (*resp.* exit) conditions to cells.

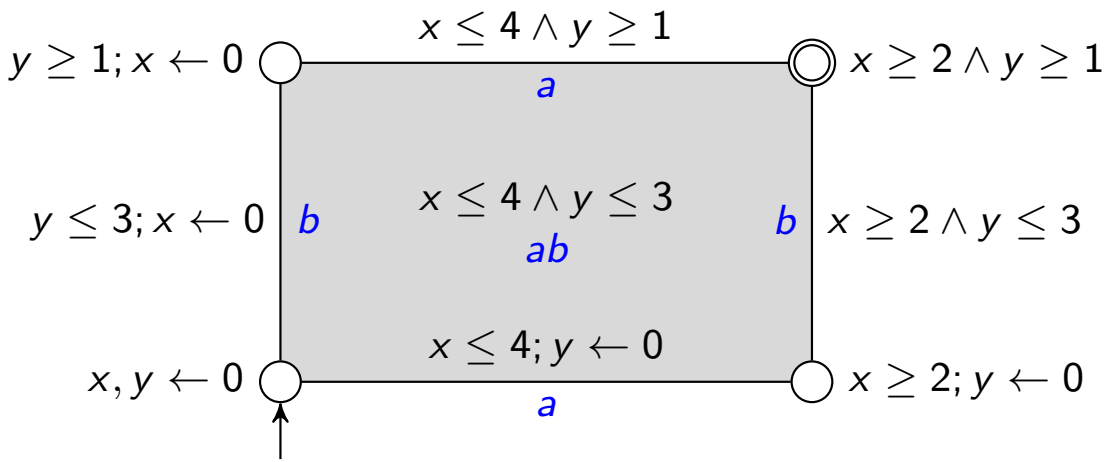
- Example with events a and b: HDA (left) of the HDTA (right)





Examples of HDTA

- First example: adding timing duration to events

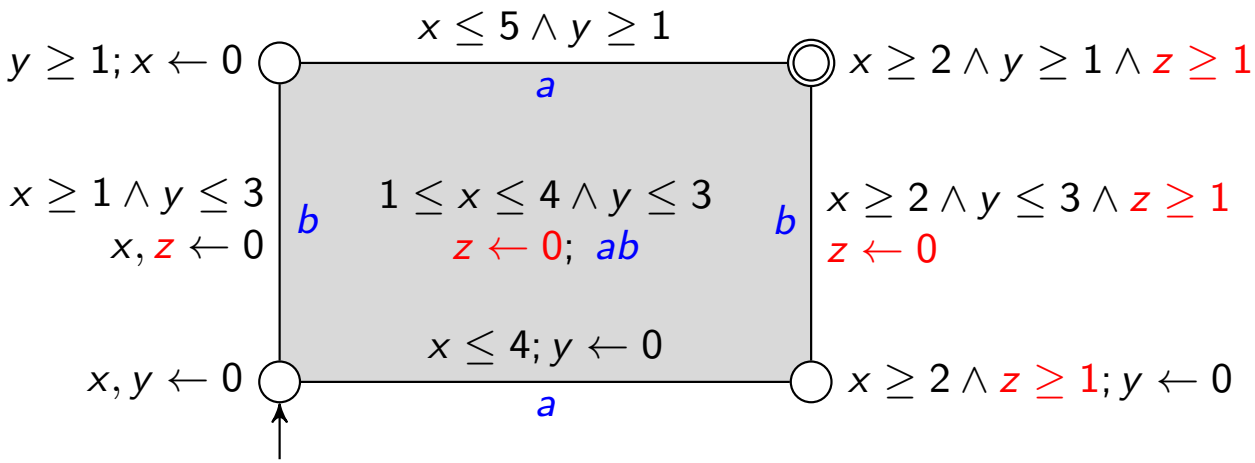


Timing duration of events:

- a: $[2, 4]$ time units
- b: $[1, 3]$ time units

Example of HDTA

- Second example: adding timing constraints between events...



Timing duration of events:

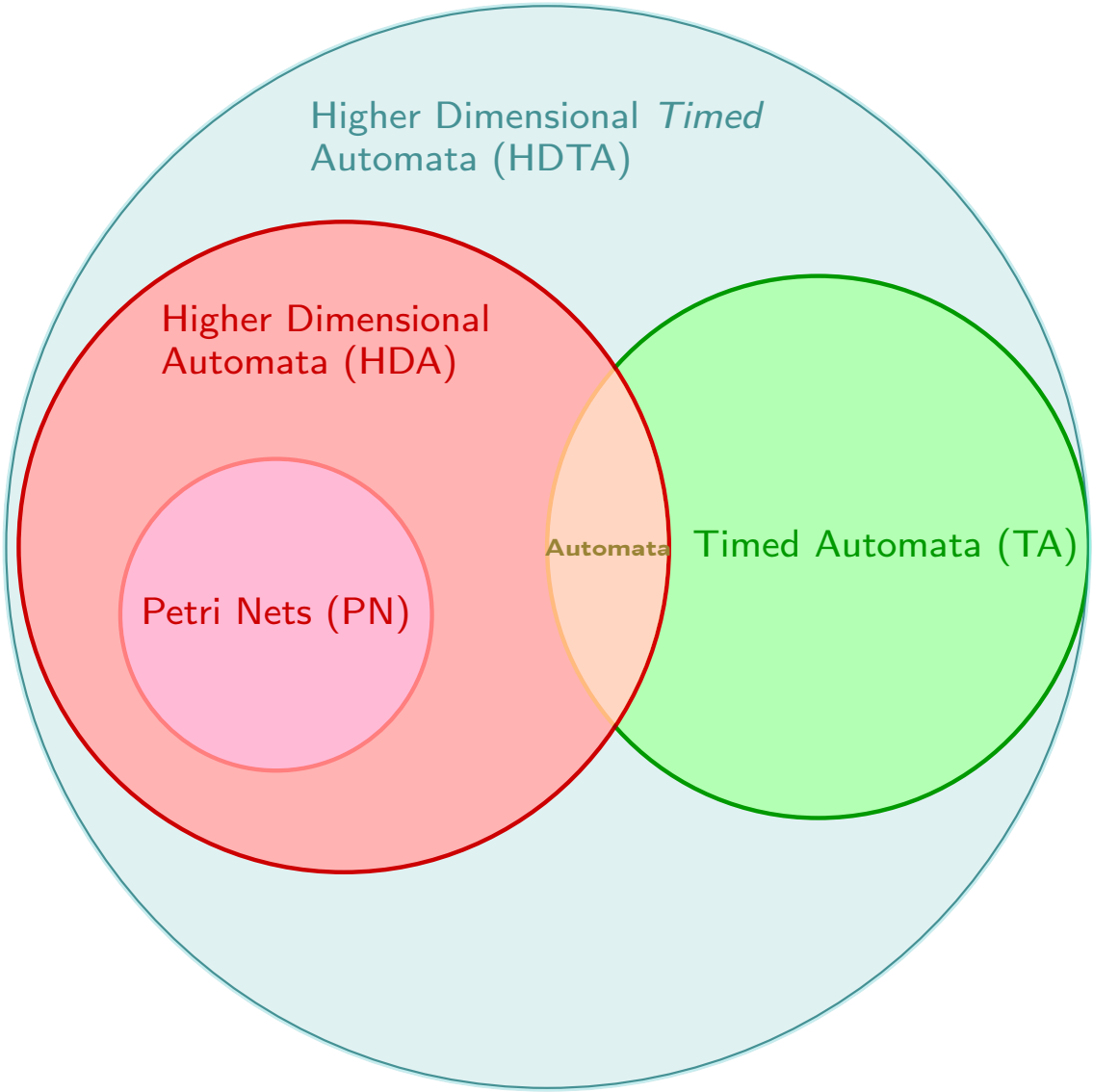
- a: $[2, 4]$ time units
- b: $[1, 3]$ time units

Constraints between starting/ending dates

- 1 time unit should elapse between b's starting date and a's starting date
- 1 time unit should elapse between b's ending date and a's ending date

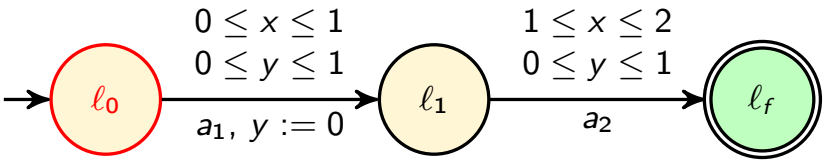
Current work: Extending properties from other models

- Language generated by HDTA
  - **TA**: timed words $((a, 0.7), (b, 1.5), (c, 2.0))...$
  - **HDA**: labeled interval orders
- Region HDTA
  - **TA**: Region Automata
- Timed bisimulation
- Future work: Robustness?

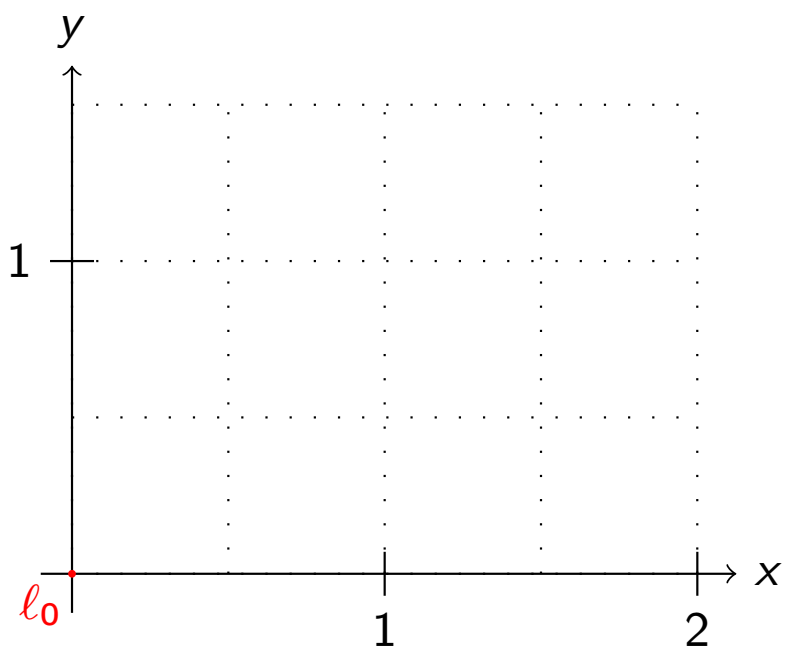


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

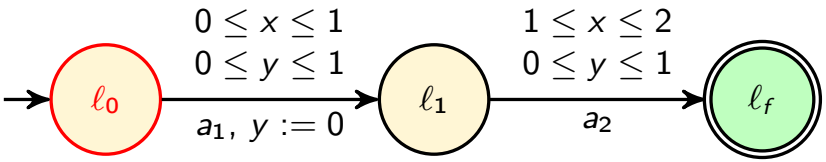


- Run with delay perturbations of at most $\delta = 0.2$

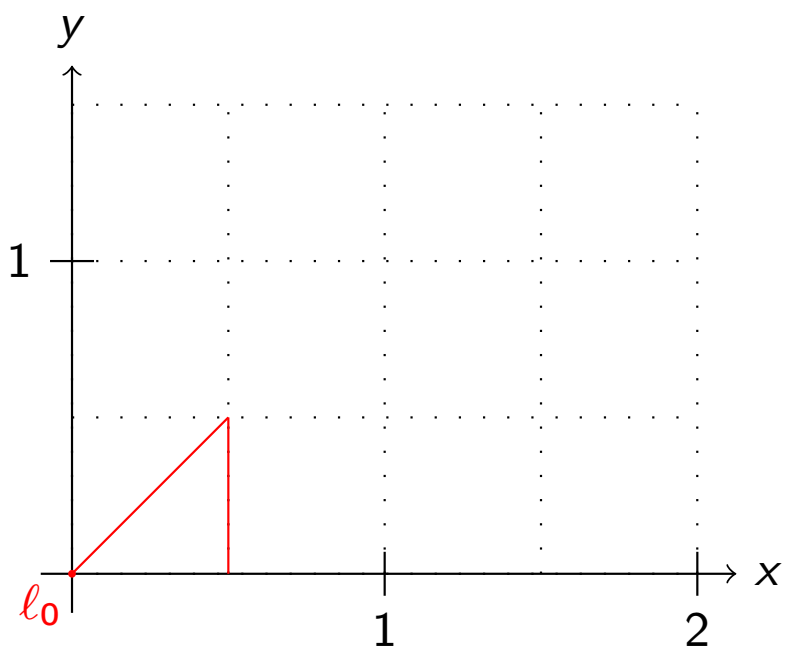


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

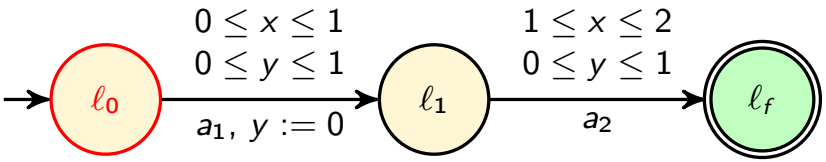


- Run with delay perturbations of at most $\delta = 0.2$

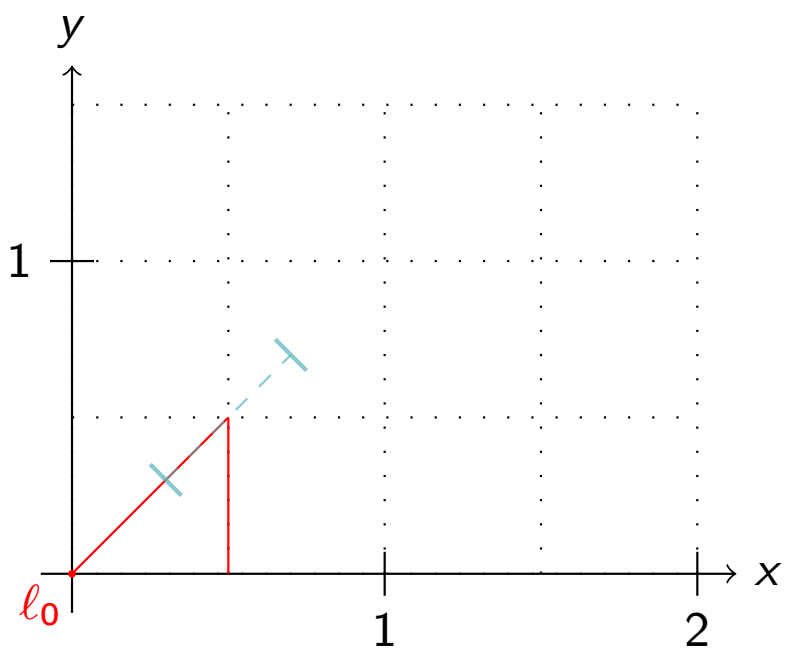


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

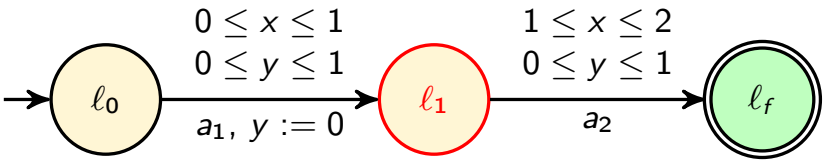


- Run with delay perturbations of at most $\delta = 0.2$

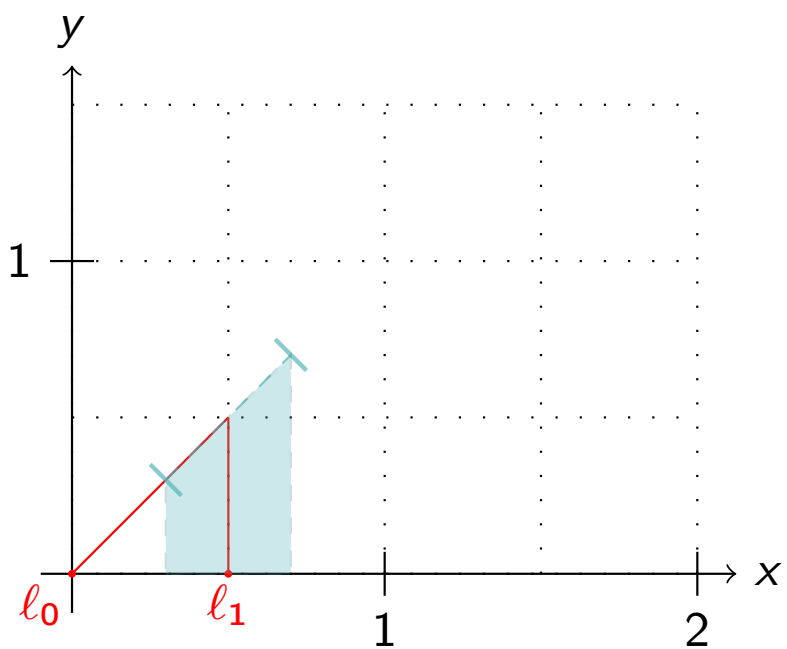


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

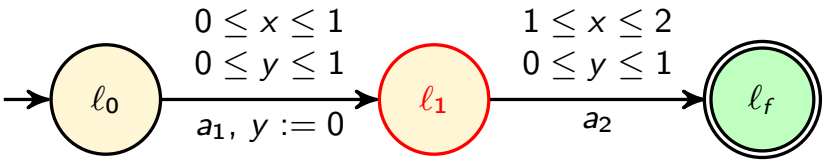


- Run with delay perturbations of at most $\delta = 0.2$

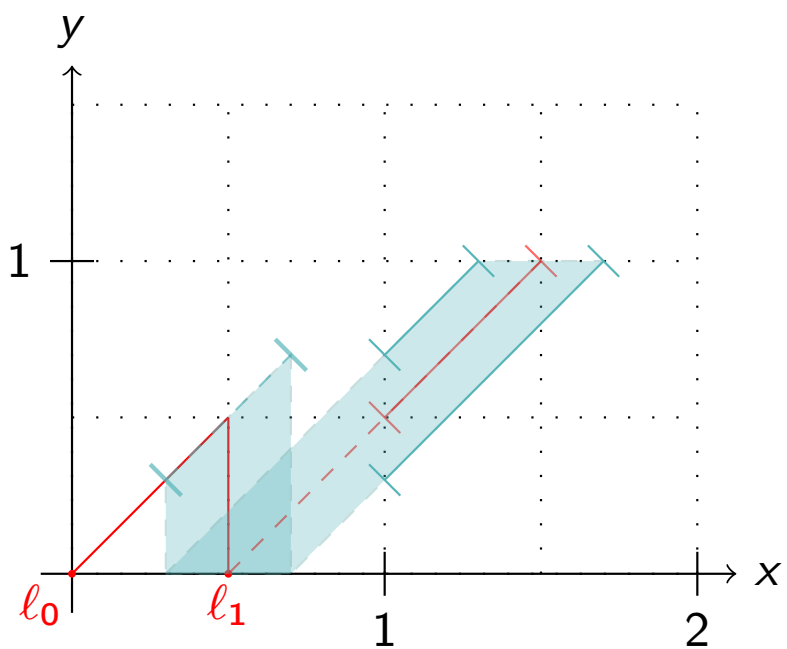


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

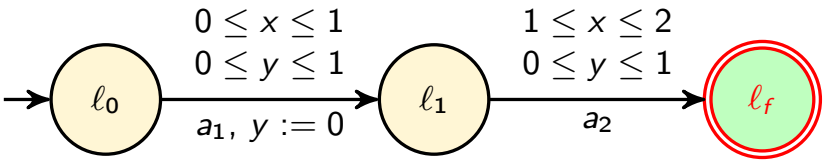


- Run with delay perturbations of at most $\delta = 0.2$

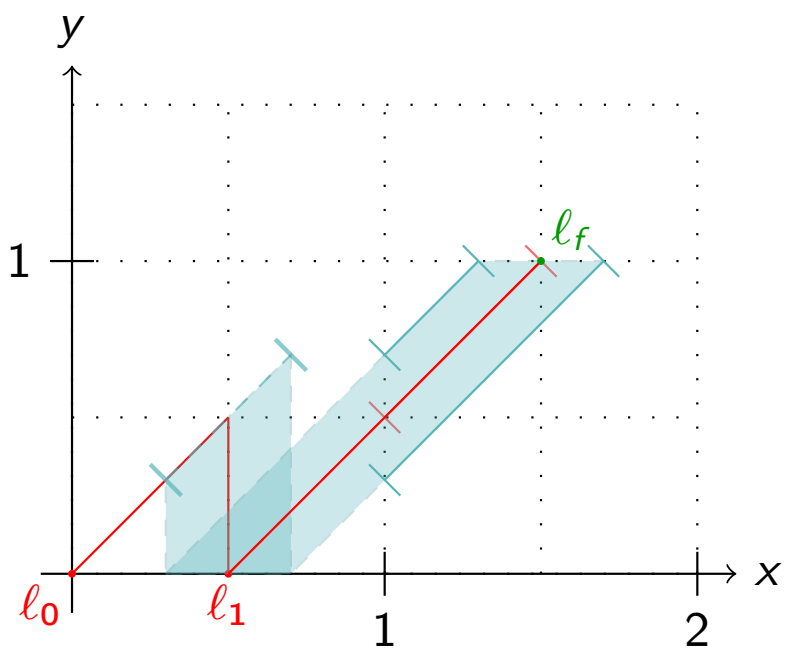


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

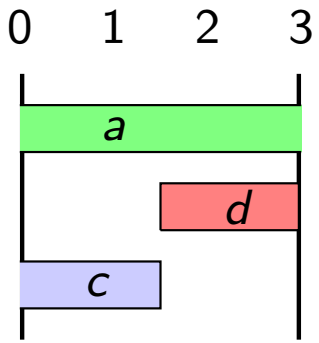


- Run with delay perturbations of at most $\delta = 0.2$



Future work: robustness on HDTA

- No timing perturbation: c and d are not in concurrency



- timing perturbation. Let us introduce a 0.1 delay on the end date of c: