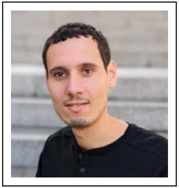
Languages of Higher-Dimensional Timed Automata

Amazigh Amrane [2] *Hugo Bazille* [2] ***Emily Clement*** [1] *Uli Fahrenberg* [2]

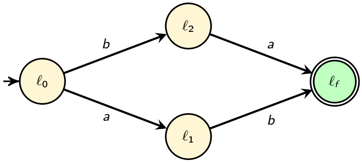
[1]Université Paris Cité, CNRS, IRIF, F-75013, Paris, France
[2]EPITA Research Laboratory (LRE), Paris, France

26th of February 2024

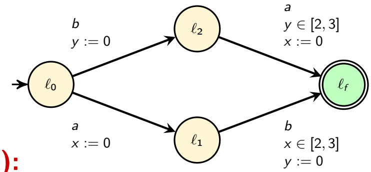



Motivation: how to represent event concurrency?

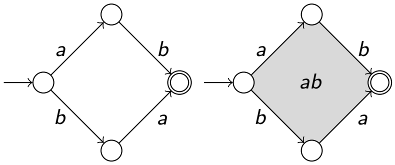
- **Automata:**
  - ▷ Inverleaving concurrency
  - ▷ No information on duration of events.



- **Timed Automata[a] (TA):**
  - ▷ Timing constraints
  - ▷ Instantaneous events
  - ▷ Interleaving concurrency.

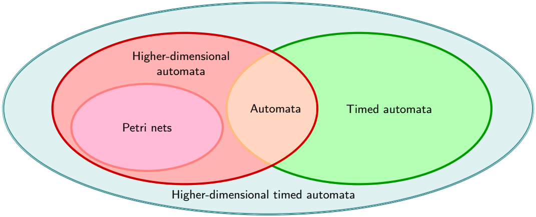


- **Higher Dimensional Automata[b] (HDA):**
  - ▷ $a \cdot b + b \cdot a$ (left) and both $a \cdot b + b \cdot a$ and $a||b$ (grey) (right)
  - ▷ No information on timing constraints or duration of events.



[a] Alur and Dill, 'A Theory of Timed Automata', 1994.
[b] Pratt, 'Modeling Concurrency with Geometry', 1991.

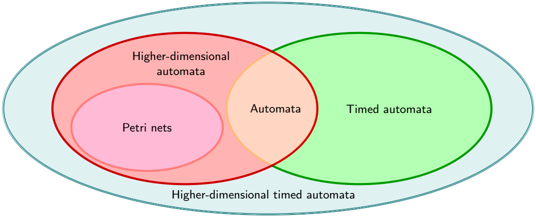
Higher Dimensional Timed Automata[1] (HDTA)



- **Timed Automata** (TA): model **Timing constraints** with set of clocks.
- **Higher Dimensional Automata** (HDA): differenciate **interleaving concurrency** and **simultaneous** events.

[1] Fahrenberg, 'Higher-Dimensional Timed and Hybrid Automata', 2022.

Higher Dimensional Timed Automata[1] (HDTA)

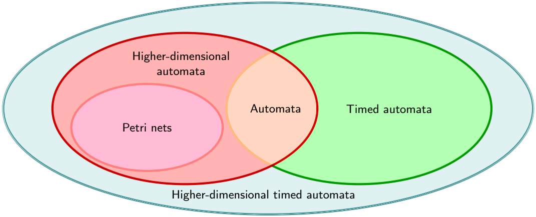


- **Timed Automata** (TA): model **Timing constraints** with set of clocks.
- **Higher Dimensional Automata** (HDA): differenciate **interleaving concurrency** and **simultaneous** events.

- Properties of HDTA
  - Event/Transition can have a **duration**
  - **Events** can occur **simultaneously**.

[1] Fahrenberg, 'Higher-Dimensional Timed and Hybrid Automata', 2022.

Higher Dimensional Timed Automata[1] (HDTA)



- ▷ **Timed Automata** (TA): model **Timing constraints** with set of clocks.
- ▷ **Higher Dimensional Automata** (HDA): differenciate **interleaving concurrency** and **simultaneous** events.

- Properties of HDTA
  - ▷ Event/Transition can have a **duration**
  - ▷ **Events** can occur **simultaneously**.
- Our Contribution
  - ▷ Express the **language of HDTA**.
  - ▷ Explain the **links between HDA, TA and HDTA**.
  - ▷ Extend some decidability/undecidability **TA results** to HDTA.

[1] Fahrenberg, 'Higher-Dimensional Timed and Hybrid Automata', 2022.

Summary of the talk

- Higher Dimensional Automata
  - ▷ **Example** of HDA
  - ▷ **Partial order of events**: Pomset with interface
  - ▷ **Definition** of HDA.

Summary of the talk

- Higher Dimensional Automata
  - ▷ **Example** of HDA
  - ▷ **Partial order of events**: Pomset with interface
  - ▷ **Definition** of HDA.
- Timed Automata
  - ▷ Example and **definition**
  - ▷ **Language** of Timed Automata.

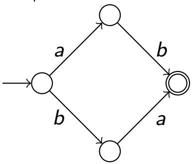
- Higher Dimensional Automata
  - ▷ **Example** of HDA
  - ▷ **Partial order of events**: Pomset with interface
  - ▷ **Definition** of HDA.
- Timed Automata
- Higher Dimensional Timed Automata
  - ▷ **Timed ipomset**
  - ▷ **Definition** and example of HDTA
  - ▷ **Language** of HDTA

Summary of the talk

- Higher Dimensional Automata
- Timed Automata
- Higher Dimensional Timed Automata
  - **Timed ipomset**
  - **Definition** and example of HDTA
  - **Language** of HDTA
- **Contribution**
  - **Language inclusion** for HDTA is **undecidable**
  - **Untimed language inclusion** for HDTA is **decidable**.
- Conclusion
  - Current and future work.

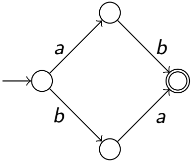
- Higher Dimensional Automata
  - ▷ **Example** of HDA
  - ▷ **Partial order of events**: Pomset with interface
  - ▷ **Definition** of HDA.

- Two-events HDA
  - $a.b + b.a$:

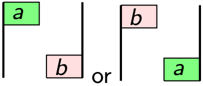


Higher Dimensional Automata: examples

- Two-events HDA
  - ▷ $a.b + b.a$:

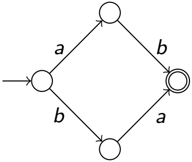


Example of traces:

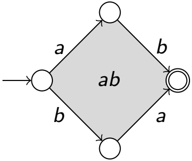


Higher Dimensional Automata: examples

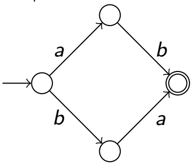
- Two-events HDA
  - $a.b + b.a$:



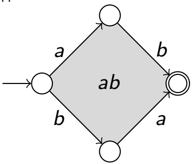
  - $a||b$:



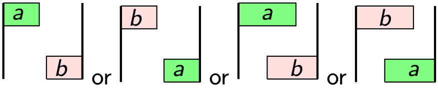
- Two-events HDA
  - $a.b + b.a$:



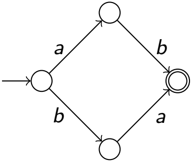
  - $a||b$:



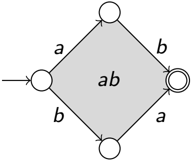
Example of traces:



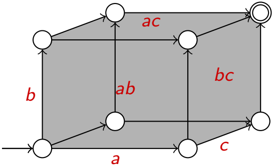
- Two-events HDA
  - ▷ $a.b + b.a$:



  - ▷ $a||b$:

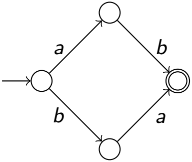


- Three-events HDA
  - ▷ $a||b + b||c + a||c$:

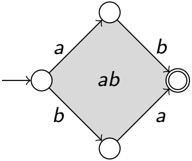


Higher Dimensional Automata: examples

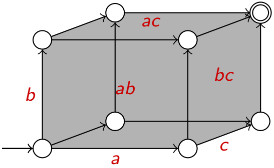
- Two-events HDA
  - ▷ $a.b + b.a$:



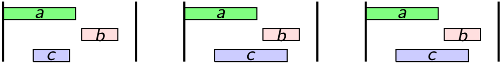
  - ▷ $a||b$:



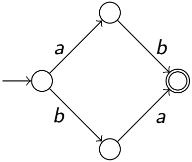
- Three-events HDA
  - ▷ $a||b + b||c + a||c$:



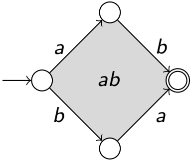
Examples of traces:



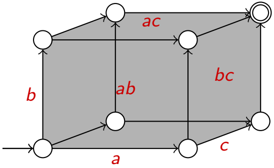
- Two-events HDA
  - ▷ $a.b + b.a$:



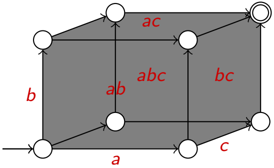
  - ▷ $a||b$:



- Three-events HDA
  - ▷ $a||b + b||c + a||c$:

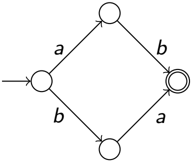


  - ▷ $a||b||c$:

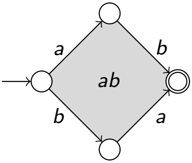


Higher Dimensional Automata: examples

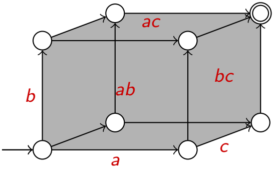
- Two-events HDA
  - ▷ $a.b + b.a$:



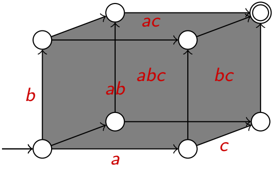
  - ▷ $a||b$:



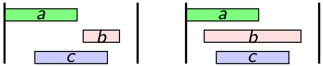
- Three-events HDA
  - ▷ $a||b + b||c + a||c$:



  - ▷ $a||b||c$:



Examples of traces:



Events representation: pomset with interfaces (ipomset)

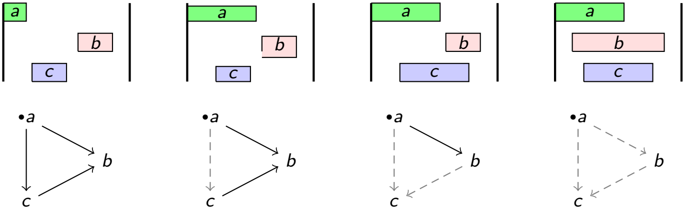
- Two partial order events
  - ▷ $<$: precedence order (rep with $\longrightarrow$) , $\dashrightarrow$: event order.
  - ▷ $< \cup \dashrightarrow$: **total** relation.

Events representation: pomset with interfaces (ipomset)

- Two partial order events
  - ▷ $<$: precedence order (rep with $\longrightarrow$) , $\dashrightarrow$: event order.
  - ▷ $< \cup \dashrightarrow$: **total** relation.
- Interfaces
  - ▷ **Source/Target interfaces**: S/T: $<$-minimal/maximal.

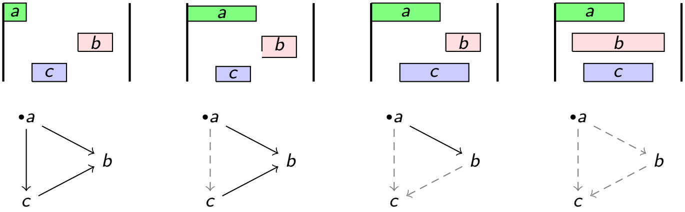
Events representation: pomset with interfaces (ipomset)

- Two partial order events
  - ▷ $<$: precedence order (rep with $\longrightarrow$) , $\dashrightarrow$: event order.
  - ▷ $< \cup \dashrightarrow$: **total** relation.
- Interfaces
  - ▷ **Source/Target interfaces**: S/T: $<$-minimal/maximal.
- Representation of events as interval

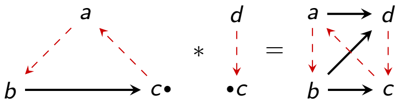


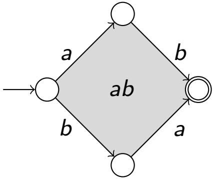
Events representation: pomset with interfaces (ipomset)

- Two partial order events
  - ▷ $<$: precedence order (rep with $\longrightarrow$) , $\dashrightarrow$: event order.
  - ▷ $< \cup \dashrightarrow$: **total** relation.
- Interfaces
  - ▷ **Source/Target interfaces**: S/T: $<$-minimal/maximal.
- Representation of events as interval



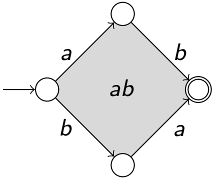
- Gluing composition:



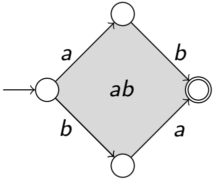


- Higher Dimensional Automata A:
  - ▷ A tuple $(X, X_\perp, X_\top)$ where X is a finite **precubical set** and $X_\perp$ (*resp.* $X_\top$) $\subseteq X$ a **start (*resp.* accept) cell**.
  - ▷ Ex: start cell $X_\perp$: $\longrightarrow\bigcirc$, accept cell $X_\top$: $\circledcirc$

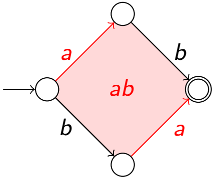
    $X : \{ \longrightarrow\bigcirc\ , \circledcirc, \bigcirc\ , \ ab\ \} \cup \left\{ \xrightarrow{\lambda} | \lambda \in \{a, b\} \right\}$



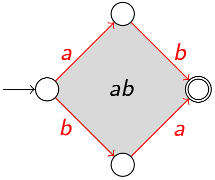
- Higher Dimensional Automata A:
  - ▷ A tuple $(X, X_\perp, X_\top)$ where X is a finite **precubical set** and $X_\perp$ (*resp.* $X_\top$) $\subseteq X$ a **start (*resp.* accept) cell**.
  - ▷ Ex: start cell $X_\perp$: $\longrightarrow\bigcirc$, accept cell $X_\top$: $\circledcirc$
    $X : \{ \longrightarrow\bigcirc\ , \circledcirc, \bigcirc\ , \ ab\ \} \cup \left\{ \xrightarrow{\lambda} | \lambda \in \{a, b\} \right\}$
- List of events
  - ▷ A **conclist** (concurrent list): a finite, totally ordered ($\dashrightarrow$) $\Sigma-$labelled set.
  - ▷ Ex: $\{a, b\}$



- Precubical set X:
  - ▷ A set of cells $\boldsymbol{X}$.
  - ▷ **List of active events** of a cell $x \in X$: a conclist $ev(x)$.
    Ex: $\{a\}$, **or** $\{b\}$ **or** $\{a, b\}$.



- Precubical set X:
  - ▷ A set of cells $\boldsymbol{X}$.
  - ▷ **List of active events** of a cell $x \in X$: a conclist $ev(x)$. Ex: $\{a\}$**, or** $\{b\}$ **or** $\{a, b\}$**.**
  - ▷ **The cells of a list of events** U : $\boldsymbol{X[U] = \{x \in X | ev(x) = U\}}$. Ex: $\boldsymbol{X[a]}$



- Precubical set X:
  - A set of cells $\boldsymbol{X}$.
  - **List of active events** of a cell $x \in X$: a conclist $ev(x)$.
    Ex: $\{a\}$, **or** $\{b\}$ **or** $\{a, b\}$.
  - **The cells of a list of events** U : $X[U] = \{x \in X | ev(x) = U\}$.
    Ex: $X[a]$
  - **Lower & Upper faces**: Let U and $A \subseteq U$ be conclists.
    $\delta_A^0 \backslash \delta_A^1$ represent **unstarting\terminating events** A:

$$\boxed{\delta_A^0 : X[U] \to X[U - A], \delta_A^1 : X[U] \to X[U - A]}$$

Path of HDA

- Paths in an HDA

Sequence $p = (x_0, \varphi_1, x_1, \cdots, x_{n-1}, \varphi_n, x_n)$ *s.t.*

- ▷ $x_i \in X$, where x_0: start cell, x_n: accept cells
- ▷ φ: face map type.
- ▷ $ev(p_1 * p_2 * \cdots * p_n) = ev(p_1) * \cdots * ev(p_n)$

- Paths in an HDA

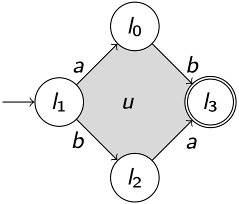
Sequence $p = (x_0, \varphi_1, x_1, \cdots, x_{n-1}, \varphi_n, x_n)$ *s.t.*

▷ $x_i \in X$, where x_0: start cell, x_n: accept cells

▷ φ: face map type.

▷ $ev(p_1 * p_2 * \cdots * p_n) = ev(p_1) * \cdots * ev(p_n)$

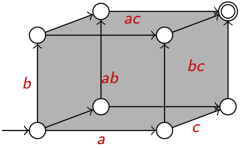
- Example of a 2−events HDA



Example of an accepting path:

$$\alpha_0 = l_0 \nearrow^{ab} u \searrow_{ab} l_3,\ ev(\alpha_0) = \left(\begin{bmatrix} a\bullet \\ b\bullet \end{bmatrix} * \begin{bmatrix} \bullet a \\ \bullet b \end{bmatrix} \right)$$

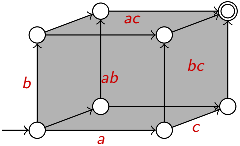
- Example of languages
  - A_1 :



$$L_1 = \{abc, acb, bac, bca, cab, cba\} \cup \left\{ \left(\begin{array}{l} a \\ b \to c \end{array} \right), \left(\begin{array}{l} a \\ c \to b \end{array} \right), \left(\begin{array}{l} b \\ a \to c \end{array} \right), \left(\begin{array}{l} b \\ a \to c \end{array} \right), \left(\begin{array}{l} b \\ c \to a \end{array} \right), \left(\begin{array}{l} c \\ a \to b \end{array} \right), \left(\begin{array}{l} c \\ b \to a \end{array} \right) \right\}$$

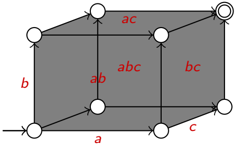
- Example of languages

  ▷ A_1 :



$$L_1 = \{abc, acb, bac, bca, cab, cba\} \cup \left\{ \begin{pmatrix} a & \\ b \to c \end{pmatrix}, \begin{pmatrix} a & \\ c \to b \end{pmatrix}, \begin{pmatrix} b & \\ a \to c \end{pmatrix}, \begin{pmatrix} b & \\ a \to c \end{pmatrix}, \begin{pmatrix} b & \\ c \to a \end{pmatrix}, \begin{pmatrix} c & \\ a \to b \end{pmatrix}, \begin{pmatrix} c & \\ b \to a \end{pmatrix} \right\}$$

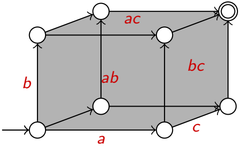
  ▷ A_2:



$$: L_2 = L_1 \cup \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\}$$

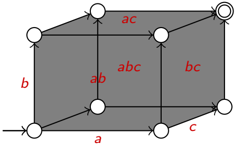
- Example of languages

  ▷ A_1 :



$$L_1 = \{abc, acb, bac, bca, cab, cba\} \cup \left\{ \begin{pmatrix} a \\ b \to c \end{pmatrix}, \begin{pmatrix} a \\ c \to b \end{pmatrix}, \begin{pmatrix} b \\ a \to c \end{pmatrix}, \begin{pmatrix} b \\ a \to c \end{pmatrix}, \begin{pmatrix} b \\ c \to a \end{pmatrix}, \begin{pmatrix} c \\ a \to b \end{pmatrix}, \begin{pmatrix} c \\ b \to a \end{pmatrix} \right\}$$

  ▷ A_2:



$$: L_2 = L_1 \cup \left\{ \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right\}$$

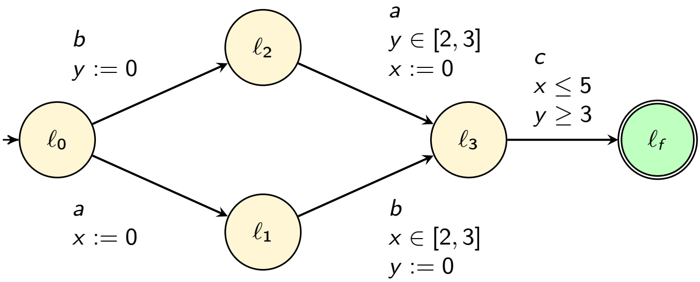
- The language of an HDA $A = (X, X_\perp, X_\top)$ is:

$$\boxed{L(A) = \{ev(\alpha) | \alpha \text{ accepting path in } X\}}$$

- Timed Automata
  - Example and **definition**
  - **Language** of Timed Automata.

- Example of Scheduling of events a, b, c

Time constraints impose that between event a and b, at least (*resp.* at most) 2 (*resp.* 3) time units elapses

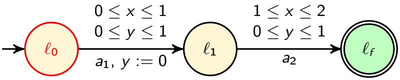


- Semantics of transitions
  - Delay transitions $(\ell, v) \xrightarrow{\delta} (\ell, v + \delta)$
  - Action transitions: $(\ell, v) \xrightarrow{a_1} (\ell_1, v[y := 0])$

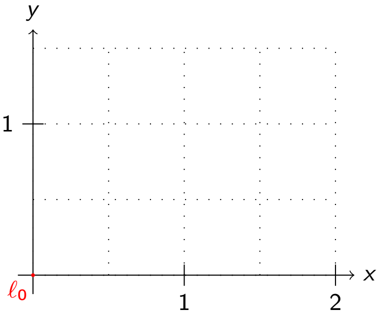
[2] Alur and Dill, 'A Theory of Timed Automata', 1994.

Clocks evolution example

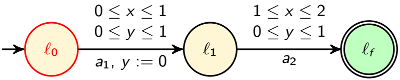
- Timed automaton $\mathcal{A}$:



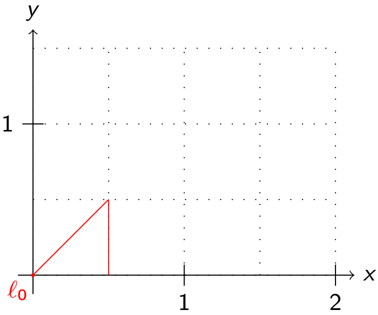
- Evolution of clocks x and y during the run



- Timed automaton $\mathcal{A}$:

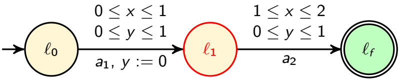


- Evolution of clocks x and y during the run

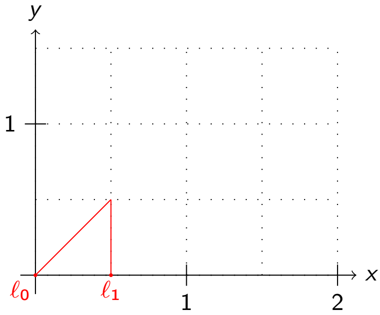


Clocks evolution example

- Timed automaton $\mathcal{A}$:

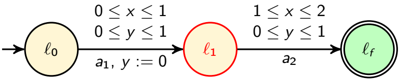


- Evolution of clocks x and y during the run

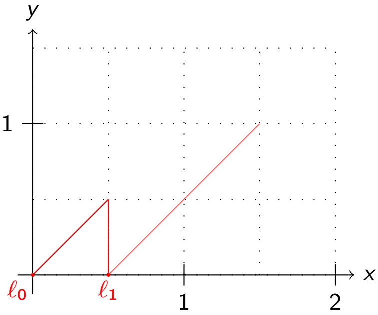


Clocks evolution example

- Timed automaton $\mathcal{A}$:

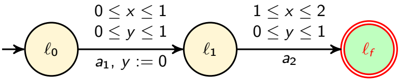


- Evolution of clocks x and y during the run

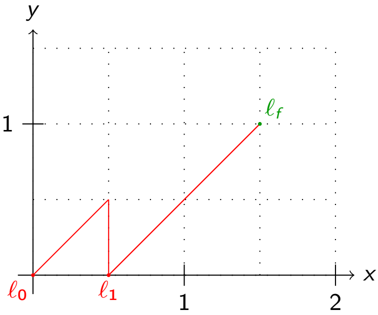


Clocks evolution example

- Timed automaton $\mathcal{A}$:



- Evolution of clocks x and y during the run



Delay move, timed words

- Delay words

Let us take a run $\pi = (\ell_0, v_0) \cdots \to \cdots (\ell_i, v_i) \cdots \to \cdots (\ell_n, v_n)$

- ▷ Delay move: $\delta : (\ell, v) \xrightarrow{d} (\ell, v + d)$
  Label of delay move: $ev(\delta) = d$
- ▷ Action move: $\delta : (\ell, v) \xrightarrow{a_1} (\ell_1, v[y := 0])$
  Label of action move: $ev(\delta) = a$
- ▷ Label of a run π:

$$ev((\ell_0, v_0) \to (\ell_1, v_1)) \cdots ev((\ell_{n-1}, v_{n-1}) \to (\ell_n, v_n))$$

- Timed words

- ▷ Definition: $TW = \{w = (a_0, t_0) \cdots (a_n, t_n) t_{n+1} | \forall i = 0, \cdots n, t_i \leq t_{i+1}\} \subseteq (\Sigma \times \mathbb{R}_{\geq 0})^* \mathbb{R}_{\geq 0}$
- ▷ Concatenation : let $w = (a_0, t_0) \cdots (a_n, t_n) t_{n+1} w' = (a'_0, t'_0) \cdots (a'_n, t'_n) t'_{n+1} \in TW$ then:

$$\boxed{ww' := (a_0, t_0) \cdots (a_n, t_n)(a'_0, t'_0) \cdots (a'_n, t'_n)(t_{n+1} + t'_{n+1}) \in TW}$$

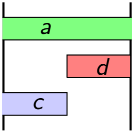
Finally : $\mathcal{L}(A)$: the set of delay words labeling accepting path in the transition system.

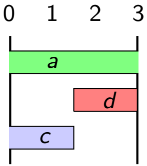
- Higher Dimensional Timed Automata
  - ▷ **Timed ipomset**
  - ▷ **Definition** and example of HDTA
  - ▷ **Language** of HDTA

Represent ipomset with timing information

- Timed ipomsets is composed of:
  - ▷ An ipomset
  - ▷ A duration d
  - ▷ A map σ labelling all events to time intervals

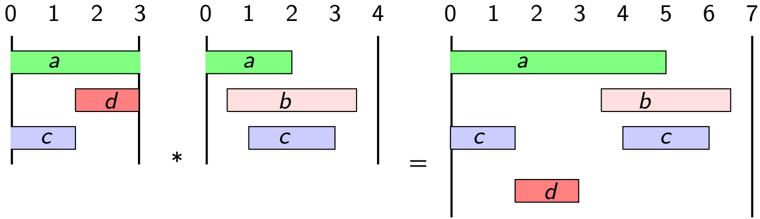
- Ipomsets (left), Timed ipomsets (right)





  - ▷ Starter: x_1, x_3 of respective label a and c
  - ▷ Target: x_2 of label b
  - ▷ $\sigma(a) = (0, 3), \sigma(b) = (0, 1.5), \sigma(c) = (1.5, 3)$
  - ▷ Total duration $d = 3$

Gluing on Timed Ipomsets

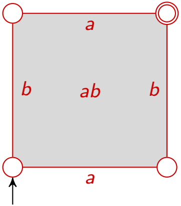


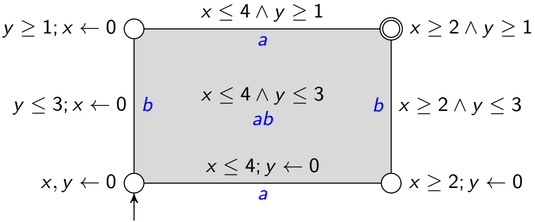
- Definition:

A HDTA is a tuple $(X, X_\perp, X_\top, \lambda, \mathcal{C}, \mathrm{inv}, \mathrm{exit})$ where:

▷ $(\boldsymbol{X}, \boldsymbol{X}_\perp, \boldsymbol{X}_\top, \boldsymbol{\lambda})$ is an HDA

- Example with events *a* and *b*: HDA (left) of the HDTA (right)



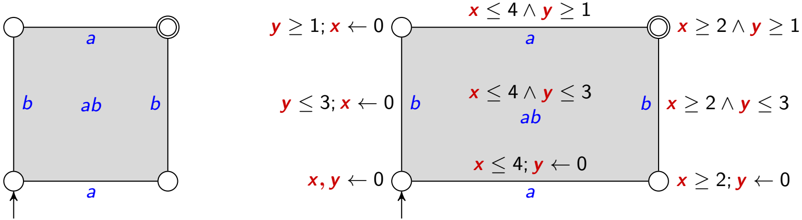


- Definition:

A HDTA is a tuple $(X, X_\perp, X_\top, \lambda, \mathcal{C}, \text{inv}, \text{exit})$ where:

- ▷ $(X, X_\perp, X_\top, \lambda)$ is an HDA
- ▷ $\mathcal{C}$: set of clocks

- Example with events a and b: HDA (left) of the HDTA (right)

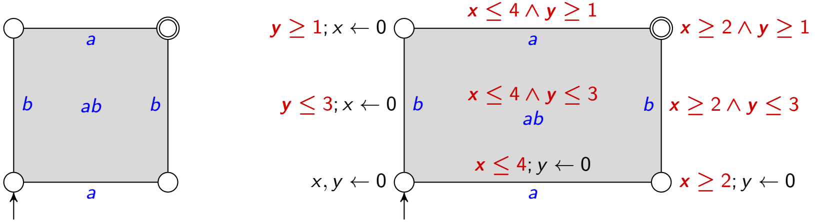


- Definition:

A HDTA is a tuple $(X, X_\perp, X_\top, \lambda, \mathcal{C}, \mathsf{inv}, \mathsf{exit})$ where:

- ▷ $(X, X_\perp, X_\top, \lambda)$ is an HDA
- ▷ $\mathcal{C}$: set of clocks
- ▷ $\mathsf{inv} : \boldsymbol{X} \rightarrow \phi(\mathcal{C})$ assign **invariant conditions** to cells.

- Example with events a and b: HDA (left) of the HDTA (right)

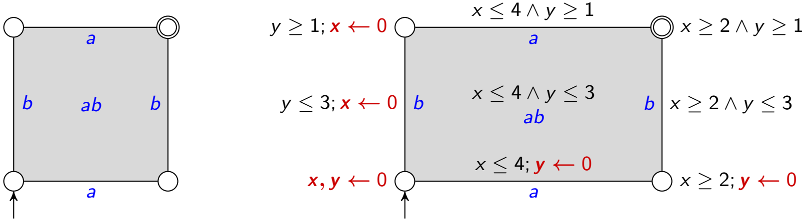


- Definition:

A HDTA is a tuple $(X, X_\perp, X_\top, \lambda, \mathcal{C}, \mathrm{inv}, \mathrm{exit})$ where:

- ▷ $(X, X_\perp, X_\top, \lambda)$ is an HDA
- ▷ $\mathcal{C}$: set of clocks
- ▷ $\mathrm{inv} : X \to \phi(\mathcal{C})$ assign invariant conditions to cells.
- ▷ $\mathrm{exit} : X \to 2^{\mathcal{C}}$ assign **exit conditions** to cells.

- Example with events a and b: HDA (left) of the HDTA (right)



Examples of HDTA

Quizz: suppose that a and b are not in concurrency

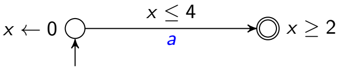
Let us draw the HDTA of a: $[2, 4]$ and b: $[1, 3]$ separately:

Timing duration of events:

- a: $[2, 4]$ time units
- b: $[1, 3]$ time units

Quizz: suppose that a and b are not in concurrency

Let us draw the HDTA of a: $[2,4]$ and b: $[1,3]$ separately:

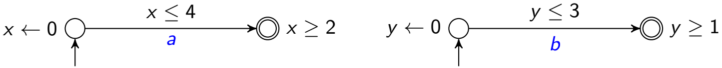


Timing duration of events:

- a: $[2,4]$ time units
- b: $[1,3]$ time units

Quizz: suppose that a and b are not in concurrency

Let us draw the HDTA of a: $[2,4]$ and b: $[1,3]$ separately:



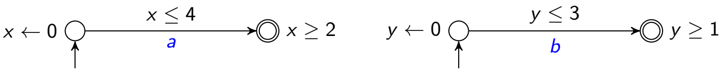
Timing duration of events:

- a: $[2,4]$ time units
- b: $[1,3]$ time units

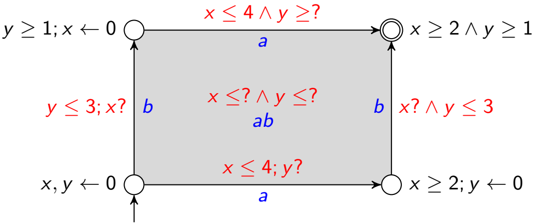
Examples of HDTA

Quizz: suppose that a and b are not in concurrency

Let us draw the HDTA of a: $[2,4]$ and b: $[1,3]$ separately:



- Let's put them together



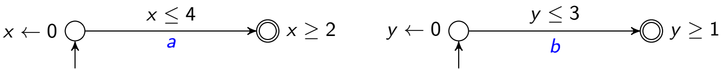
Timing duration of events:

- a: $[2,4]$ time units
- b: $[1,3]$ time units

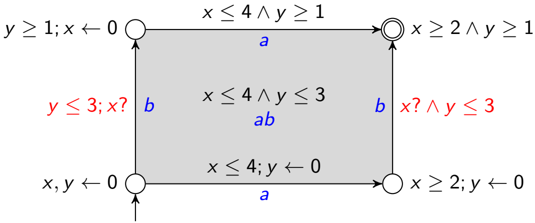
Examples of HDTA

Quizz: suppose that a and b are not in concurrency

Let us draw the HDTA of a: $[2, 4]$ and b: $[1, 3]$ separately:



- Let's put them together

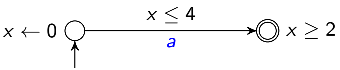


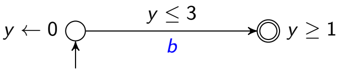
Timing duration of events:

- a: $[2, 4]$ time units
- b: $[1, 3]$ time units

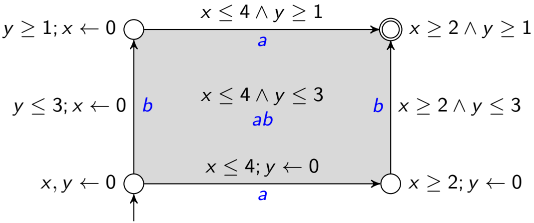
Quizz: suppose that a and b are not in concurrency

Let us draw the HDTA of a: $[2,4]$ and b: $[1,3]$ separately:





- Let's put them together

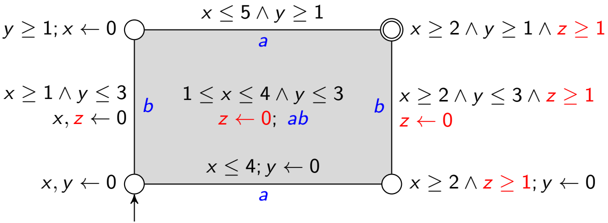


Timing duration of events:

- a: $[2,4]$ time units
- b: $[1,3]$ time units

Example of HDTA

- Second example: adding timing constraints between events...



Timing duration of events:

- a: $[2, 4]$ time units
- b: $[1, 3]$ time units

Constraints between starting/ending dates

- 1 time unit should elapse between b's starting date and a's starting date
- 1 time unit should elapse between b's ending date and a's ending date

Differences between TA and HDTA

- Cells
  - 0-cells: location,
  - $1-$cell: edges,
  - $d-$cell, $d > 1$.

- Differences

	TA	HDTA
Difference between locations, edges	Yes	No
Exit conditions	Edges	On $d-$cells, $\forall d$
Invariants	Locations	On $d-$cells, $\forall d$
Reset	Edges	On $d-$cells, $\forall d$
Events	Instantaneous	With duration
Concurrency	Interleaving	Possibly simultaneous

Interval Delay words, untimed of Timed Ipomsets

- Timed Ipomsets and Interval delay words
  - Timed Ipomsets: (P, σ_P, d_P).

Interval Delay words, untimed of Timed Ipomsets

- Timed Ipomsets and Interval delay words
  - Timed Ipomsets: (P, σ_P, d_P).
  - Steps sequence (HDA)

$$(S_{Q_0}, Q_0, T_{Q_0}) * (S_{Q_1}, Q_1, T_{Q_1}) * \cdots * (S_{Q_n}, Q_n, T_{Q_n}) \ s.t \ T_{Q_i} = S_{Q_{i+1}}$$

- Timed Ipomsets and Interval delay words
  - Timed Ipomsets: (P, σ_P, d_P).
  - Steps sequence (HDA)

$$(S_{Q_0}, Q_0, T_{Q_0}) * (S_{Q_1}, Q_1, T_{Q_1}) * \cdots * (S_{Q_n}, Q_n, T_{Q_n}) \ s.t \ T_{Q_i} = S_{Q_{i+1}}$$

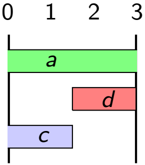
  - Interval delay words: steps sequence interspersed with delays (start/termination of events).

- Timed Ipomsets and Interval delay words
  - ▷ Timed Ipomsets: (P, σ_P, d_P).
  - ▷ Steps sequence (HDA)

$$(S_{Q_{\mathbf{0}}}, Q_0, T_{Q_{\mathbf{0}}}) * (S_{Q_{\mathbf{1}}}, Q_1, T_{Q_{\mathbf{1}}}) * \cdots * (S_{Q_n}, Q_n, T_{Q_n}) \ s.t \ \ T_{Q_i} = S_{Q_{i+\mathbf{1}}}$$

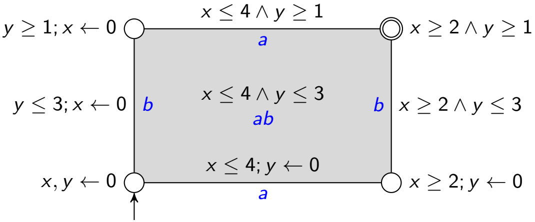
  - ▷ Interval delay words: steps sequence interspersed with delays (start/termination of events).

- Untimed of Timed Ipomsets
  - ▷ Untimed $unt((P, \sigma_P, d_P)) = (P, <_P, \dashrightarrow_P, S, T, \lambda)$

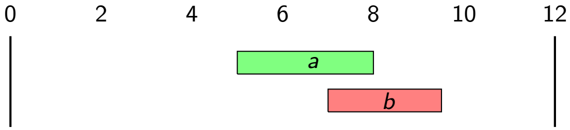
Ex: T = 

$, \ unt(T) = \begin{bmatrix} & \bullet a \bullet & \\ \bullet c & \rightarrow & d \end{bmatrix}$

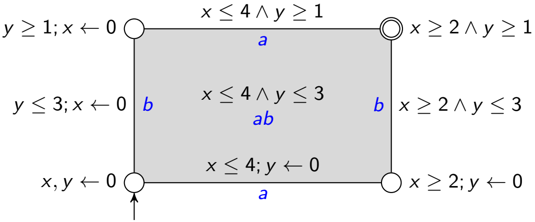
- Example of accepting path



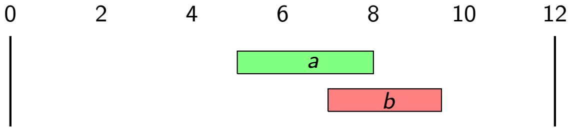
Accepting path:



- Example of accepting path



Accepting path:



- The language of an HDTA A is:

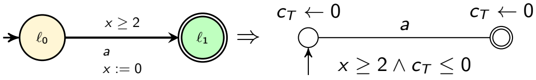
$$\boxed{L(A) = \{ev(\alpha) | \alpha \text{ accepting path in } X\}}$$

- **Contribution**
  - ▷ **Language inclusion** for HDTA is **undecidable**
  - ▷ **Untimed language inclusion** for HDTA is **decidable**.

- Contribution: **Embedding of TA into HDTA**

Let $\mathcal{A}$ be a timed automaton, we can transform it to express it as an HDTA, providing:

- ▷ **forcing immediate transition** : add a global clock x, for any transition
- ▷ Examples : TA(left), HDTA (right)



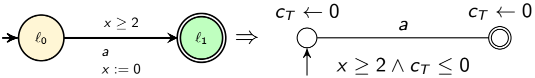
- Contribution: **Corollary**

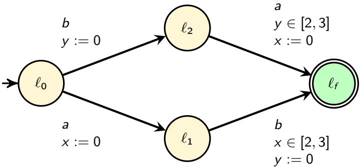
Language inclusion of HDTA is **undecidable**.

• Contribution: **Embedding of TA into HDTA**

Let $\mathcal{A}$ be a timed automaton, we can transform it to express it as an HDTA, providing:

- ▷ **forcing immediate transition** : add a global clock x, for any transition
- ▷ Examples : TA(left), HDTA (right)





$\Rightarrow$? (quizz if time)

• Contribution: **Corollary**

Language inclusion of HDTA is **undecidable**.

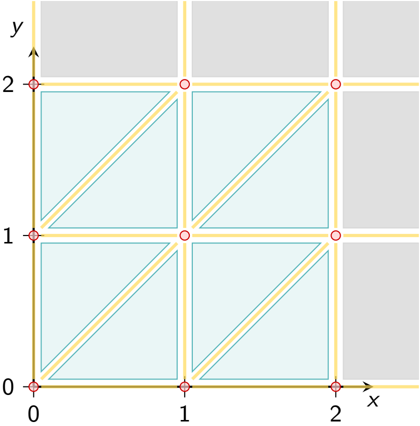
Untimed language inclusion is decidable

- Contribution: **Express Region Automata for HDTA**

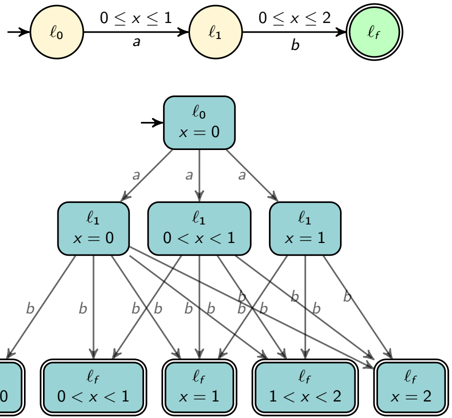
Untimed language inclusion is decidable

Recall: Region Automata

- Ex: Region of the constraint $0 \leq x, y \leq 2$



- A timed Automaton and its region automaton



- Reachability problem for TA

PSPACE (Alur et al, 1994): correspondance between runs of TA and the one of the corresponding region automata.

Untimed language inclusion is decidable

- **Region equivalence**

Let $A = (\Sigma, C, L, \bot_L, \top_L, inv, exit)$ be an HDTA

- ▷ $M :=$ the maximal constant appearing in *inv*
- ▷ $\cong$: region equivalence on $\mathbb{R}^C_{\geq 0}$ defined as follows: for any $v, v' : C \to \mathbb{R}_{\geq 0}$:
  - $\lfloor v(x) \rfloor = \lfloor v'(x) \rfloor$ or $v(x), v'(x) > M$, $\forall x \in C$,
  - $\{v(x)\} = 0 \Leftrightarrow \{v'(x)\} = 0, \forall x \in C$
  - $\{v(x)\} \leq \{v(y)\} \Leftrightarrow \{v'(x)\} \leq \{v'(y)\}$

- Contribution: **Express untimed language**

For any HDTA A, $(unt(L(A))) = R(A)$.

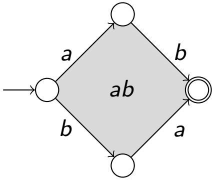
- Consequence: **Untimed language inclusion is decidable**

Conclusion

Current & Future work

- HDA/HDTA
  - ▷ Collaborators: Amazigh Amrane, Hugo Bazille, Uli Fahrenberg, Marie Fortin, Krzysztof Ziemański, Jeremy Ledent, Enzo Erlich
  - ▷ **Temporal logic for HDA**.
  - ▷ **Robustness of HDTA**: guard enlargment, delay perturbation, distance between timed ipomsets, etc.
  - ▷ **Timed simulation and bisimulation**: HDTA model checking.
- Robustness for Timed Automata
  - ▷ Collaborator: Damien Bussato-Gaston
  - ▷ **Robustness with operators**
  - ▷ **Optimal dynamic strategies for cyclic TA**.
- Complexity, well-order set
  - ▷ Collaborator: Sylvain Schmitz
  - ▷ **Complexity of algorithm for Groebner bases**.

Definition of Higher Dimensional Automata



- **Precubical set**
  - Sets $(X_n)_n$
  - A set of functions $\left(\delta^{\varepsilon}_{i,n} : X_n \mapsto X_{n-1}\right)_{n>0, i\in\{1,\cdots,n\}, \varepsilon\in\{0,1\}}$ such that :

$$\boxed{\delta^{\varepsilon'}_{j,n} \circ \delta^{\varepsilon}_{i,n+1} = \delta^{\varepsilon}_{i-1,n} \circ \delta^{\varepsilon}_{j,n+1}, \forall i, j}$$

.

- **Application in HDA**

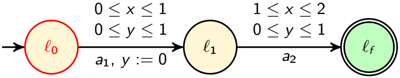
A precubical set on a finite alphabet Σ:

$$X = \left(X, ev, \{\delta^0_{A,U}, \delta^1_{A,U} | U \in C, A \subseteq U\}\right)$$

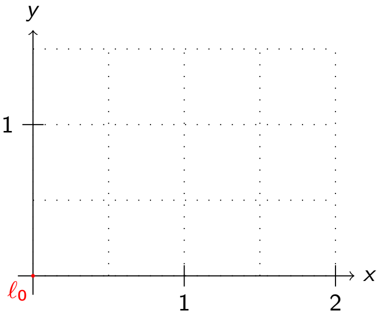
where C is the set of conclist over Σ

Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

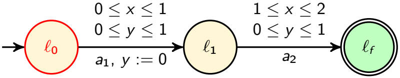


- Run with delay perturbations of at most $\delta = 0.2$

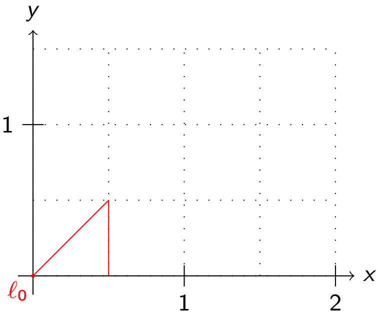


Future work: What about the robustness?

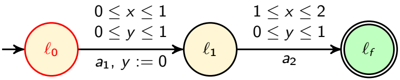
- Timed automaton $\mathcal{A}$:



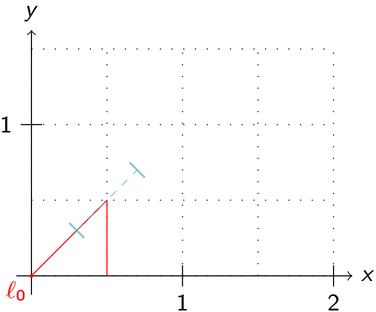
- Run with delay perturbations of at most $\delta = 0.2$



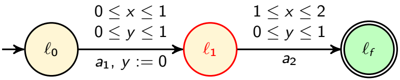
- Timed automaton $\mathcal{A}$:



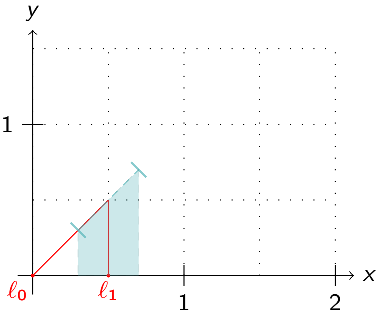
- Run with delay perturbations of at most $\delta = 0.2$



- Timed automaton $\mathcal{A}$:

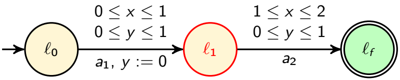


- Run with delay perturbations of at most $\delta = 0.2$

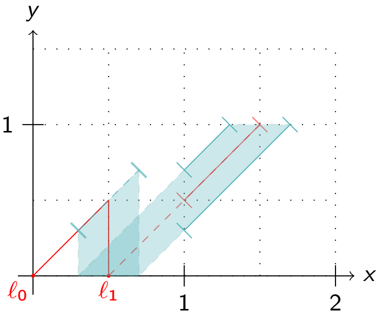


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

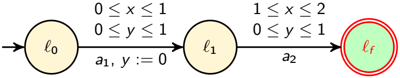


- Run with delay perturbations of at most $\delta = 0.2$

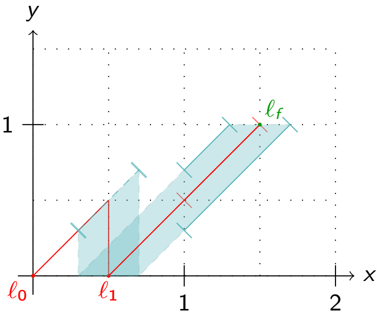


Future work: What about the robustness?

- Timed automaton $\mathcal{A}$:

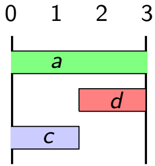


- Run with delay perturbations of at most $\delta = 0.2$



Future work: robustness on HDTA

- No timing perturbation: c and d are not in concurrency



- timing perturbation. Let us introduce a 0.1 delay on the end date of c: