Conference Hopf algebras, operads, deformations for singular dynamics

Yvain Bruned[1]

IECL (UMR 7502), Université de Lorraine
Email: yvain.bruned@univ-lorraine.fr

Abstract

Conference organised in Nancy from 21 to 23 of June 2023, funded by the ANR via the project LoRDeT (Dynamiques de faible regularité via les arbres décorés) https://anr.fr/Projet-ANR-23-ERCS-0011. The PI of the project is Yvain Bruned.

Wednesday 21 June 2023

Talks took place at IECL, Salle Döblin.

- 14h00-14h50: Bruno Vallette.

  Effective integration of Lie type algebras (including pre-Lie algebras)

  I will review the various formulas defining the gauge group and its action on Maurer—Cartan elements of algebraic structures of Lie type. The case of pre-Lie algebras is made up of combinatorics of labeled rooted trees, for instance via the seminal pre-Lie exponential map. This case is motivated by direct applications in the deformation theory of algebra structures over operads. The other applications in deformation theory require to deal with new interesting cases (bialgebraic structures, homotopy Lie algebras). This talk will be based on joint works with Ricardo Campos, Vladimir Dotsenko, Daniel Robert-Nicoud, and Sergei Shadrin.

- 14H50-15h40: Dominique Manchon.

  Post-groups and post-Lie algebras in differential geometry

  It is well-known that the Lie algebra of vector fields is pre-Lie when equipped with a flat torsion-free connection. It has been recently remarked that, for a flat connection with constant torsion, the Lie algebra of vector fields is a post-Lie algebra. I will explain how post-Lie algebras also play a prominent role in the general setting in presence of curvature and torsion. Based on joint works with Mahdi Al-Kaabi, Kurusch Ebrahimi-Fard, Hans Z. Munthe-Kaas and Yuanyuan Zhang.

- 15h40-16h10: Coffee break.
- 16h10-17h00: Yvain Bruned.

  Post-Lie algebras in Regularity Structures.

  In this talk, we will present recent progress on the understanding of the Hopf algabraic structures at play in the context of singular SPDEs. Th key object is a natural post-Lie algebra on decorated trees that has its counterpart in the context of multiindices, an alternative approach for describing solutions of singular SPDEs. This is a joint work with Foivos Katsetsiadis.

- 19h30-22h30: Conference dinner, Grand Café Foy.

Thursday 22 June 2023

Talks took place at IECL, Salle Döblin.

- 9h30-10h20: Dotsenko Vladimir.

  The inexhaustible beauty of pre-Lie algebras

  The remarkable class of nonassociative algebras known as pre-Lie algebras is well known to many participants of this workshop. I shall discuss several recent new results about these algebras (some proved by myself, some obtained in collaboration with Oisín Flynn-Connolly and Ualbai Umirbaev).

- 10h20-10h50: Coffee break.
- 10h50-11h40: Frédéric Fauvet.

  Resurgence and alien calculus with multiple critical times

  We shall present and implement in simple situations Ecalle's alien calculus when several critical times are involved, within the mechanism of acceleration. The talk will be essentially self-contained, all relevant notions will be introduced, with elementary examples.

- 11h40-12h30: Markus Tempelmayr.

  Multi-index based regularity structures for quasi-linear SPDEs.

  We give an overview of the solution theory for singular SPDEs in case of a quasi-linear equation, following the recent approach of Otto, Sauer, Smith and Weber. The basic idea is to parametrize the model, which captures the local solution behaviour, by partial derivatives w.r.t. the non-linearity. This allows for an efficient bookkeeping and an inductive construction of the model. We outline the construction of the structure group, used to "re-center" the model, based on a Lie algebra consisting of infinitesimal generators of actions in the space of non-linearities. Although the approach is tree-free, we show morphism properties w.r.t. well-known tree-based structures in branched rough paths and regularity structures. Based on joint work with Pablo Linares and Felix Otto.

- 12h30-14h30: Lunch.

  Talks took place at Salle de réunion 5ème étage.

- 14h30-15h20: Jean-David Jacques.

  Post-Lie algebra of derivations and regularity structures

  Post-Lie algebra structures are a generalization of Pre-Lie algebras. They have their roots in geometry and correspond to the algebraic properties satisfied by the covariant derivative in the case of a flat and constant torsion connection. In my talk, I will provide a brief overview of the new theory of regularity structures developed by F. Otto and colleagues, and discuss how post-Lie algebra structures arise in this framework.

- 15h20-15H50: Coffee break.

- 15H50-16h40: Jacob Armstrong-Goodall.

- 16h40-17h30: Martin Bordemann.

  A gentle introduction to Drinfel'd's associator'

  This is a pedagogical talk about the famous associator invented by V.G.Drinfel'd in 1989/90 and a proof of its hexagon and pentagon identities. It is well-known that this associator has applications to the quantization of (quasi) Lie bialgebras (P.I.Etingof-D.A.Kazhdan and B.Enriquez-G.Halbout, and P.Ševera), deformation quantization (D.E.Tamarkin) and number theory (multiple zeta values). Drinfel'd's exposition in the original articles (but also in most textbooks) had been quite sketchy, and the mortal reader usually does not immediatley see to what extent s/he has to know deep complex analysis or algebraic topology. We give a more elementary and explicit approach using limits of (formal) parallel transports along concrete paths in explicit star-shaped domains of the real line and plane with respect to flat (formal) connections derived from the Knizhnik-Zamolodchikov connection (In collaboration with Andrea Rivezzi and Thomas Weigel, University of Milano-Bicocca, Italy).

Friday 23 June 2023

Talks take place at IECL, Salle Conférence.

- 9h00-9h50: Paul Laubie.

  Combinatorics of pre-Lie products sharing the Lie bracket

  Pre-Lie products sharing the Lie bracket are controlled by the $\bigvee^n_{\text{Lie}} \text{PreLie}$ operad, the coproduct of n copies of PreLie in the category of operads over Lie. From a first inspection on the dimension of $\bigvee^2_{\text{Lie}} \text{PreLie}$, one may find that it is related to the so called Greg trees. We define an operad structure on the species of

Greg trees, show that it is a binary, quadratic and Koszul operad verifying the Nielsen-Scheier property.

- 9h50-10h20: Coffee break.

- 10h20-11h10: Nils Berglund.

Perturbation theory for the Φ_3^4 measure, revisited with Hopf algebras

The talk will address the existence of a perturbation expansion for the partition function of the properly renormalised Phi-four measure on the three-dimensional torus. While proofs of this result go back to works by Glimm and Jaffe, and several different mathematical methods have been used by different authors, we will argue that the proofs can be streamlined even more using algebraic methods. Based on joint work with Tom Klose (Berlin).

- 11h10-12h00: Frédéric Chapoton.

About groups of power series attached to operads

I will recall how one can associate to every operad P a group that behaves very much like formal power series under composition, and that acts on the completed free P-algebra. This construction is a functor, hence every morphism of operads induces a morphism of groups. One can therefore send rooted-trees-indexed series (from the Pre-Lie operad) to other kinds of similar series, indexed for example by planar binary trees or permutations. Particular elements in these groups could be thought of as "special functions", for example the very useful pre-Lie exponential and logarithm. I will also introduce the notion of "rooted-operad" and explain how this structure gives a refinement of the group structure. This can be useful in combinatorics, for example when using rooted-trees-indexed series to enumerate "combinatorial structures on rooted trees".