Sujet de Thèse

- **Titre : The Smith conjecture in low regularity**
- **Unité de recherche :** IRMAR, UMR-6625
- **Thème : Low-dimensional geometry and topology**
- **Mots clefs : 3-manifolds, wild homeomorphisms, Smith conjecture**
- **Les noms, prénoms, courriel, établissement des directeurs ou directrices de thèse**

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Objectif de la thèse

Smith proved in 1939 that the fixed point set of an orientation preserving self-homeomorphism $f : \mathbf{S}^3 \to \mathbf{S}^3$ of finite order is either empty or a knot and conjectured that the knot had to be trivial. This was disproved in 50's by Montgomery and Zippin: they constructed an involution of $\mathbf{S}^3$ whose fixed point set is a wild knot. On the other hand, Smith's conjecture was proved in the early 80's for diffeomorphisms using Thurston's work on geometrisation. The goal of the present project is to prove the Smith conjecture for *maps of low regularity*, that is Lipschitz or quasi-conformal. Some related questions and/or extensions, asked earlier by various people (Freedman, Church and Hemmingsen, Heinonnen,...):

(1) Are finite groups of low-regularity homeomorphisms of $\mathbf{S}^3$ conjugated into O_4?
(2) Construct counterexamples to Smith's conjecture in the Sobolev space $W^{1,n}$.
(3) Prove that quasi-regular self-maps of $\mathbf{S}^3$ with branch set of topological dimension 0 are homeomorphisms.
(4) Suppose that $f : \mathbf{S}^5 \to \mathbf{S}^5$ is quasiconformal of finite order and that its fixed point set is a knot K. Prove that $\mathbf{S}^5 \setminus K$ is simply connected.