Layered controller synthesis for dynamic multi-agent systems

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Septembre 19 2023

Introduction

Dynamic multi-agent system's verification

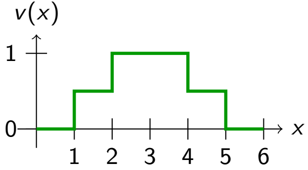
Our objectives

- A running example

https://perso.eleves.ens-rennes.fr/people/Emily.Clement/Videos/example_episodes/ex_0.mp4

	Timed Automata	Reinforcement Learning
Model	Abstract representation (acceleration)	
Waekness	Time of execution	Combinatorial or Continuous aspects

Our layered approach

- Our assumptions
  - ▷ **Speed:** 

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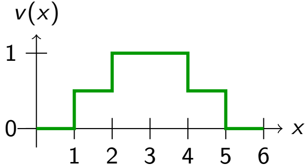

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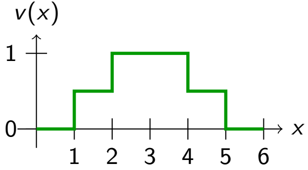
  - ▷ **Speed:** 

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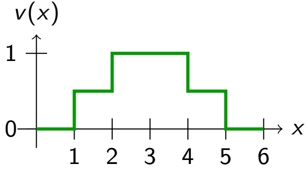
  - **Speed:** 

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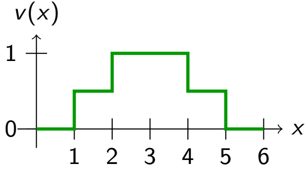
  - **Speed:** 

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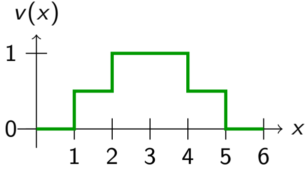
  - ▷ **Speed:** 

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- Our contribution: Three-layered Controller synthesis

SWA-SMT Solver

Stage 1: *Reachability algorithm on a simplified ISWA model*

SWA

Our layered approach

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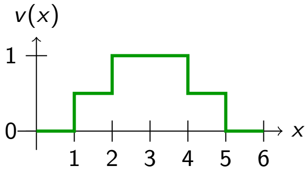

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SWA-SMT Solver

Stage 1: *Reachability algorithm on a simplified ISWA model*

SWA

Stage 2: *Refine the model of the speed*

SMT

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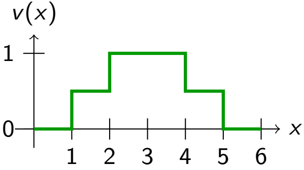

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SWA-SMT Solver

Stage 1: *Reachability algorithm on a simplified ISWA model*

SWA

Stage 2: *Refine the model of the speed*

SMT

RL training

Generate *a dataset for random initial positions*

Dataset

Stage 3: *Train an RL algorithm with our dasaset*

RL

SWA-SMT solver

SWA solver

Stage 1: Reachability algorithm on system of ISWA

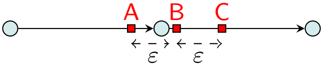
SWA

***Stage 2**: Model the accelleration and deceleration*

SMT

Rules to model collision avoidance

• #1: security distance when driving in the same direction and between neighbouring sections



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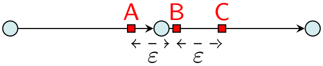

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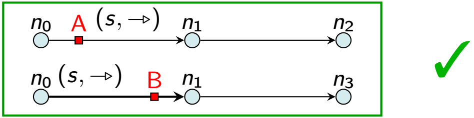

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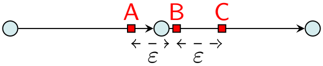

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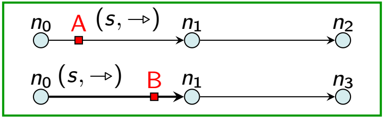
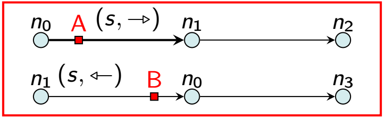


✓



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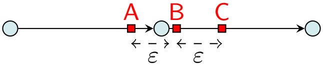

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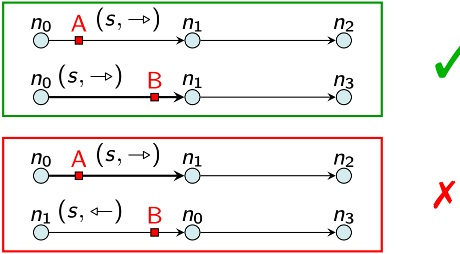
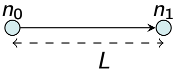


- #3: No Overtaking between cars

- A point in $\mathbb{R}^2$: a node 

- A section $s_{[n_{\mathbf{0}},n_{\mathbf{1}}],L}$ of the road: 

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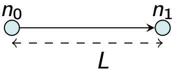
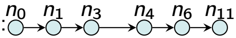
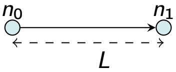
- A section $s_{[n_0,n_1],L}$ of the road: 

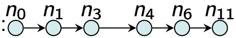
- A path: p_0 : 

Model for a car traffic

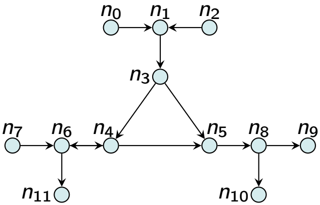
- A point in $\mathbb{R}^2$: a node n_0
- A section $s_{[n_0,n_1],L}$ of the road: $n_0 \xrightarrow{} n_1$, L
- A path: $p_0 : n_0 \to n_1 \to n_3 \to n_4 \to n_6 \to n_{11}$
- Car: (position, speed, trajectory)

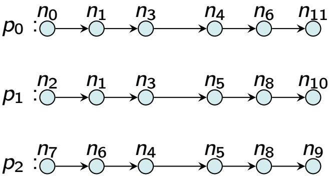
- A point in $\mathbb{R}^2$: a node 

- A section $s_{[n_0,n_1],L}$ of the road: 

- A path: p_0 : 

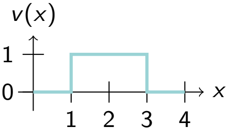
- Car: (position, speed, trajectory)
- A car traffic: c_0, c_1, c_2 are each assigned paths p_0, p_1, p_2:





What type of Timed Automata to use to model this?

- Needs
  - ▷ **Stopwatch Timed Automata**:



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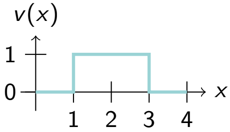
  - **Stopwatch Timed Automata**: 

  - **Clocks of TA**: Monitor each car's progress.

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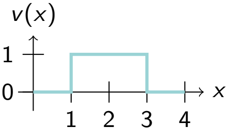
  - **Stopwatch Timed Automata**: 

  - **Clocks of TA**: Monitor each car's progress.
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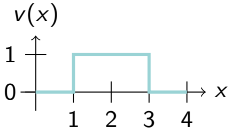
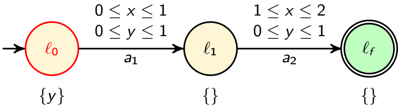
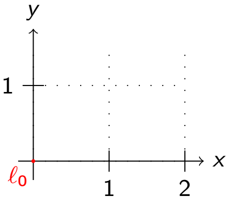
  - **Stopwatch Timed Automata**: 

  - **Clocks of TA**: Monitor each car's progress.
  - **Synchronised action**: Compute distance between each cars.
  - **FiFo channels**: A car cannot overtake another car.

- Example of a two-clocks Stopwatch Timed Automata





Initialized Stopwatch Timed Automata with bounded channels

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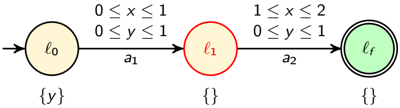

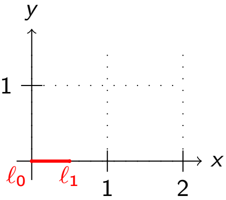

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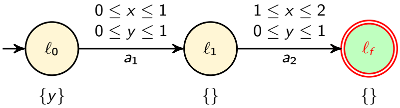

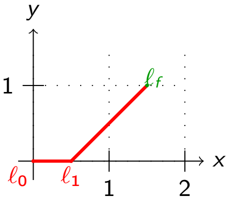

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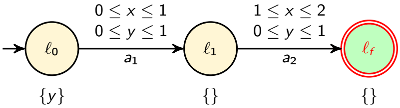

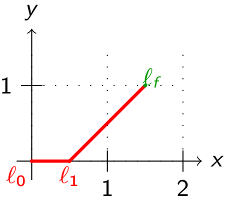

▷ Reachability is Undecidable in general cases.

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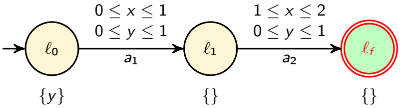

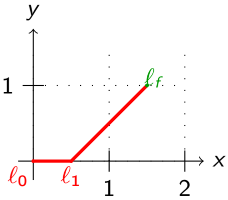

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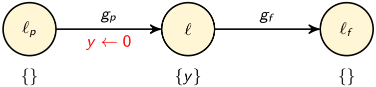
- Initialized Stopwatch Timed Automata
  - ▷ Reset the stopped clock in the previous or following transition:



  - ▷ **Reachability** becomes Decidable for this fragment of SWA.

Initialized Stopwatch Timed Automata with bounded channels

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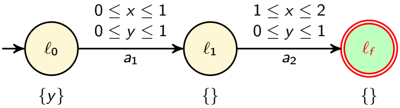

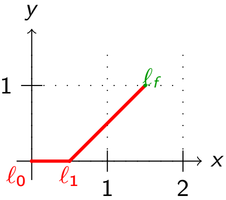

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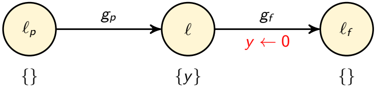

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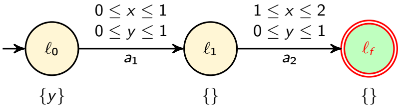

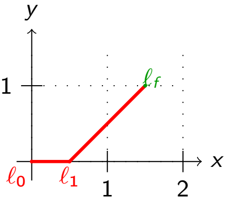

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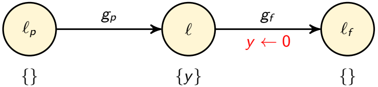

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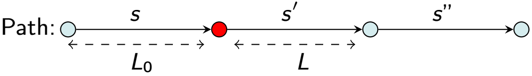
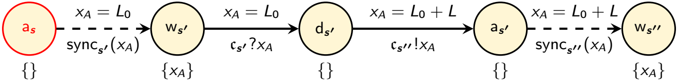
- Bounded channels
  - ▷ Channels: FiFo queue of symbols (actions) to be pushed/read

Model the car progress

- Car A progress along its paths



- Car A Timed automaton:



  ▷ **Clock** x_A: distance travelled along its paths

  ▷ **Stopwatches** $\{x_A\}$: the car A stops instantly.

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  ▷ **Intersection**: use classical synchronized action to activate *intersection automata*

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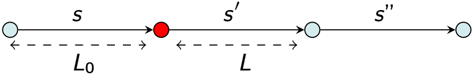
Path: 

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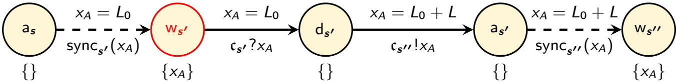

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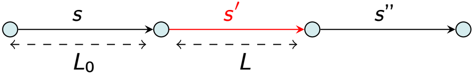
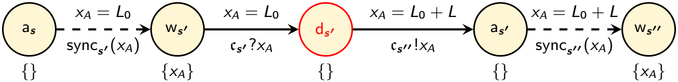


- Car *A* Timed automaton:



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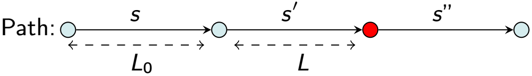

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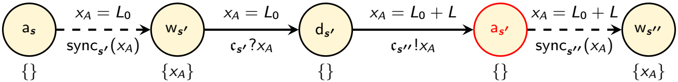

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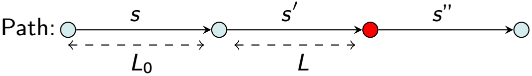
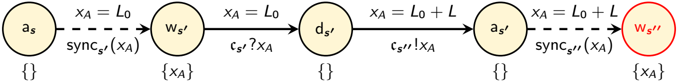


- Car A Timed automaton:



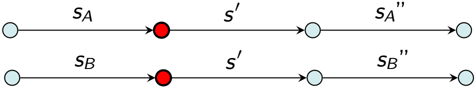
▷ **Clock** x_A: distance travelled along its paths

▷ **Stopwatches** $\{x_A\}$: the car A stops instantly.

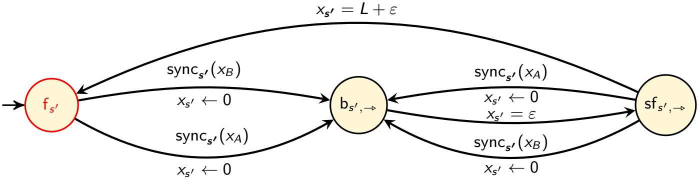
▷ **Channels** $\mathfrak{c}_{s'}!x_A/\mathfrak{c}_{s'}?x_A$: respect the order of cars in a section $s \Rightarrow$ no overtaking.

▷ **Intersection**: use classical synchronized action to activate *intersection automata*

Path of car A:
Path of car B:

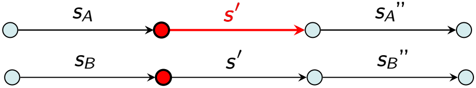


- Intersection automaton



Model distance between cars: intersection

Path of car A:
Path of car B:



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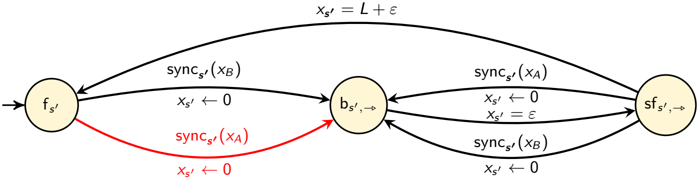

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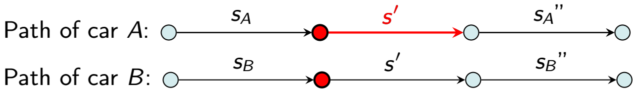

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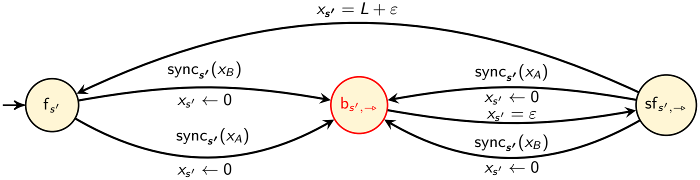

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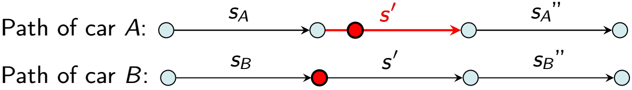

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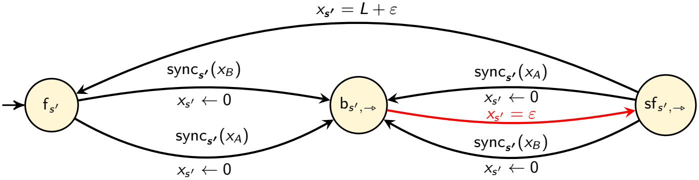

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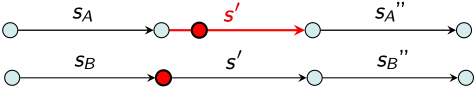

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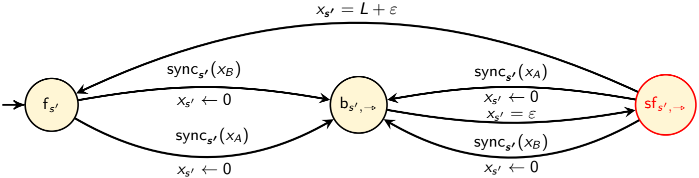

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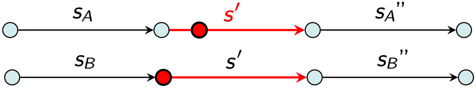

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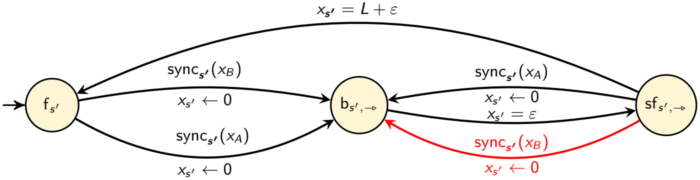

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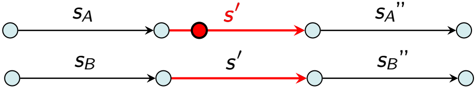

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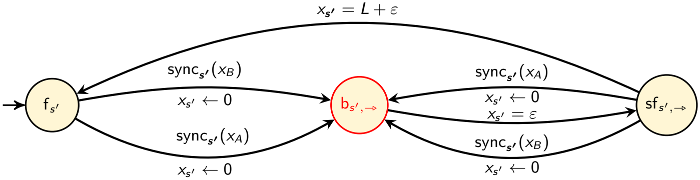

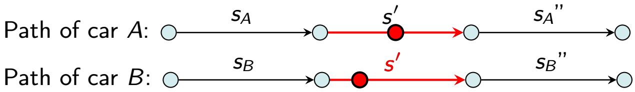

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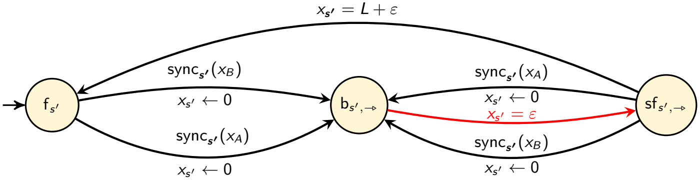

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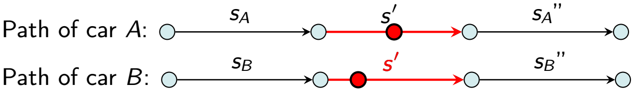

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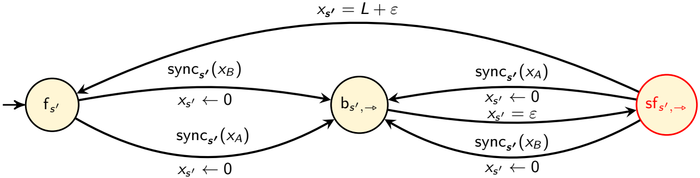

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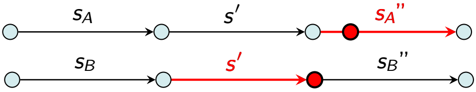

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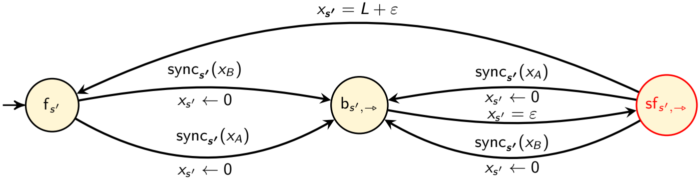

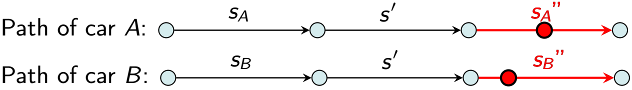

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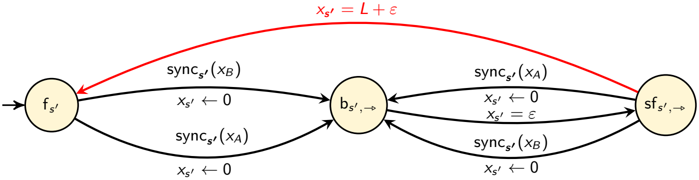

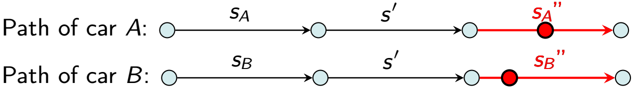

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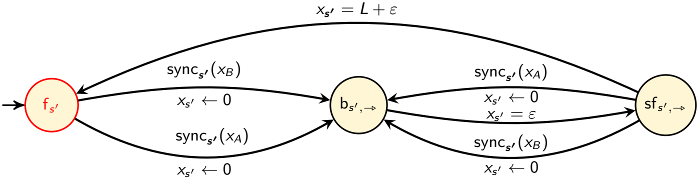
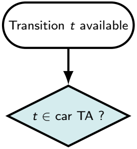


Our Algorithm: a DFS with an optimised succ function



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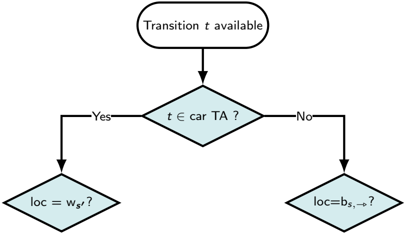

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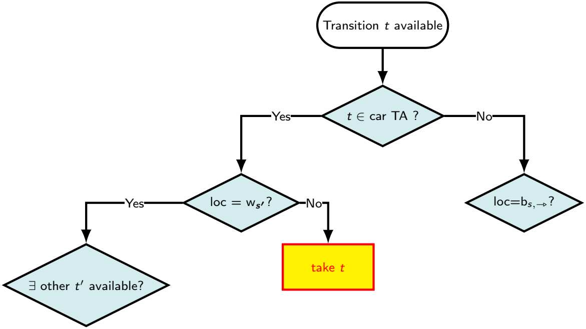

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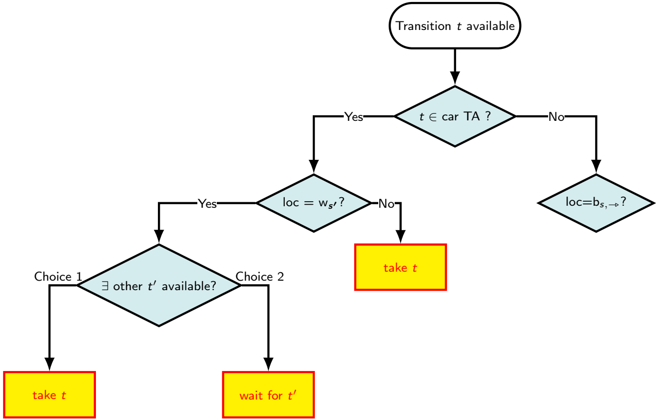

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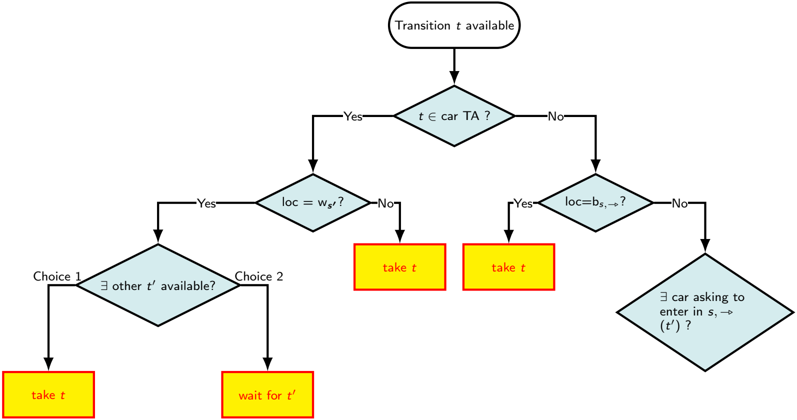

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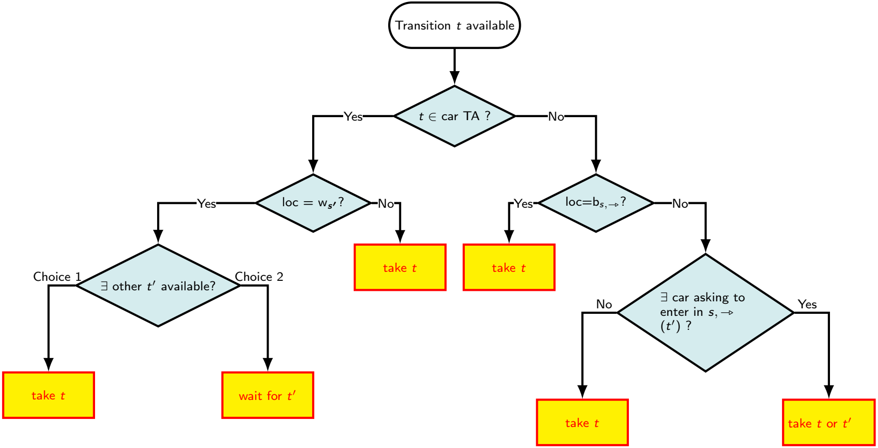

SWA-SMT solver

SMT solver

Stage 1: Reachability algorithm on system of ISWA

SWA

***Stage 2**: Model the accelleration and deceleration*

SMT

Why use of SMT solver?

DFS algorithm

Stage 1: *Reachability algorithm on a simplified ISWA model*

SWA

Solved: combinatorial aspect of the problem. Results: Important events and their relative order

Drawback: A very abstract model of speed

SMT Solver

Stage 2: *Refine the model of the speed*

SMT

RL training

Generate *a dataset for random initial positions*

Dataset

Stage 3: *Train an RL algorithm with our dasaset*

RL

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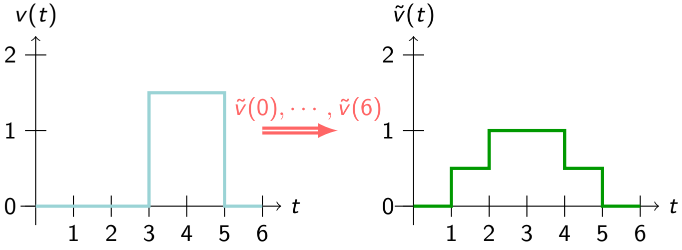
__Stage 3__: Train an RL algorithm with our dasaset

RL

- SMT solver
  - The **continuous** aspect of the problem
  - Introduce a more **realistic** model of speed

- A constant piecewise affine function
  - ▷ A more realistic model that takes into account the **dynamic of the system**
  - ▷ **Different** car speeds
  - ▷ **Bounds** on deceleration and acceleration

$$\begin{array}{ccc} v_i(t) & \Rightarrow & \tilde{v}_i(0), \cdots, \tilde{v}_i(k-1) \\ x(t) & \Rightarrow & \tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l) \end{array}$$



How to preserve security distance ?

- New positions/speeds
  - $\tilde{x}_i(k) = \sum_{l=0}^{k-1} \tilde{v}_i(l)$
  - $\tilde{v}_i(0), \cdots, \tilde{v}_i(k-1)$

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- Example of SMT solver's inequalities

For each step k :

  - $\tilde{v}_i(k) - d_{\max} \leq \tilde{v}_i(k+1) \leq \tilde{v}_i(k) + a_{\max}$

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SMT Solver

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SMT

A more realistic model of speed
Results: traces that takes into account the dynamical aspect of the problem
Drawback: runtime execution

SWA-SMT solver

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Generate *a dataset for random initial positions*

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RL

Drawback: our problem has both combinatorial and continuous aspects
Goal: get an intuition from dataset to avoid unsuccessful choices

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- RL training dataset
  - Create random initial positions/speeds for cars
  - Generate traces with the SWA-SMT solver

Model

- Markov Decision Process
  - **Deterministic running example**: deterministic transition function.

Model

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  - **Deterministic running example**: deterministic transition function.
  - **State** s_i. For each section s, if a car c is in s: $v_{i,c}, \mathsf{pos}_{i,c}, \mathsf{id}_\mathsf{c}, 1$

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  - **Action** act_i: $(\text{acc}_{i,c})_{c \in \text{Cars}}$

$$\underset{i}{(\text{pos}_{i,c}, v_{i,c})} \dashrightarrow \underset{i+1}{(\text{pos}_{i,c} + v_i, v_{i,c} + \text{acc}_{i,c})}$$

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  - **Trajectories** s_i, Obs_i, act_i

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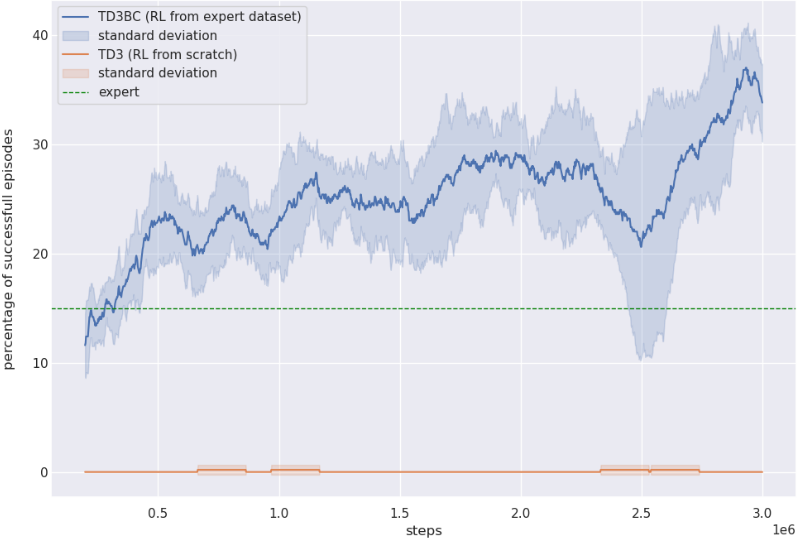
  - **Trajectories** s_i, Obs_i, act_i
  - **Reward**:
    - $+2000$ if goals are achieved
    - -100 if distance rules are not respected
    - $\nearrow$ with speed
    - $\nearrow$ with the increase of distance between cars

Results with SWA-SMT solver, post SWA-SMT solver RL and single RL training



Steps of the layered method

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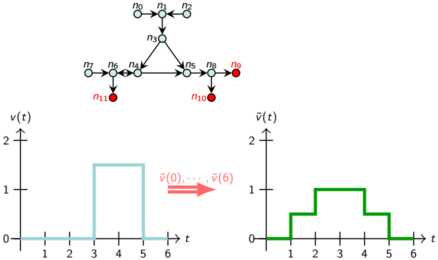
Method: get an intuition from dataset to avoid unsuccessful choices
MDP model to reward short-time episode and distance between cars

- SWA-SMT Solver



Automata-based model

Efficient algorithm
Abstract model with unrealistic speed model

Piecewise-affine speed graph

Bounded accelleration and deceleration
Different speed
SMT solver to model and solve the distance constraints

Conclusion

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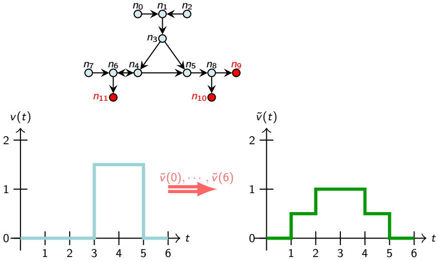


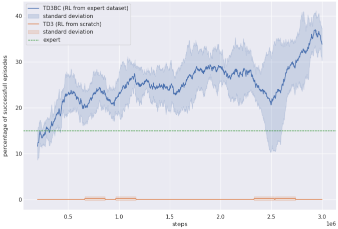
- RL training

Dataset

Trace generated with SWA-SMT solver
Random positions & speeds

Performance of RL (helped with SWA-SMT solver)

Better than single RL
Better than SWA-SMT solver
Runtime: $\sim$ *2 days*



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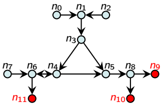

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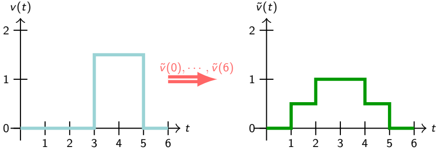

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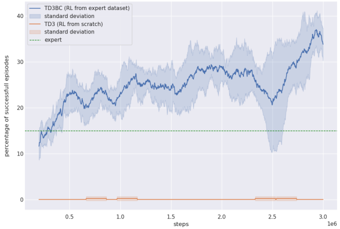

- Future work: Decentralized multi-agent systems