



DOCTORAL POSITION IN MATHEMATICS: OPEN POSITION

DESCRIPTION

The PhD student will work in the framework of the ERC Starting Grant project LoRDeT (Low Regularity Dynamics via Decorated Trees), directed by Professor Yvain Bruned. Below one has a summary of the project and its main objectives:

Low regularity dynamics are used for describing various physical and biological phenomena near criticality. The low regularity comes from singular (random) noise or singular (random) initial value. The first example is Stochastic Partial Differential Equations (SPDEs) used for describing random growing interfaces (KPZ equation) and the dynamic of the Euclidean quantum field theory (stochastic quantization). The second concerns dispersive PDEs with random initial data which can be used for understanding wave turbulence. A recent breakthrough is the resolution of a large class of singular SPDEs through the theory of Regularity Structures invented by Martin Hairer. Such resolution has been possible thanks to the help of decorated trees and their Hopf algebras structures to perform the crucial renormalisation procedures. Decorated trees are used for expanding solutions of these dynamics. They also appear for describing resonance schemes for a large class of dispersive PDEs at low regularity. The aim of this project is to push forward the scope of resolution given by decorated trees and their Hopf algebraic structures. One of the main ideas is to develop algebraic tools by the mean of algebraic deformations. We want to see the Hopf algebras used for SPDEs as deformation of those used in various fields such as numerical analysis and perturbative quantum field theory. This is crucial to work in interaction with these various fields in order to get the best result for singular SPDEs and dispersive PDEs.

We will focus on the following long-term objectives:

1. Give a notion of existence and uniqueness of two classes of singular SPDEs: the quasilinear and the dispersive SPDEs.
2. Identify the process whose dynamic has the Brownian loop measure as invariant measure via an extension of the resolution of SPDEs to discrete dynamics.
3. Develop the algebraic structures for singular SPDEs in connection with Numerical Analysis, Perturbative Quantum Field Theory and Rough Paths.

4. Use decorated trees for dispersive PDEs with random initial data and provide a systematic way to derive wave kinetic equations in Wave Turbulence.

5. Develop a software platform for decorated trees and their Hopf algebraic structures that appear in singular SPDEs and dispersive PDEs.

The PhD student will work on one or several of the project's axes, depending on her/his own skills.

Major publications connected to the ERC project LoRDeT:

- Y. Bruned, D. Manchon. "Algebraic deformation for (S)PDEs". *Journal of the Mathematical Society of Japan*, 2023, Volume 75, Issue 2, pp 485-526. arXiv:2011.059. doi:10.2969/jmsj/88028802.
- Y. Bruned, K. Schratz, "Resonance based schemes for dispersive equations via decorated trees", *Forum of Mathematics, Pi*, 2022, Volume 10, e2, pp 1-76. arxiv:2005.01649. doi:10.1017/fmp.2021.13.
- Y. Bruned, F. Gabriel, M. Hairer, L. Zambotti, "Geometric stochastic heat equations" , *Journal of the American Mathematical Society*, 2022, Volume 35, Issue 1, pp 1-80. arxiv:1902.02884. doi:10.1090/jams/977.
- Y. Bruned, A. Chandra, I. Chevyrev, M. Hairer, "Renormalising SPDEs in regularity structures" , *Journal of the European Mathematical Society*, 2021, Volume 23, Issue 3, pp 869-947. arxiv:1711.10239. doi:10.4171/JEMS/1025.
- Y. Bruned, M. Hairer, L. Zambotti, "Algebraic renormalisation of regularity structures", *Inventiones Mathematicae*, March 2019, Volume 215, Issue 3, pp 1039-1156, arxiv:1610.08468. doi:10.1007/s00222-018-0841-x.

Candidate qualification:

- Master's degree in mathematics or applied mathematics, with a strong background in probability and stochastic calculus. Strong profile in Algebra, Numerical Analysis, PDEs and Physics will also be considered.
- Programming skills will be appreciated but are not mandatory.

TERMS AND TENURE

These 36 months Doctoral position will be based in Nancy, at Institut Elie Cartan de Lorraine (IECL), Faculté des Sciences et Technologies, Campus, Boulevard des Aiguillettes, 54506 Vandœuvre-lès-Nancy. The target start date for the position is September 2025, with some flexibility on the exact start date.

HOW TO APPLY

Applicants may contact Yvain Bruned (yvain.bruned@univ-lorraine.fr) for any questions.

Applicants are requested to submit the following materials:

- A cover letter applying for the position.
- Full CV.
- Academic transcripts (unofficial versions are fine)
- One reference letter.

Deadline for application is 15/11/2024. Pre-selected candidates will be interviewed online.

Applications are only accepted through email.

All document must be sent to Yvain Bruned (yvain.bruned@univ-lorraine.fr).

JOB LOCATION

Institut Elie Cartan de Lorraine (IECL), Université de Lorraine, Faculté des Sciences et Technologies, Campus, Boulevard des Aiguillettes, 54506 Vandœuvre-lès-Nancy.