

Operads, Symmetries for QFT and Singular SPDEs

Yvain Bruned¹

Universite de Lorraine, CNRS, IECL, F-54000 Nancy, France

Email: yvain.bruned@univ-lorraine.fr

Abstract

Conference organised in Nancy from 3 to 5 of December 2025, funded by the ERC Starting Grant LoRDeT (Low Regularity Dynamics via Decorated Trees) <https://cordis.europa.eu/project/id/101075208>. The PI of the project is Yvain Bruned.

Wednesday 3 December 2025

Talks take place at IECL, Salle de Conférences.

- 14h00-14h50: Vladimir Dotsenko.

- 14h50-15h40: Benoit Fresse.

Partition Lie algebras of graphs and Maurer-Cartan elements associated to E_n -operads

I will revisit the results of previous research, carried out in collaborations with Victor Turchin and Thomas Willwacher, which give an interpretation of certain Lie algebras of graphs in terms of the deformation theory of E_n -operads. These Lie algebras of graphs are variations of the graph complexes introduced by Kontsevich, with ideas coming from the theory of Feynman diagrams, in subjects at the interface of geometric topology and mathematical physics.

In the first part of the talk, I will review the definition of the Lie algebras of graphs considered in our work, of the E_n -operad interpretation of certain graph complex models, and I will give a survey of known results in characteristic zero, including the connection of graph complex models with the theory of associators and the Grothendieck-Teichmüller theory in the E_2 -operad case.

Then I will tackle a work in progress about the definition of complexes of graphs equipped with a partition Lie algebra structure in the odd characteristic setting (partition Lie algebras are derived versions of usual restricted Lie algebra structures). I will explain that classical formality constructions can be

adapted in order to associate, to certain models of E_n -operads, a solution of the Maurer-Cartan equation in these partition Lie algebras of graphs.

- 15h40-16h10: Coffee break.
- 16h10-17h00: Muriel Livernet.

- 19h30-22h30: Conference dinner at Grand Café Foy.

Thursday 4 December 2025

Talks take place at IECL, Salle de Conférences.

- 9h30-10h20: Adrien Busnot Laurent.

The algebraic structure of exotic (Lie)-Butcher series for the foundations of stochastic geometric numerical integration

The exotic Butcher series are an algebraic object used in numerical analysis for representing Taylor expansions of the law of stochastic flows. Their use became unavoidable as the calculations of order conditions for high order become increasingly tedious, especially in the recent creation of intrinsic numerical methods on manifolds. In this talk, we will motivate the use of exotic B-series from concrete numerical questions and we will draw deep links between Hopf algebra structures and crucial numerical theories, with an emphasis on backward error analysis. We will further justify the use of such algebraic object with numerical experiments, universal geometric characterisations, and the characterisation of measure-preserving methods. This is joint work with Eugen Bronasco (Chalmers University), Baptiste Huguet (Univ Rennes), and Hans Munthe-Kaas (University of Bergen and University of Tromsø).

- 10h20-10h50: Coffee break.
- 10h50-11h40: Harprit Singh.

- 11h40-12h30: Nguyen Viet Dang.

- 12h30-14h30: Lunch.

Talks take place at IECL, Salle de Conférences.

- 14h30-15h20: Alberto Cattaneo.

Yang–Mills theory from a topological theory

Four-dimensional Yang–Mills theory can be obtained from a topological field theory of Schwarz type via a procedure known as BV pushforward, which amounts in integrating out some gauge-fixed fields. This turns out to be an equivalence in the sense that it establishes an isomorphism between the observables of the two theories, which in turns implies that their expectation values can be computed in either theory with the same outcome. I will give an introduction to the basic concept, including the BV pushforward, and discuss in details an easier case.

- 15h20-15h50: Coffee break.
- 15h50-16h40: Nils Berglund.

Perturbative renormalisation of the $\Phi_{d-\varepsilon}^4$ model via generalized Wick maps

We consider the perturbative renormalisation of the Φ_d^4 model from Euclidean Quantum Field Theory for any, possibly non-integer dimension $d < 4$. The so-called BPHZ renormalisation, named after Bogoliubov, Parasiuk, Hepp and Zimmermann, is usually encoded into extraction-contraction operations on Feynman diagrams, which have a complicated combinatorics. We show that the same procedure can be encoded in the much simpler algebra of polynomials in two unknowns X and Y , which represent the fourth and second Wick power of the field. In this setting, renormalisation takes the form of a "Wick map" which maps monomials into Bell polynomials. The construction makes use of recent results by Bruned and Hou on multiindices, which are algebraic objects of intermediate complexity between Feynman diagrams and polynomials. Joint work with Tom Klose and Nikolas Tapia.

- 16h40-17h30: Alberto Bonicelli.

Representation of the semigroup for Itô diffusions via (multi-indices) exotic B-series and Feynman diagrams

In this talk, after justifying the expansion of the semigroup of a one-dimensional Itô diffusion as a power series in time, I will build on previous results on expansions labelled by exotic rooted trees to derive an explicit expression for the combinatorial factors involved. A key step is the extension of the notion of tree factorial and Connes-Moscovici weights to this richer family of rooted trees. As a result, we obtain an exotic Butcher series representation of the semigroup, suitable for a comparison with the perturbative path integral construction of the statistics of the diffusion, known in the literature as Martin-Siggia-Rose formalism. Computations in the latter framework are based on the erroneous assumption that the measure of the path integral can be seen as a perturbation

of a Gaussian measure. Resorting to multi-indices to represent pre-Feynman diagrams, I will shed some light on why, even if starting from such an assumption, the results happen to be correct.

Friday 5 December 2025

Talks take place at IECL, Salle de Conférences.

- 9h00-9h50: Nurlan Ismailov.

Symmetric elements in a free Novikov algebra

Let $\mathbb{F}$ be a field of characteristic zero. An algebra is called *right-symmetric* if it satisfies the identity

$$(a, b, c) = (a, c, b)$$

where $(a, b, c) = (ab)c - a(bc)$ is the associator of a, b, c .

A right-symmetric algebra is said to be *Novikov* if it satisfies the additional identity

$$a(bc) = b(ac).$$

Let $P_n = \mathbb{F}[x_1, \dots, x_n]$ be the polynomial algebra over $\mathbb{F}$ in the variables $x_1, \dots, x_n$. Denote by $W_n = \{u\partial_i \mid u \in P_n, 1 \leq i \leq n\}$ the Witt algebra of index n (the Lie algebra of all derivations of P_n). Define

$$u\partial_i \cdot v\partial_j = (v\partial_j(u))\partial_i$$

Then $L_n = (W_n, \cdot)$ is right-symmetric, called the right-symmetric Witt algebra. If $n = 1$, then L_1 becomes a Novikov algebra, but if $n > 1$, it is not Novikov. The right-symmetric Witt algebra L_1 , more explicitly Novikov-Witt algebra, generates the variety of all Novikov algebras over $\mathbb{F}$ [1], [2]. The commutator of L_n corresponds to the Witt algebra W_n of index n . Finding a criterion for determining Lie elements and solving the identity problem for W_1 are well-known open questions [3].

In our talk, we consider the symmetrization product $a \circ b = a \cdot b + b \cdot a$ of the multiplication of L_1 . We focus on a subalgebra of $L_1^{(+)}$ generated by a set X and refer to its elements as symmetric elements of L_1 . We find a basis for the space of symmetric elements in terms of differential polynomials. Furthermore, we provide a criterion for identifying elements of the space of symmetric elements in L_1 . Finally, we give a complete description of the module structure of this space over the symmetric group.

The talk is based on the results of joint work with A Dzhumadil'daev.

References

- [1] A. Dzhumadil'daev, C. Löfwall, *Trees, free right-symmetric algebras, free Novikov algebras and identities*, Homology Homotopy Appl., Vol. 4(2), (2002), 165–190.

- [2] L. Makar-Limanov L., U. Umirbaev *The Freiheitssatz for Novikov algebras*, TWMS J. Pure Appl. Math., Vol. 2(2), (2011), 228–235.
- [3] Yu. Razmyslov *Identities of Algebras and Their Representations*. Translations of Mathematical Monographs. Vol. 138., Amer. Math. Soc., Providence, RI, 1994.

- 9h50-10h20: Coffee break.
- 10h20-11h10: Carlo Bellingeri.

- 11h10-12h00: Dominique Manchon.

Rough paths and controlled rough paths: a general Hopf-algebraic setting

We present rough paths and controlled rough paths associated to a broad class of combinatorial Hopf algebras, encompassing T. Lyons' original formulation, M. Gubinelli's branched presentation, and the planarly branched presentation associated to the Munthe-Kaas–Wright Hopf algebra of planar forests. Joint work with Xing Gao, Nannan Li and Zhicheng Zhu.