Higher-Dimensional (Timed) Automata

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20th of November 2024

Higher Dimensional Automata : non-interleaving concurrency

- **<u>Goal</u>** : represent non-interleaving concurrency : $\boxed{a||b \neq a.b + b.a}$
- **<u>Dimension</u>** : maximal number of simultaneous events.
- Higher dimensional Automata of <u>**dimension** 1</u> ($\mathcal{A}_1$, left), and <u>**dimension** 2</u> ($\mathcal{A}_2$, right) :

Example :

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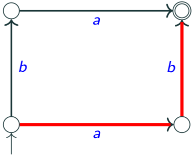
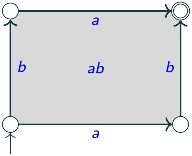
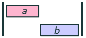




Example :



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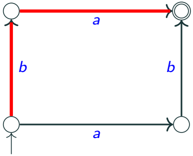

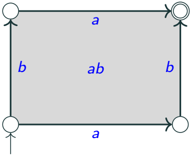

Example :

- **Language of the HDA** :

$$L(\mathcal{A}_1) = \{ (\ a \rightarrow b\ ), (\ b \rightarrow a\ ) \}$$

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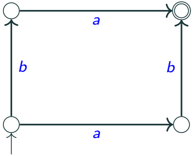

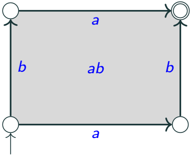

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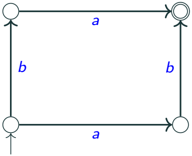

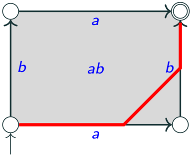

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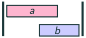


- **Language of the HDA** :

$$L(\mathcal{A}_2) = \left\{ \begin{pmatrix} a \\ b \end{pmatrix}, \left(a \to b \right), \left(b \to a \right) \right\}$$

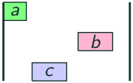
iiPomset : Interval-order partially ordered multisets with interfaces

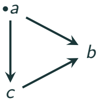
- **iiPomset** : Representation of events as **intervals** with **starting/ending interfaces** :

  $\longrightarrow$: precedence

  $\dashrightarrow$: event order

- **Examples** :





iiPomset : Interval-order partially ordered multisets with interfaces

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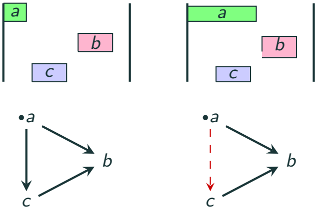

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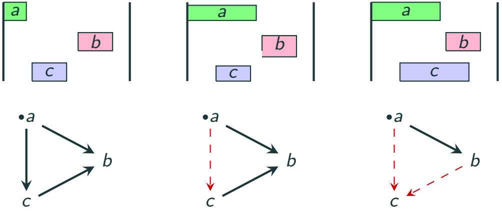

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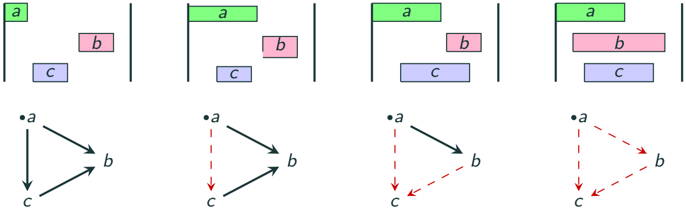

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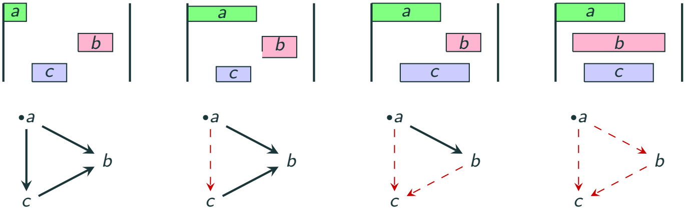
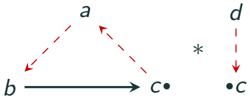


- **Gluing composition** between two iiPomset



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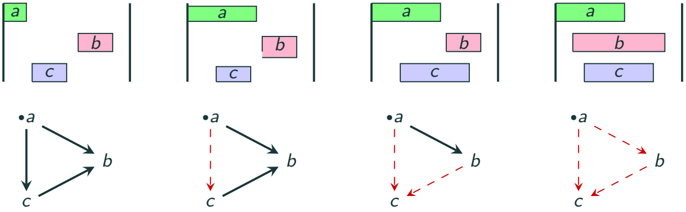

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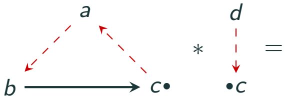

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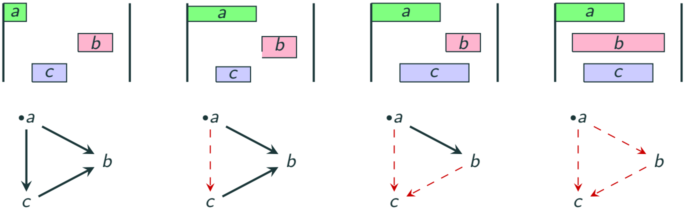

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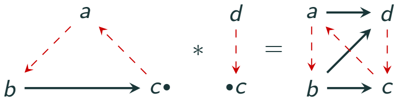
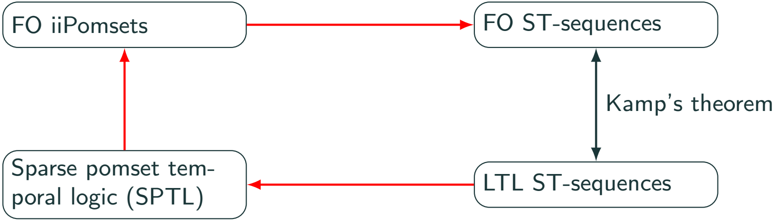


Logic over iiPomset

- Define **logics over iiPomsets** : First Order logic & LTL-like logic (SPTL)
- Prove the **equivalence** between logics :



FO iiPomsets

FO ST-sequences

Kamp's theorem

Sparse pomset temporal logic (SPTL)

LTL ST-sequences

How to link FO over word to FO over Pomset ?

- **Discrete Pomsets** :

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Conclist

$\begin{bmatrix} a \\ b \end{bmatrix}$

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Conclist	Starter	Terminator
$\begin{bmatrix} a \\ b \end{bmatrix}$	$\begin{bmatrix} a\bullet \\ b\bullet \end{bmatrix}$	$\begin{bmatrix} \bullet a \\ \bullet b\bullet \end{bmatrix}$

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- **ST-sequence** :

$$[a\bullet] * \begin{bmatrix} \bullet a\bullet \\ b\bullet \end{bmatrix}, \begin{bmatrix} a\bullet \\ b\bullet \end{bmatrix}, [b\bullet] * \begin{bmatrix} a\bullet \\ \bullet b\bullet \end{bmatrix}$$

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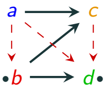

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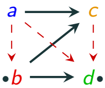

$$= \begin{bmatrix} a\bullet \\ \bullet b\bullet \end{bmatrix} * \begin{bmatrix} \bullet a\bullet \\ \bullet b \end{bmatrix}$$

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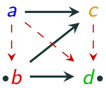

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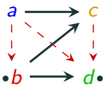

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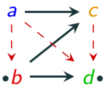

$$= \begin{bmatrix} a\bullet \\ \bullet b\bullet \end{bmatrix} * \begin{bmatrix} \bullet a\bullet \\ \bullet b \end{bmatrix} * \begin{bmatrix} \bullet a\bullet \\ d\bullet \end{bmatrix} * \begin{bmatrix} \bullet a \\ \bullet d\bullet \end{bmatrix} * \begin{bmatrix} c\bullet \\ \bullet d\bullet \end{bmatrix} * \begin{bmatrix} \bullet c \\ \bullet d\bullet \end{bmatrix} \text{sparse}$$

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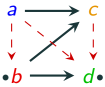

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First Order logic over iiPomsets (dim $\leq k$)

- FO for iiPomsets :

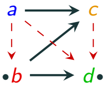
$$\varphi, \psi ::= \neg\varphi \mid \varphi \wedge \psi \mid \exists x.\varphi \mid \overset{a \in \Sigma}{a}(x) \mid \overset{\text{starter}}{\mathbf{s}}(x) \mid \underset{\text{terminating}}{\mathbf{t}}(x) \mid x \underset{\text{precedence}}{<} y \mid x \overset{\text{event order}}{\dashrightarrow} y$$

- FO over ST-sequence (same as words) :

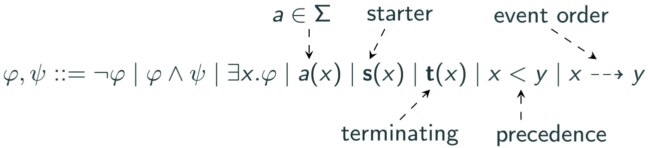
$$\varphi, \psi ::= \neg\varphi \mid \varphi \wedge \psi \mid \exists x.\varphi \mid ST(x) \mid x < y$$

Examples

$$\varphi, \psi ::= \neg\varphi \mid \varphi \wedge \psi \mid \exists x.\varphi \mid \overset{a \in \Sigma}{a(x)} \mid \overset{\text{starter}}{\mathbf{s}(x)} \mid \underset{\text{terminating}}{\mathbf{t}(x)} \mid x \underset{\text{precedence}}{<} y \mid x \overset{\text{event order}}{\dashrightarrow} y$$

-  $\overset{?}{\models} \forall x.b(x) \Rightarrow \exists y.(\mathbf{t}(y) \wedge x < y)$

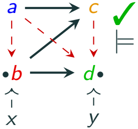
Examples



$a \in \Sigma$ starter event order

$$\varphi, \psi ::= \neg\varphi \mid \varphi \wedge \psi \mid \exists x.\varphi \mid a(x) \mid \mathbf{s}(x) \mid \mathbf{t}(x) \mid x < y \mid x \dashrightarrow y$$

terminating precedence

- 

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Examples

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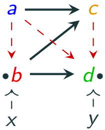
- 

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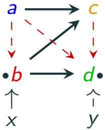
- 

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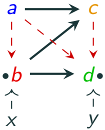
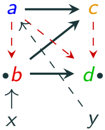
- 

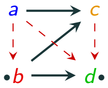
✓ $\models \forall x.b(x) \Rightarrow \exists y.(\mathbf{t}(y) \wedge x < y)$

- 

✓ $\models \forall x.b(x) \Rightarrow \exists y.(x \dashrightarrow y)$

ST-sequence vs Pomset : how to link the variables ?

- **Variables** do not represent the same **objects** in Pomset and in ST-sequence :



$$\begin{bmatrix} a\bullet \\ \bullet b \bullet \end{bmatrix} * \begin{bmatrix} \bullet a \bullet \\ \bullet b \end{bmatrix} * \begin{bmatrix} \bullet a \bullet \\ d \bullet \end{bmatrix} * \begin{bmatrix} \bullet a \\ \bullet d \bullet \end{bmatrix} * \begin{bmatrix} c \bullet \\ \bullet d \bullet \end{bmatrix} * \begin{bmatrix} \bullet c \\ \bullet d \bullet \end{bmatrix}$$

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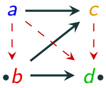

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Example : $(\begin{bmatrix} a\bullet \\ \bullet b \bullet \end{bmatrix}, 1) = (\begin{bmatrix} \bullet a \bullet \\ d \bullet \end{bmatrix}, 1) = a$

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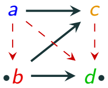

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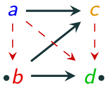

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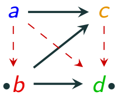
LTL-like logic for iiPomset : SPTL

- Conclist decomposition :



$$b \mid \begin{smallmatrix} a \\ b \end{smallmatrix} \mid a \mid \begin{smallmatrix} a \\ d \end{smallmatrix} \mid d \mid \begin{smallmatrix} c \\ d \end{smallmatrix} \mid d$$

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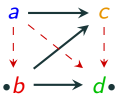

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$$\varphi, \psi ::= \mathcal{C} \mid \neg\varphi \mid \varphi \wedge \psi \mid X\varphi \mid \varphi U \psi$$

conclist

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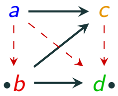

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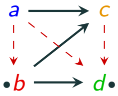
- 

$\overset{?}{\models} XX a$,

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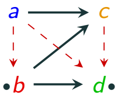

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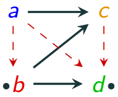
- 

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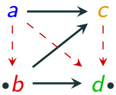

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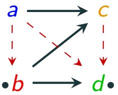
- 

✓ $\models XXa$, $a \longrightarrow c \overset{?}{\models} aUc$

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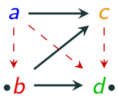

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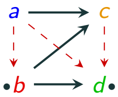
- 

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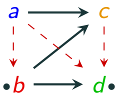

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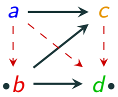
- 

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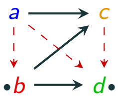

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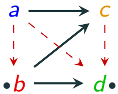
- 

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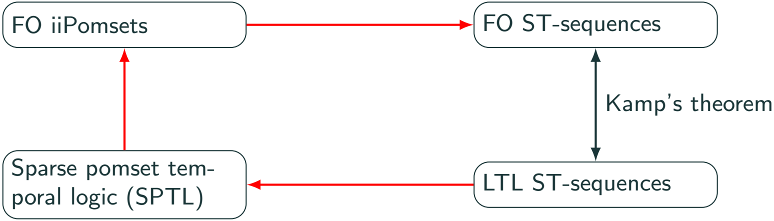
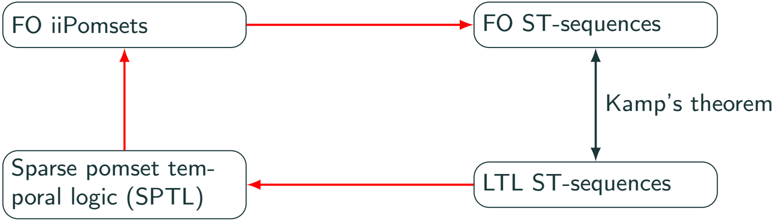
FO=SPTL



Enzo Erlich's **current work** :

FO=SPTL



FO iiPomsets

FO ST-sequences

Kamp's theorem

LTL ST-sequences

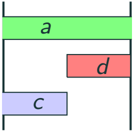
Sparse pomset temporal logic (SPTL)

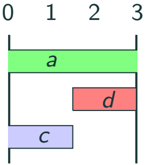
Enzo Erlich's **current work** :

- Exploring over possibilities for LTL-like logics
- Compute the cost of this FO → SPTL (and ←) translation.

Higher Dimensional Timed Automata : Let's add time with Timed iiPomsets

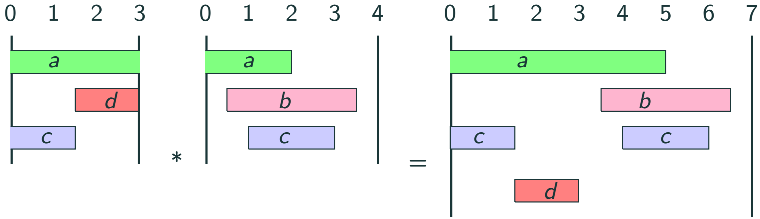
- **Timed iiPomsets** is composed of :
  - An **iiPomset**
  - A **duration** d
  - A map σ labelling all events to **time intervals**.
- iiPomsets (left), Timed iiPomsets (right)





  - **Starter** : x_1, x_3 of respective label a and c
  - **Target** : x_2 of label b
  - $\sigma(a) = (0, 3), \sigma(b) = (0, 1.5), \sigma(c) = (1.5, 3)$
  - **Total duration** $d = 3$

Gluing on Timed iiPomsets



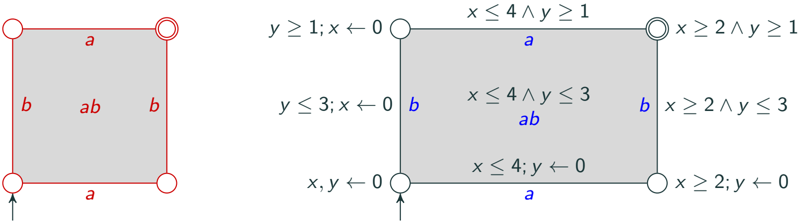
Higher Dimensional Timed Automata : intuition

- Definition :

A HDTA is a tuple $(X, X_\perp, X_\top, \lambda, \mathcal{C}, \mathrm{inv}, \mathrm{exit})$ where :

- ▷ $\boldsymbol{(X, X_\perp, X_\top, \lambda)}$ is an HDA

- Example with events *a* and *b* : HDA (left) of the HDTA (right)



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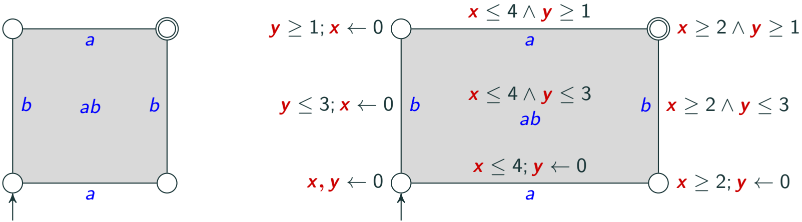

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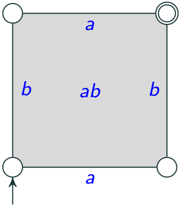

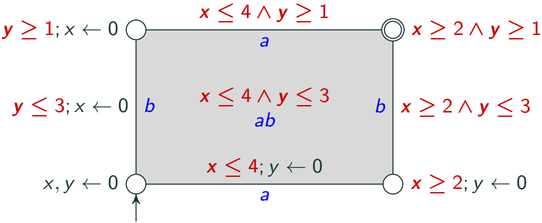

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- ▷ exit **:** $\boldsymbol{X} \to 2^{\boldsymbol{\mathcal{C}}}$ assign **exit conditions** to cells.

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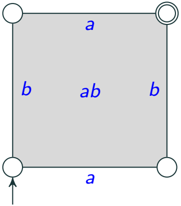

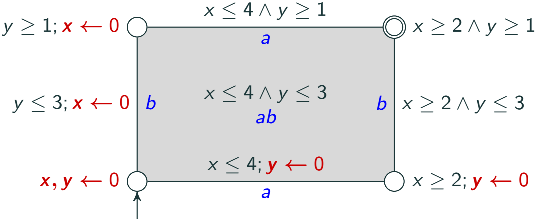

Examples of HDTA

Quizz : suppose that a and b are not in concurrency

Let us draw the HDTA of $a : [2, 4]$ and $b : [1, 3]$ separately :

Timing duration of events :

- $a : [2, 4]$ time units
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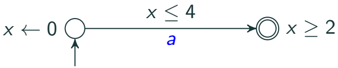

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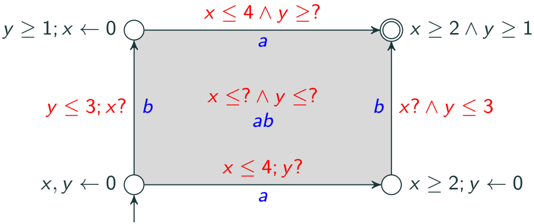

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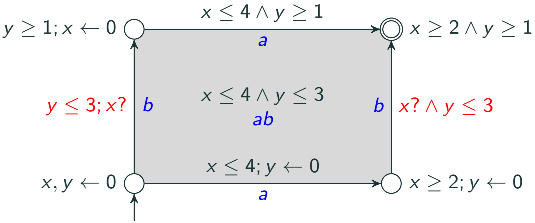

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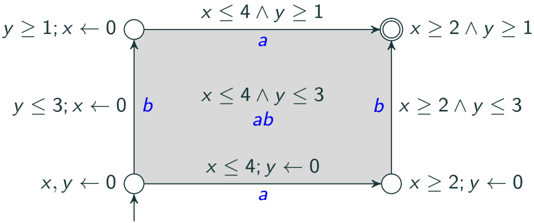

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Future works

Higher Dimensional Automata :

- **Model Checking** for HDA (Enzo Erlich, Jeremy Ledent)
- **Implementation** (Philipp Schlehuber-Caissier in Telecom)

Higher Dimension Timed Automata :

- Logics for **Timed iiPomsets**
- **Distance** between Timed iiPomsets
- **Quantitative MC, Monitoring**, etc for HDTA