

Noncrossing partitions and a generalization of COXETER sortable elements in type \tilde{A}

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Part I

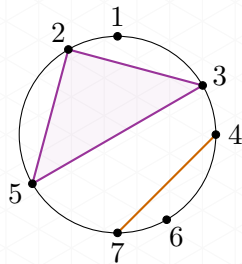
Introduction & Motivation

Noncrossing partitions and c -sortable elements

- ▶ Let W be a COXETER group, c a COXETER element and \leq_R the absolute order on W . c -noncrossing partitions of $W = [1, c]_{\leq_R}$.

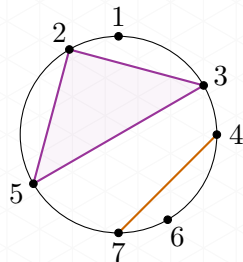
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- ▶ When W is finite : COXETER-CATALAN combinatorics. E.g. $W = \mathfrak{S}_n$, $|[1, c]_{\leq_R}| = C_n = \frac{1}{n+1} \binom{2n}{n}$, combinatorial model, ...



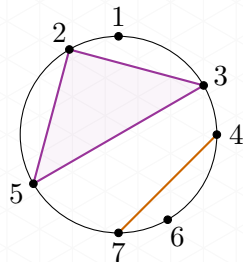
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- ▶ READING (2005) : c -sortable elements of W defined using a particular reduced expression of an element $w \in W$. When W is finite, $nc_c : \text{Sort}_c(W) \rightarrow [1, c]_{\leq_R}$ is a **bijection**.



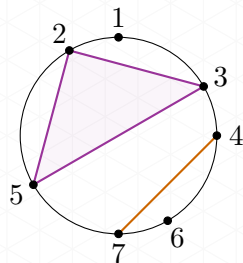
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- ▶ When W is infinite, nc_c is still defined but **not surjective**.
- ▶ **Goal** : generalize the c -sortable elements and the map nc_c to obtain a bijection.



Some notation about the affine symmetric group

▷ An **affine permutation** σ is a permutation of \mathbb{Z} such that :

$$(i) \quad \forall i \in \mathbb{Z}, \sigma(i + n) = \sigma(i) + n$$

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▷ The **one-line notation** of an affine permutation is infinite

i	...	-3	-2	-1	0	1	2	3	4	5	6	7	...
$\sigma(i)$...	-7	0	1	-4	3	4	-1	6	7	2	9	...

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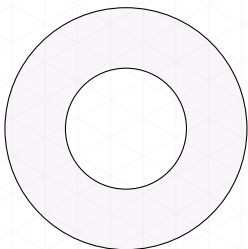
- ▶ An **inversion** of σ is a pair $(i, j) \in \mathbb{Z}^2 / (n, n)\mathbb{Z}$ such that $i < j$ and j appears before i in the one-line notation. A **cover reflection** of σ is an inversion where j and i are consecutive in the one-line notation of σ .
- ▶ Example : σ above has 4 inversions $\{(2, 3), (2, 4), (2, 6), (2, 7)\}$ and 1 cover reflection, $(2, 7)$.

Part II

READING's map nc_c in type \tilde{A}

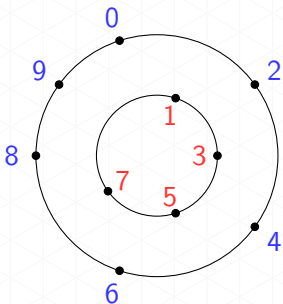
Noncrossing partitions in $\widehat{\mathfrak{S}}_n$

- ▷ Combinatorial model by DIGNE (2006) and READING (2024) : **noncrossing partitions of an annulus.**



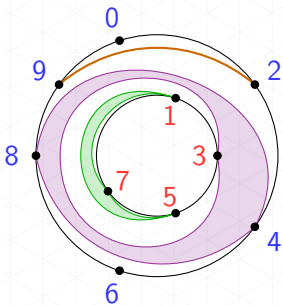
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- ▷ A COXETER element of $\widehat{\mathfrak{S}}_n$ is characterized by a **nontrivial partition** in two parts $L_c \sqcup R_c = \llbracket 0, n-1 \rrbracket$, e.g. $c = \{0, 2, 4, 6, 8, 9\} \sqcup \{1, 3, 5, 7\}$.



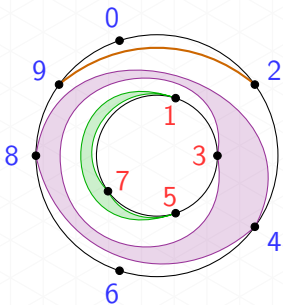
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Theorem 1 (DIGNE, 2006).

The set of c -noncrossing partitions of an annulus is in bijection with the set of c -noncrossing partitions of $\widehat{\mathfrak{S}}_n$.

- ▷ **Difficulties:** labels, curved polygons and annular blocks.

c -sortable elements of $\widehat{\mathfrak{S}}_n$

- ▷ Let $L_c \sqcup R_c$ be a nontrivial partition of $\llbracket 0, n-1 \rrbracket$ defining a COXETER element c .

Theorem 2 (A., 2026).

An affine permutation $\sigma \in \widehat{\mathfrak{S}}_n$ is c -sortable if and only if its one-line notation **avoids the patterns** kij with $i < j < k$ and $j \bmod n \in L_c$ and the patterns jki with $i < j < k$ and $j \bmod n \in R_c$.

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- ▷ Example : $L_c = \{1, 3\}$, $R_c = \{0, 2\}$, $\sigma = [-1, 5, 4, 2] \in \widehat{\mathfrak{S}}_4$.

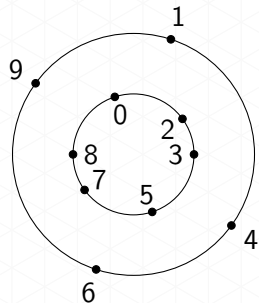
$\dots, -5, 1, 0, -2[-1, 5, \mathbf{4}, \mathbf{2}]\mathbf{3}, 9, 8, 6, \dots$

There is a pattern kij with $j \equiv 3 \pmod 4 \in L_c$, thus σ is not a c -sortable element of $\widehat{\mathfrak{S}}_4$.

From c -sortable elements to c -noncrossing partitions

▷ Let $\sigma \in \widehat{\mathfrak{S}}_n$ be a c -sortable element.

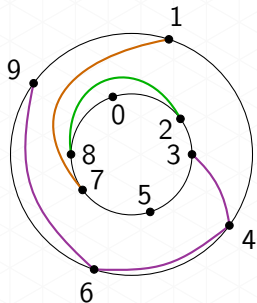
▷ Example : $L_c = \{1, 4, 6, 9\}$, $R_c = \{0, 2, 3, 5, 7, 8\}$,
 $\sigma = [-2, 0, 9, 6, 4, 3, 5, 11, 7, 12] \in \widehat{\mathfrak{S}}_{10}$.



From c -sortable elements to c -noncrossing partitions

- ▷ Let $\sigma \in \widehat{\mathfrak{S}}_n$ be a c -sortable element. We list the **cover reflections** of σ , and draw a line connecting the coefficients of each cover reflection in the annulus.

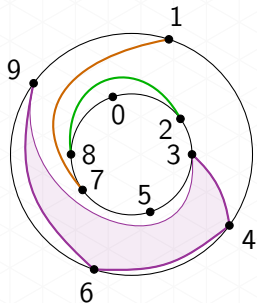
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 reflections of σ are $(6, 9)$, $(4, 6)$, $(3, 4)$, $(7, 11)$ and
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From c -sortable elements to c -noncrossing partitions

- ▶ Let $\sigma \in \widehat{\mathfrak{S}}_n$ be a c -sortable element. We list the **cover reflections** of σ , and draw a line connecting the coefficients of each cover reflection in the annulus. We then take the « **convex hull** » of each connected component to obtain a c -noncrossing partition.

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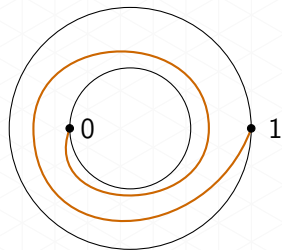
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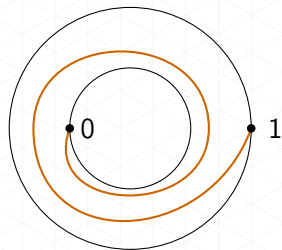


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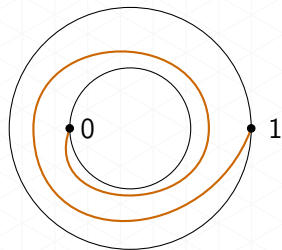


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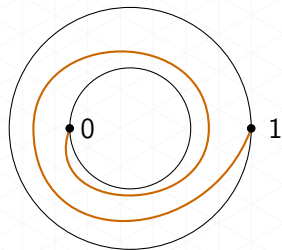


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Part III

Generalization of c -sortable elements in type \tilde{A}

Translation-invariant total orders

- ▷ An affine permutation can be viewed as a **total order** on \mathbb{Z} such that for all $x, y \in \mathbb{Z}$:
1. $x \prec y \Leftrightarrow x + n \prec y + n$
 2. $x \prec x + n$
 3. $d(x, y) = |\{z \in \mathbb{Z} \mid x \prec z \prec y \text{ or } y \prec z \prec x\}| < +\infty$

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▷ Example : let $n = 5$.

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and is of **shape** $[\{2, 5\}][\underline{\{1, 3\}}][\{4\}]$.

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▶ For technical reasons, we also keep condition 2. for all $x \in \mathbb{Z}$ such that $d(x, x + n) = 1$.

c -sortable TITOs

- ▶ Taking inspiration from c -sortable affine permutation, we define **c -sortable TITOs** using pattern avoidance.

Definition 4.

Let \prec be a TITO on \mathbb{Z} , we say that \prec is c -sortable if it avoids the patterns $k \prec i \prec j$ with $j \bmod n \in L_c$ and $j \prec k \prec i$ with $j \bmod n \in R_c$

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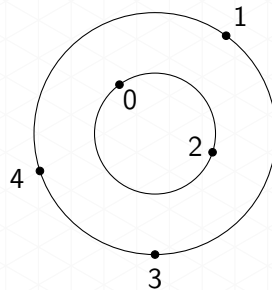
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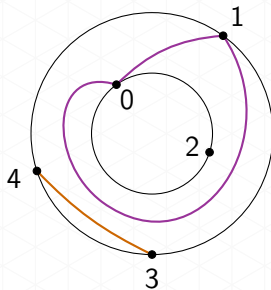
The map $nc_c^{\tilde{A}}$

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- ▶ Let $L_c = \{1, 3, 4\}$, $R_c = \{0, 2\}$ and $\prec = [4, 3][\underline{5}, 1][2]$. It is a c -sortable TITO



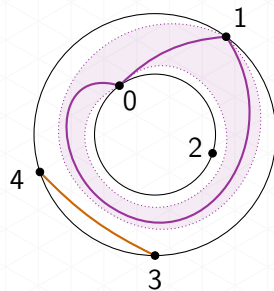
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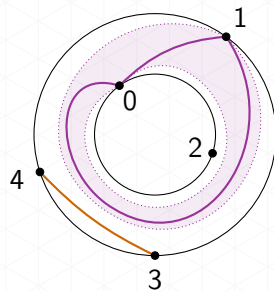
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Theorem 5 (A., arXiv:2605.02668).

The map $nc_c^{\tilde{A}}$ is a bijective map between the set of c -sortable TITOs on \mathbb{Z} and the set of c -noncrossing partitions of $\tilde{\mathfrak{S}}_n$.

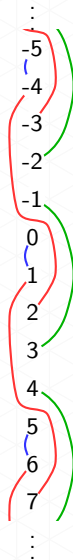
Moreover, if \prec is a TITO on \mathbb{Z} whose one-line notation is the one-line notation of an affine permutation σ , then $nc_c^{\tilde{A}}(\prec) = nc_c(\sigma)$.

Part IV

Tools and main idea for the proof of the generalization

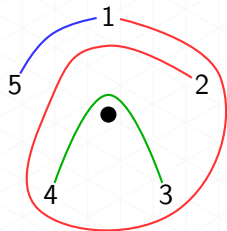
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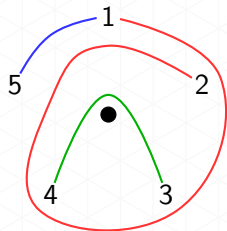


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Theorem 6 (BARKLEY, 2025).

Cyclic noncrossing arc diagrams are in bijection with TITOs with no two consecutive waxing blocks.

- ▶ Moreover, under this bijection, we have that (p, q) is a **cover reflection** of a TITO \prec if and only if there exists an **arc** from p to q in the cyclic noncrossing arc diagram.



Cyclic c -noncrossing arc diagrams

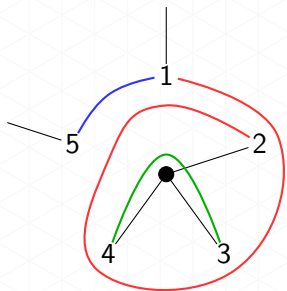
- ▷ Under BARKLEY's bijection, what is the image of the c -sortable TITOs ?

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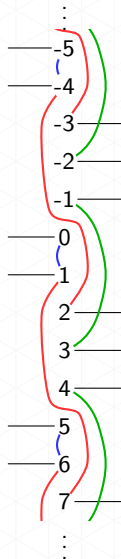
- ▷ Under BARKLEY's bijection, what is the image of the c -sortable TITOs ?
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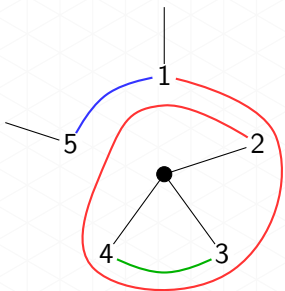


- ▶ We represent them with horizontal half lines going left (resp. right) from each point in L_c (resp. R_c) modulo n .
- ▶ In a cyclic c -noncrossing arc diagram, an arc is **uniquely defined** by its endpoints.

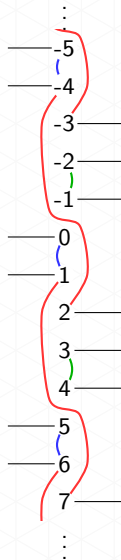


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Cyclic c -noncrossing arc diagrams and c -noncrossing partitions

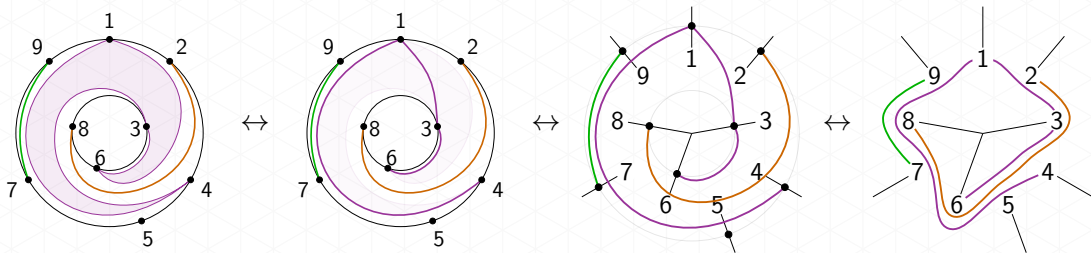
Theorem 7 (A., 2026).

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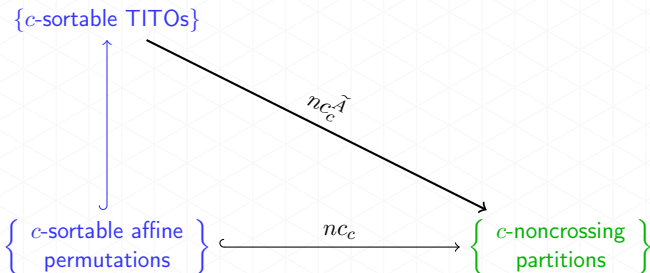


- Under this bijection, each part of a c -noncrossing partition is the convex hull of a maximal chain of arcs.

Proof of injectivity

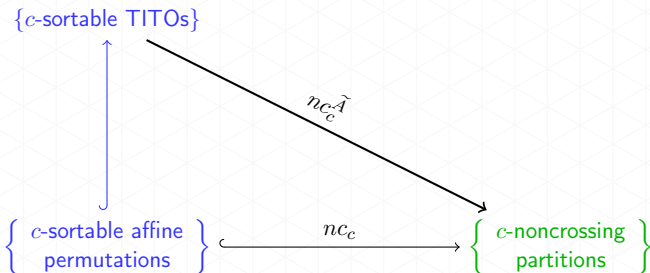


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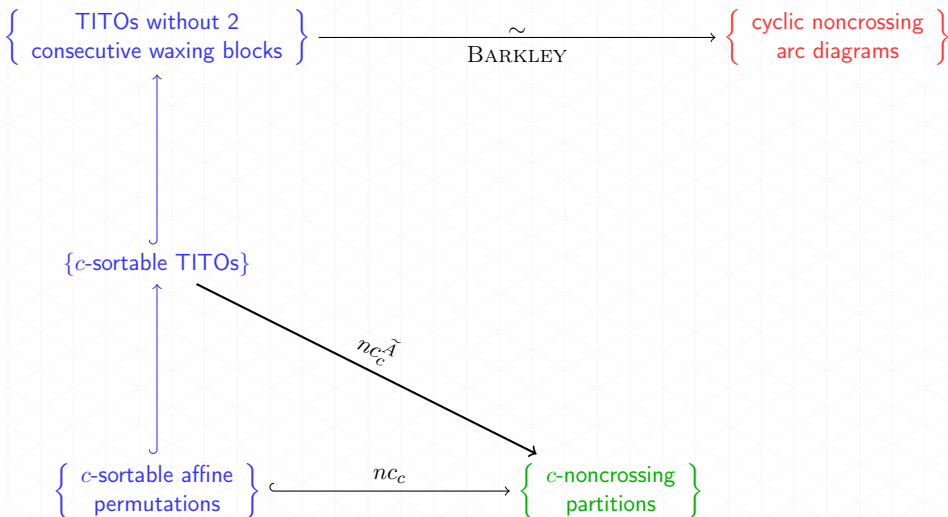


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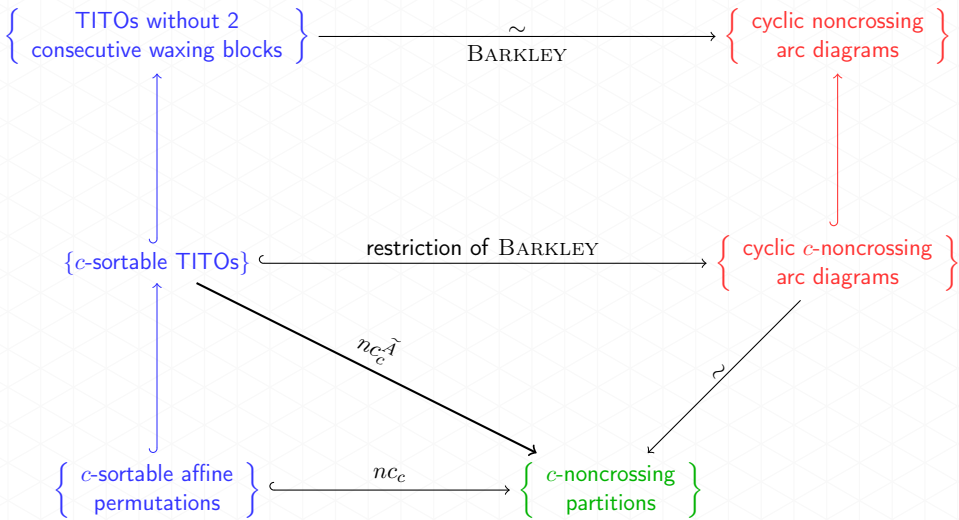
{ cyclic noncrossing
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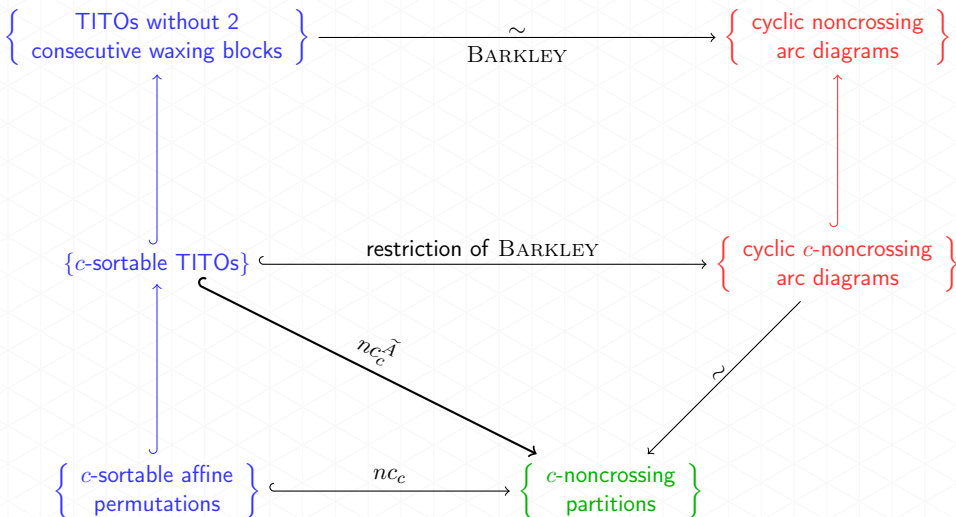
Proof of injectivity



Proof of injectivity



Proof of injectivity



**Thank you very much
for your attention!**