Reaching Your Goal Optimally by Playing at Random with no Memory

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TickTac meeting
December 4, 2020

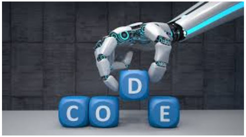
Motivation : game theory for synthesis

Classic approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents : environment and controller



Code synthesis

Correct by construction : synthesis of controller

Different sorts of games

Qualitative games

Reach or avoid some (sequences of) states

Quantitative games

- Consider quantitative parameters : energy consumption...
- Compare distinct strategies

Different sorts of games

Qualitative games

Reach or avoid some (sequences of) states

Quantitative games

- Consider quantitative parameters : energy consumption...
- Compare distinct strategies

Shortest-Path games

- Combination of a qualitative with a quantitative objective
- Reach a target with a minimum cost

Different sorts of games

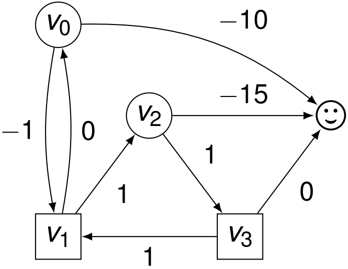
Timed games

- Consider timed issues : receive a message...
- Infinite games

Weighted Timed games

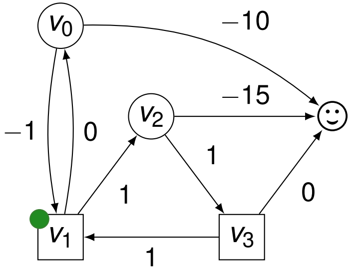
Combination of timed games and shortest path games.

Shortest Path Game



Shortest Path Game

□ Adam ○ Eve

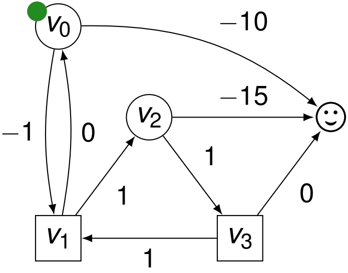


How to play?

Move a token along an edge

$$\pi = v_1$$

Shortest Path Game



How to play?

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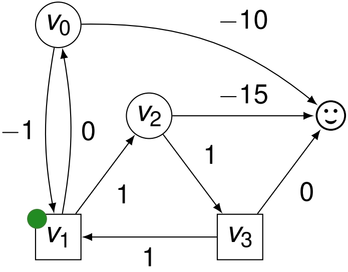

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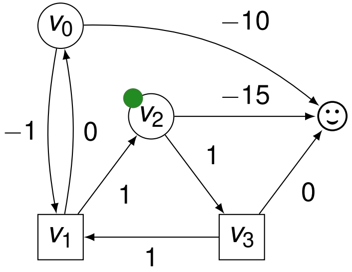

How to play?

Move a token along an edge

$$\pi = v_1 v_0 v_1 v_2$$

Shortest Path Game

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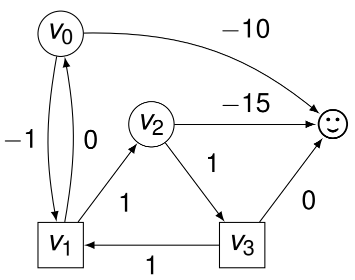

Play

Infinite path or reach the target

$$\pi = (v_i)_i \in V^\omega \qquad \pi = (v_i)_i ☺$$

How to play?

Move a token along an edge

$$\pi = v_1 v_0 v_1 v_2 v_3 ☺$$

Shortest Path Game

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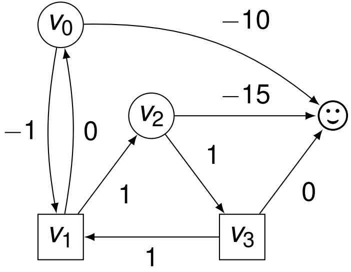

Play

Infinite path or reach the target

$$\pi = (v_i)_i \in V^\omega \qquad \pi = (v_i)_i \smiley$$

How to play?

Move a token along an edge

$$\pi = v_1 v_0 v_1 v_2 v_3 \smiley$$

Shortest Path payoff of a play π

$$\mathbf{SP}(\pi) = \begin{cases} \sum_{i=0}^{n-1} w((\pi_i, \pi_{i+1})) & \text{if } \exists n \text{ (the smallest) s.t. } \pi_n = \smiley \\ +\infty & \text{if } \pi \text{ does not reach } \smiley \end{cases}$$

Shortest Path Game

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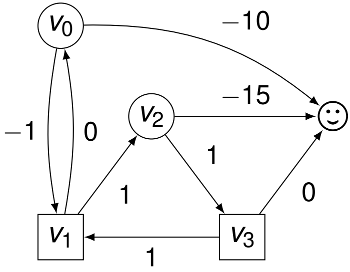

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How to play?

Move a token along an edge

$$\pi = v_1 v_0 v_1 v_2 v_3 ☺$$
$$\mathbf{SP}(\pi) = 0 + (-1) + 1 + 1 + 0 = 1$$

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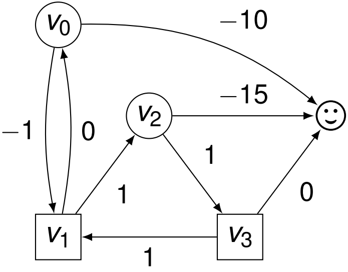
Shortest Path Game

Adam Eve



Objectives

Eve maximise the payoff

Adam minimise the payoff

Shortest Path payoff of a play π

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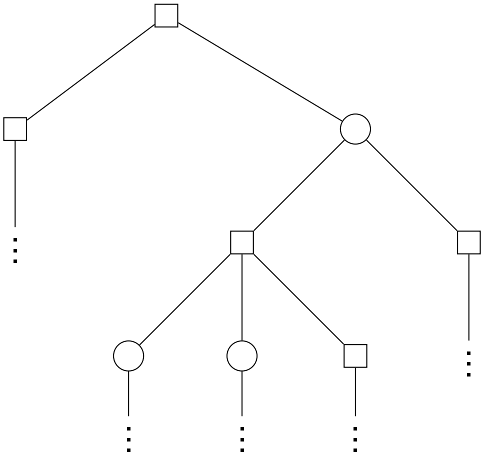
Strategies for Adam



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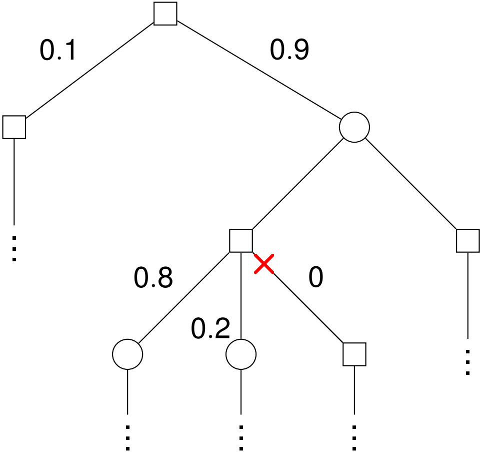


A strategy

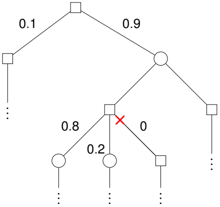
$\sigma : V^* V_{Adam} \rightarrow \Delta(V)$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Strategies for Adam

Infinite memory

$\sigma : V^* V_{Adam} \to \Delta(V)$



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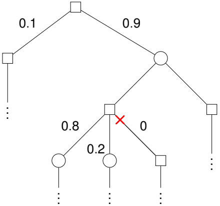
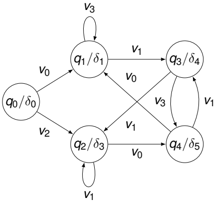


Finite memory

Moore machine



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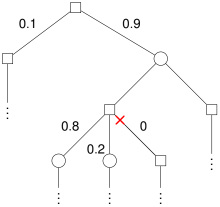

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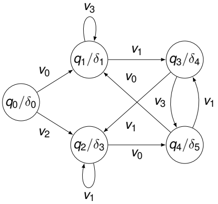
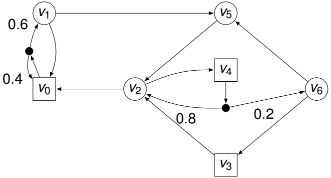


Memoryless

$\sigma : V_{Adam} \to \Delta(V)$



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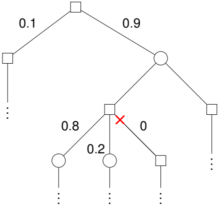

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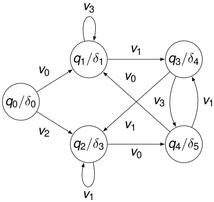

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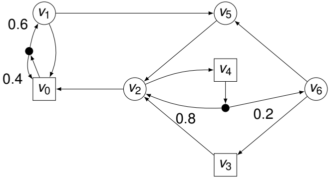
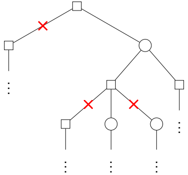


Deterministic

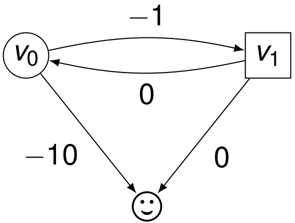
$$\sigma : V^* V_{Adam} \to V$$



Deterministic Strategies

$\sigma : V^* \times V_{\mathsf{Adam}} \to V$

$\boxed{\sigma}$ Adam τ Eve



Value

$$\overline{\mathsf{dVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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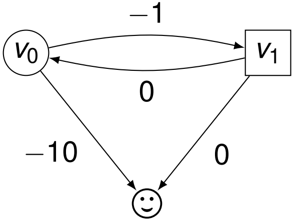

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Determinacy

$$\mathrm{dVal}(v) = \overline{\mathrm{dVal}}(v) = \underline{\mathrm{dVal}}(v)$$

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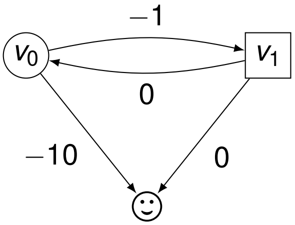

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Bellman equation

$$\text{dVal}(v) = \begin{cases} 0 & \text{if } v = ☺ \\ \max_{v'} (w(v, v') + \text{dVal}(v')) & \text{if } v \in V_{\text{Eve}} \\ \min_{v'} (w(v, v') + \text{dVal}(v')) & \text{if } v \in V_{\text{Adam}} \end{cases}$$

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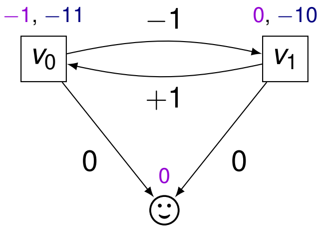

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Unicity

Bellman equation may have many solutions.

Bellman equation

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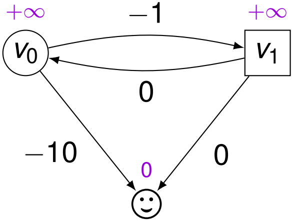

Value

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Value iteration

- Compute dVal as a greatest fixpoint

Bellman equation

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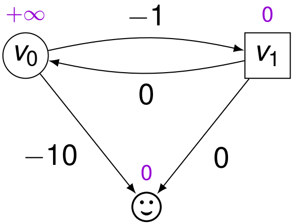

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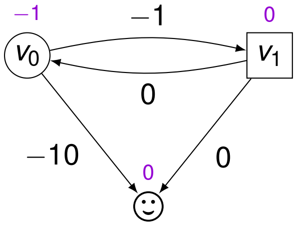

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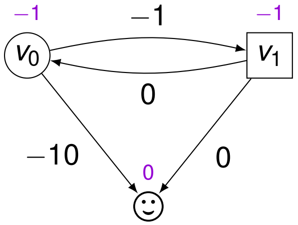

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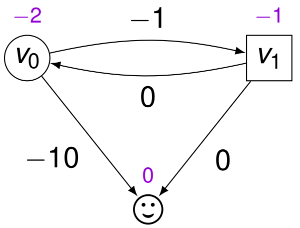

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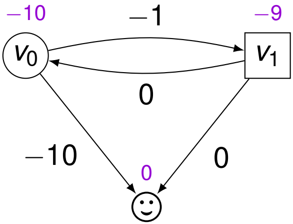

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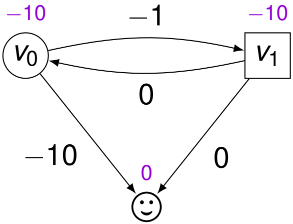

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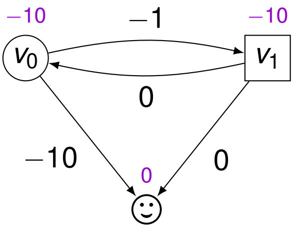

Value

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Value iteration

- Compute dVal as a greatest fixpoint
- Complexity: pseudo-polynomial

Bellman equation

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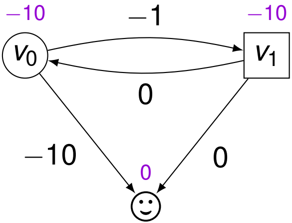


Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy

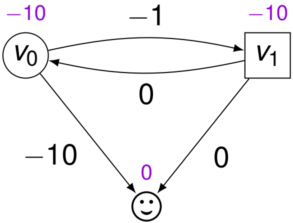
$$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\boxed{\sigma}$ Adam $\quad \tau$ Eve

$\sigma : V^* \times V_{\mathrm{Adam}} \to V$



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

An optimal strategy for Adam may require finite memory.

Optimal strategy

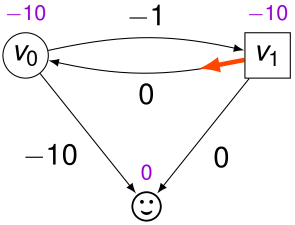
$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\mathsf{Adam}} \to V$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle

Optimal strategy

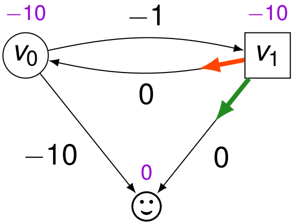
$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\mathrm{Adam}} \to V$

$\boxed{\sigma}$ Adam τ Eve



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle
- σ_2 : reach ☺

Optimal strategy

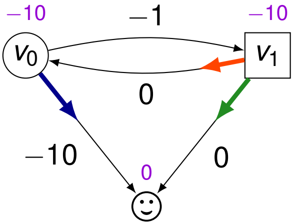
$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\text{Adam}} \to V$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\mathrm{dVal}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^\sigma(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle
- σ_2 : reach ☺

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Optimal strategy for Eve

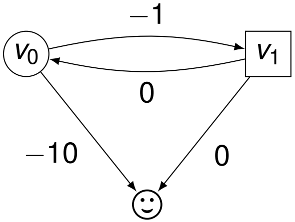
Eve has a memoryless optimal strategy.

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Memoryless strategies

$\sigma : V_{\mathrm{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam τ Eve

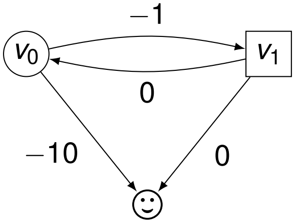


Value

$$\overline{\mathrm{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



$\boxed{\sigma}$ Adam $\quad$ τ Eve

Value

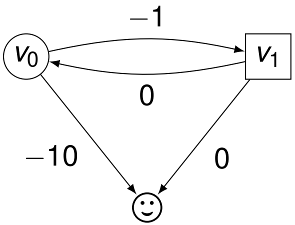
$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Determinacy

$$\text{mVal}(v) = \overline{\text{mVal}}(v) = \underline{\text{mVal}}(v)$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



$\boxed{\sigma}$ Adam $\quad$ τ Eve

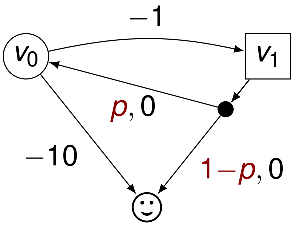
Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



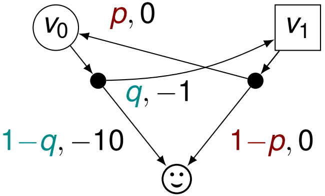
Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam τ Eve



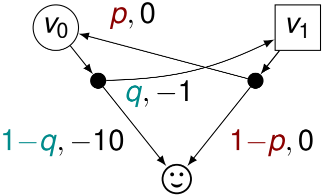
Value

$$\mathrm{mVal}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^\sigma(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

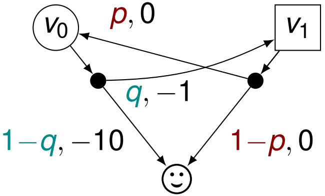
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam τ Eve



Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Unicity

A unique fixpoint : $\text{mVal}^{\sigma,\tau}$

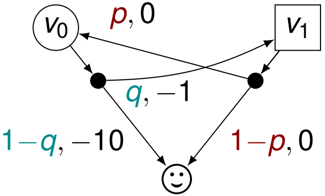
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam τ Eve



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p \times \mathrm{mVal}^{\sigma,\tau}(v_0)$$
$$\mathrm{mVal}^{\sigma,\tau}(v_0) = q(\mathrm{mVal}^{\sigma,\tau}(v_1) - 1) - 10(1 - q)$$

Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

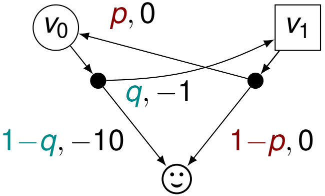
Memoryless strategies

$\sigma : V_{\mathrm{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve

Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

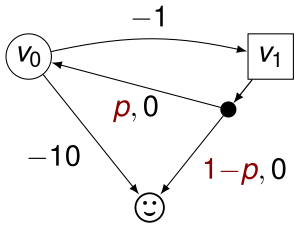
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

$\boxed{\sigma}$ Adam $\quad$ τ Eve

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
- If $p \geqslant \frac{9}{10}$

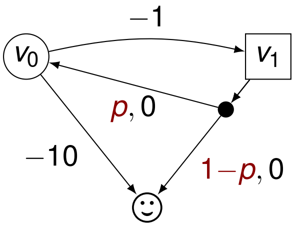
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\boxed{\sigma}$ Adam $\quad$ τ Eve

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
- If $p \geqslant \frac{9}{10}$, then $q = 0$:

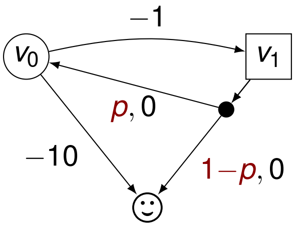
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad \tau$ Eve



Value

$$\mathrm{mVal}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^\sigma(v)}$$

Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^σ

- If $p < \frac{9}{10}$, then $q = 1$:

  $\mathrm{mVal}^\sigma(v_1) = \frac{-p}{1-p}$

  $\mathrm{mVal}^\sigma(v_0) = \frac{-1}{1-p}$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

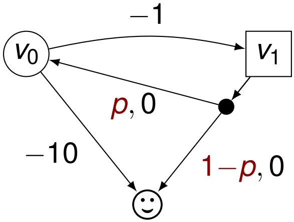
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
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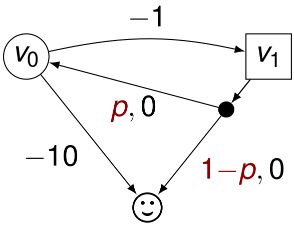
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam τ Eve



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

$$\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$$

$$\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

$$\mathrm{mVal}^{\sigma}(v_1) = -10p$$

$$\mathrm{mVal}^{\sigma}(v_0) = -10$$

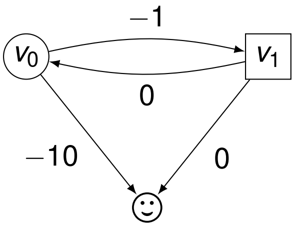
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
  $$\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$$
  $$\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$
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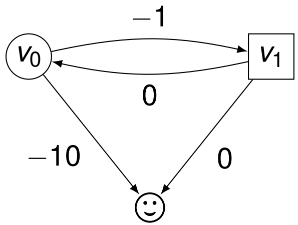
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\boxed{\sigma}$ Adam τ Eve

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $$\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} > -9$$

  $$\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $$\mathrm{mVal}^{\sigma}(v_1) = -10p$$

  $$\mathrm{mVal}^{\sigma}(v_0) = -10$$

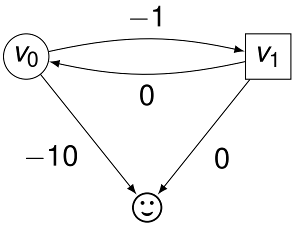
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad \tau$ Eve



Value

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Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $$\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} > -9$$

  $$\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p} > -10$$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $$\mathrm{mVal}^{\sigma}(v_1) = -10p$$

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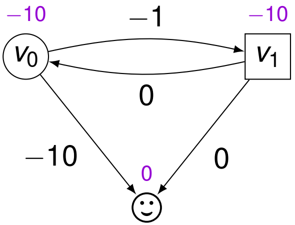
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

$\boxed{\sigma}$ Adam τ Eve

Value

$$\text{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
  $$\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} > -9$$
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- If $p \geqslant \frac{9}{10}$, then $q = 0$:
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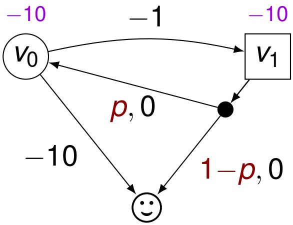
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam τ Eve



Value

$$\mathrm{mVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}_v^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Value in a MDP

Computable in polynomial time

Bellman equation in a Markov Chain

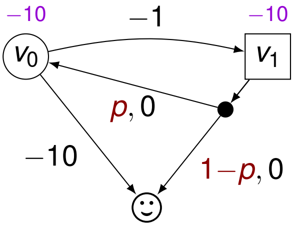
$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Stochastic Shortest Paths and Weight-Bounded Properties in Markov Decision Processes, C. Baier, N. Bertrand, C. Dubslaff, D. Gburek and O. Sankur, 2018, LICS.

Memoryless strategies

$\boxed{\sigma}$ Adam $\quad \tau$ Eve

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Value

$$\text{mVal}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbb{E}_v^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^\sigma(v)}$$

Value in a MDP

Computable in polynomial time

ε-optimal strategy

$$\text{mVal}^{\sigma^*}(v) \leqslant \text{mVal}(v) + \varepsilon$$

Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Stochastic Shortest Paths and Weight-Bounded Properties in Markov Decision Processes, C. Baier, N. Bertrand, C. Dubslaff, D. Gburek and O. Sankur, 2018, LICS.

Contribution

$$\text{dVal} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

$$\text{dVal} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives

$$\mathrm{dVal} \quad = \quad \mathrm{mVal}$$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives
- Reachability Timed Games

$$\mathrm{dVal} \quad = \quad \mathrm{mVal}$$

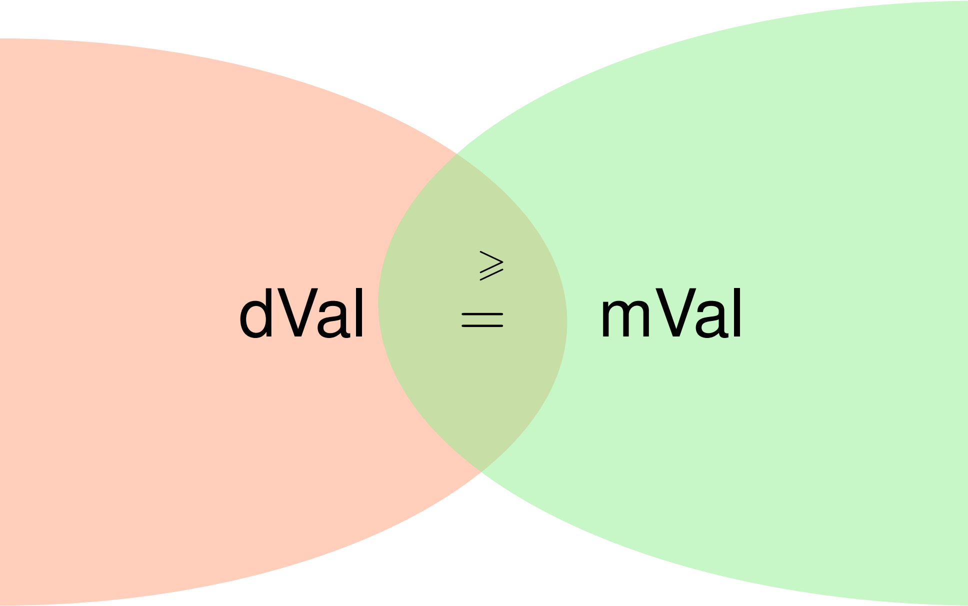
Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

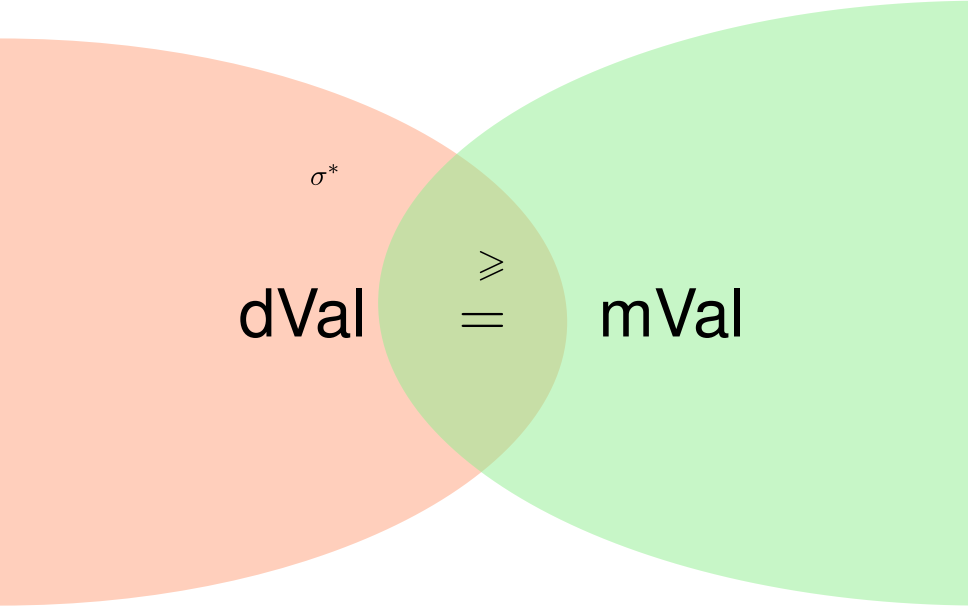
Contribution

dVal = mVal

Contribution



Contribution



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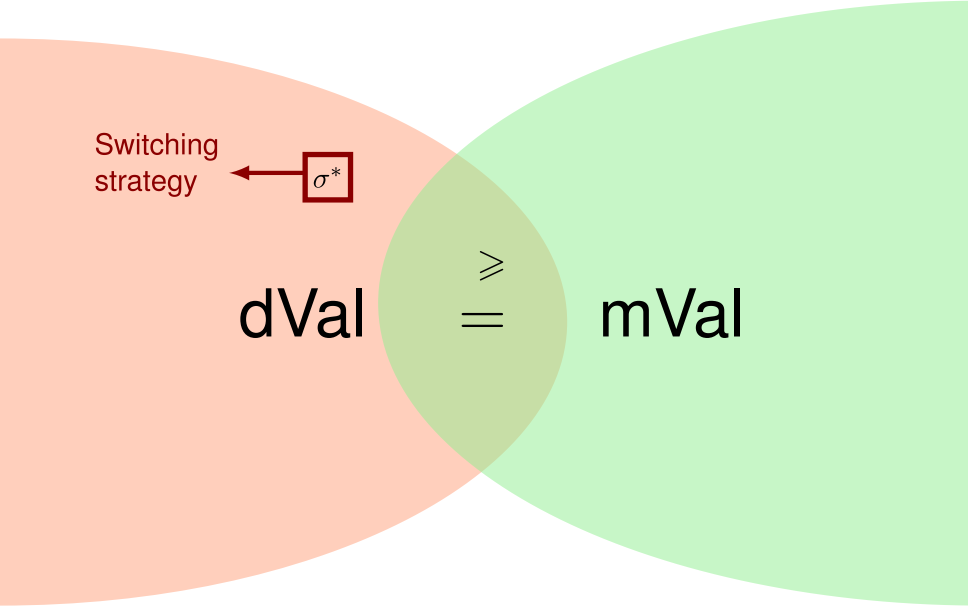
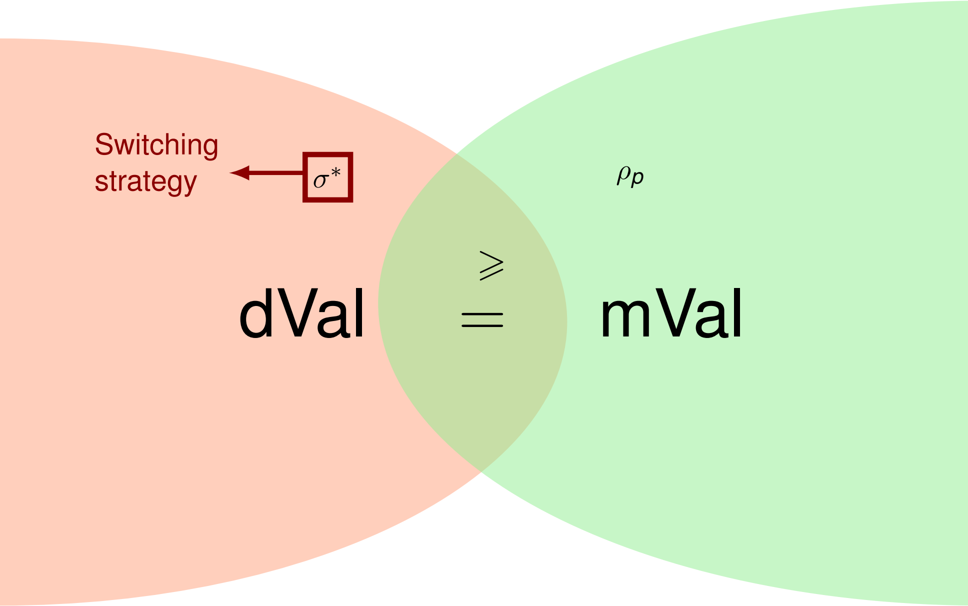


Contribution



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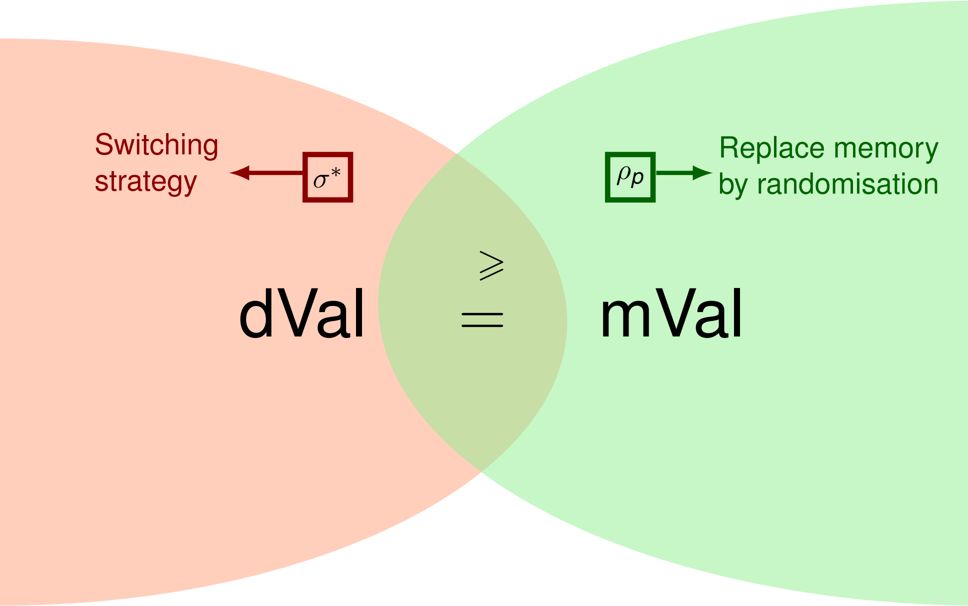
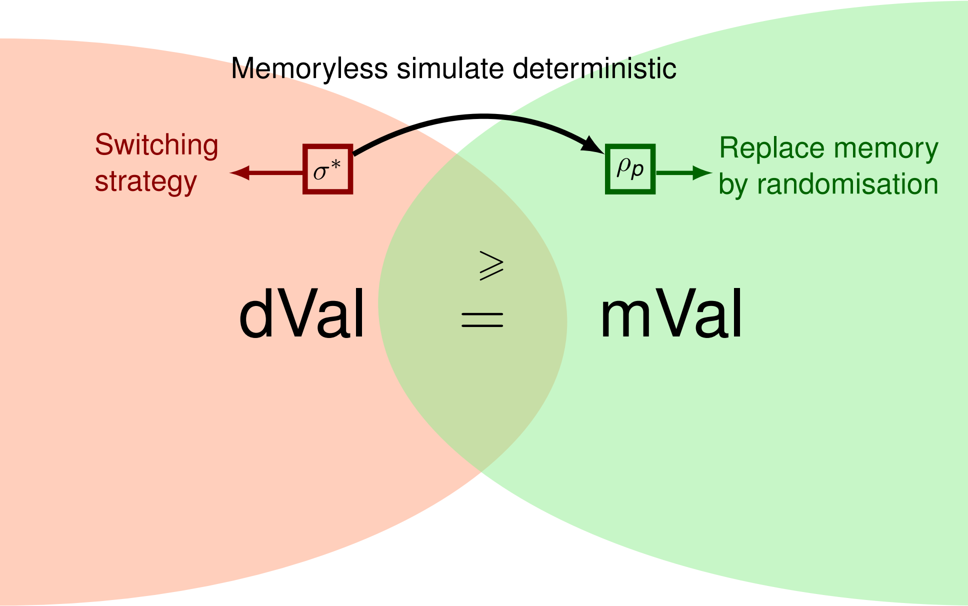
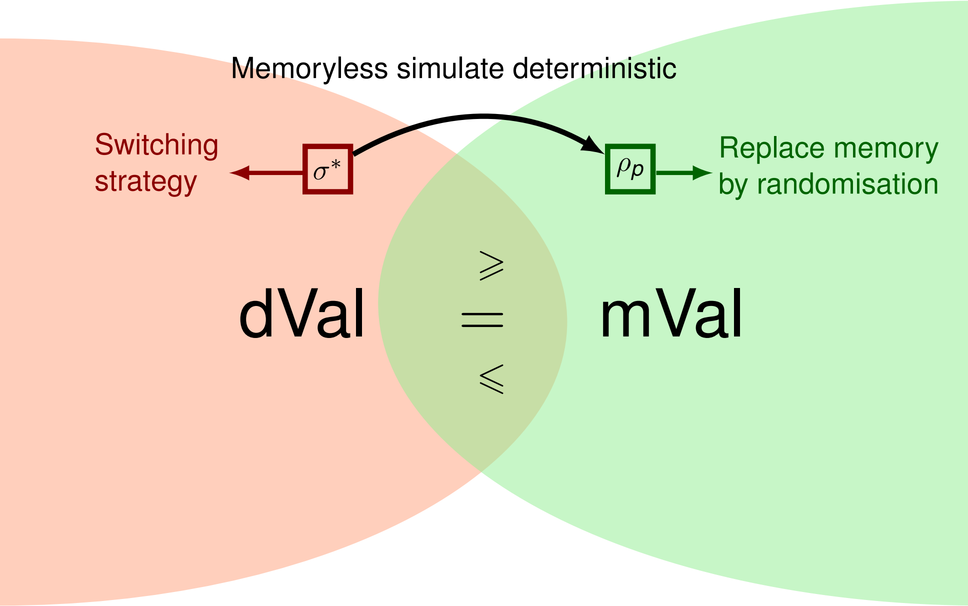


Contribution



Contribution



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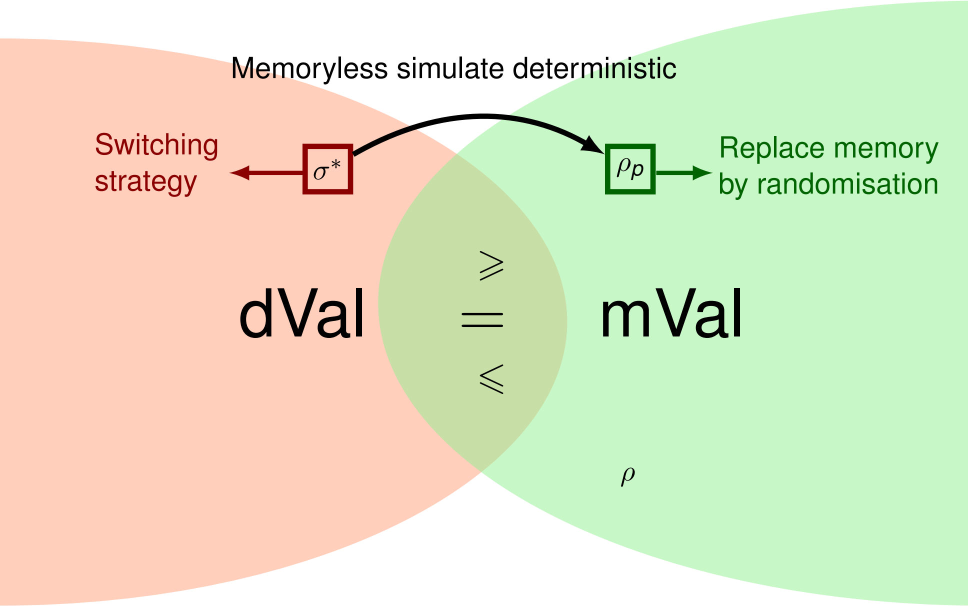
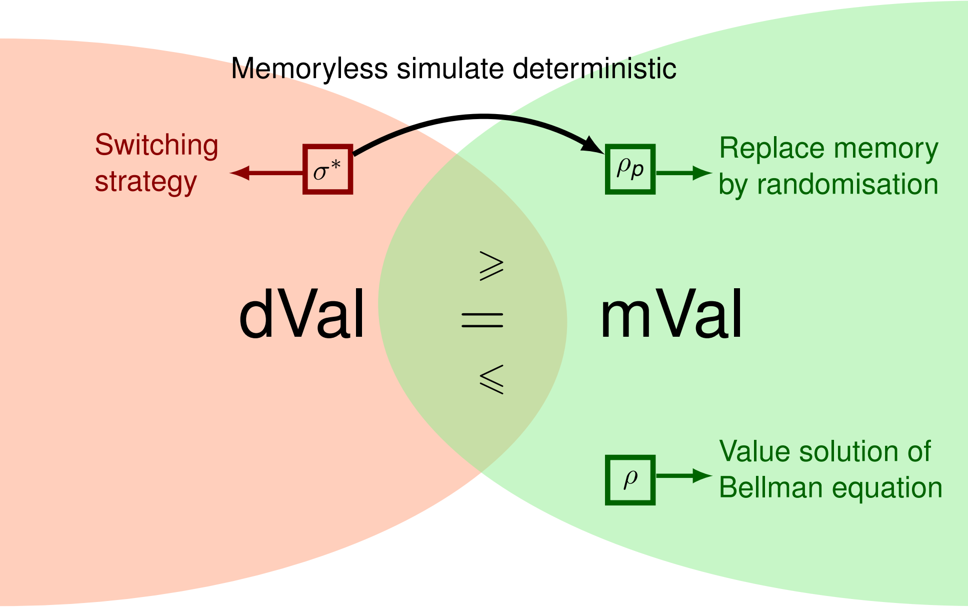
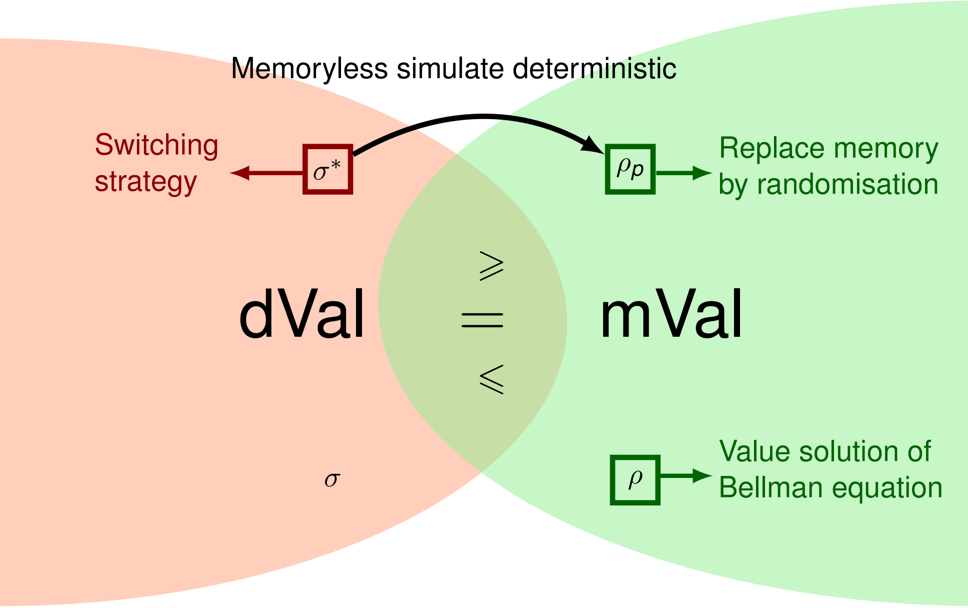


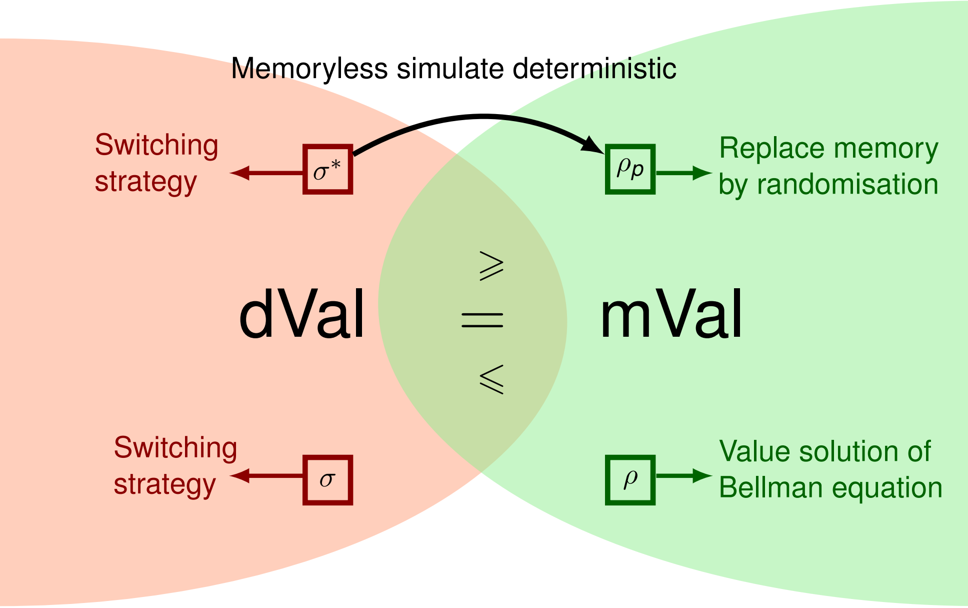
Contribution



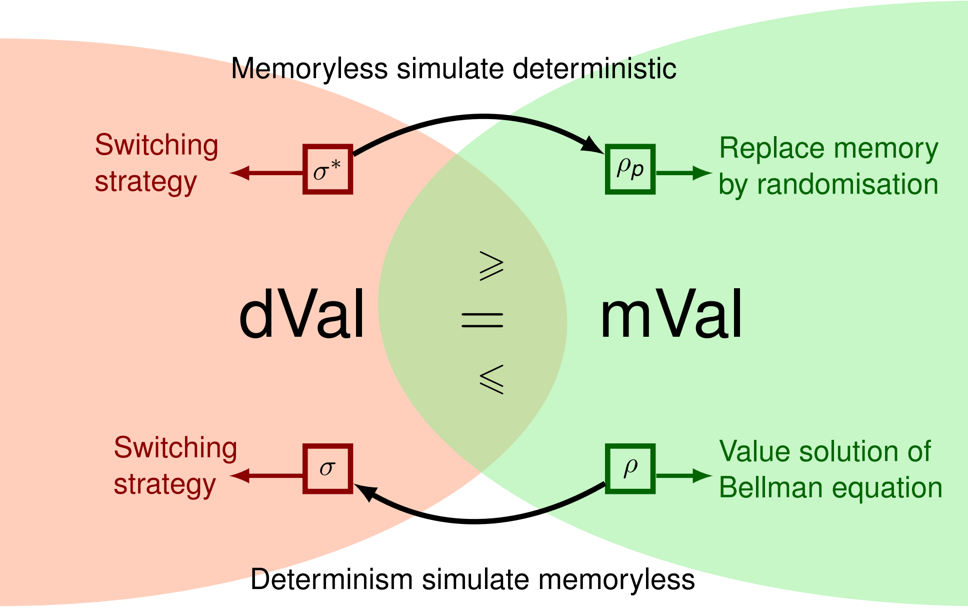
Contribution



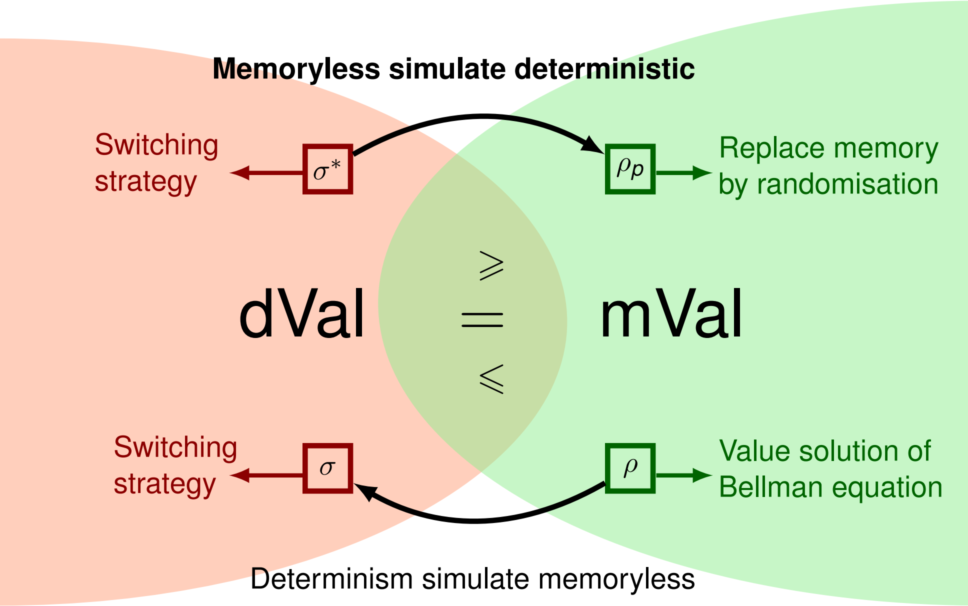
Contribution



Contribution



Contribution

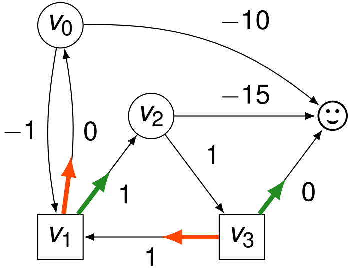


Memoryless simulate deterministic

Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Memoryless simulate deterministic



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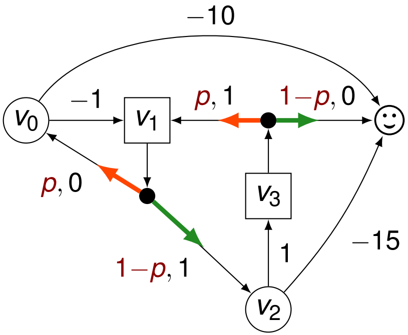
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy,

Memoryless simulate deterministic



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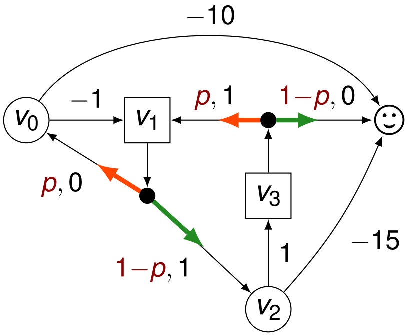
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy, for all $p \in (0, 1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Memoryless simulate deterministic

□ Adam ○ Eve



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Properties of ρ_p

- For all τ, $\mathbb{P}^{\rho_p,\tau}(\diamond ☺) = 1$

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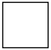
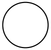
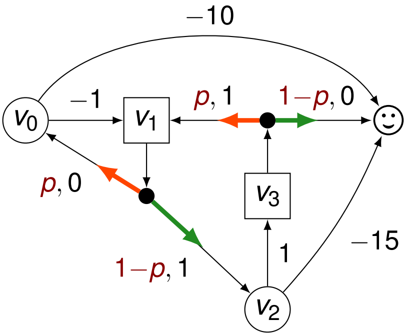
 Adam  Eve



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- For all τ, $\mathbb{P}^{\rho_p,\tau}(\diamond☺) = 1$
- Eve has an optimal memoryless deterministic strategy.

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An analysis of stochastic shortest path problems, D. Bertsekas and J. Tsitsiklis, 1991, Mathematics of Operations Research.

Memoryless simulate deterministic

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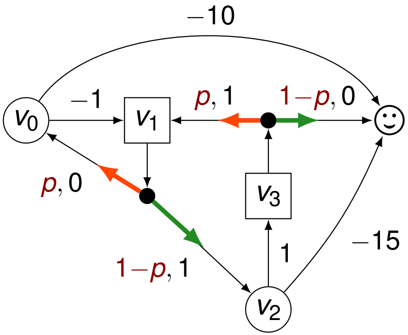

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Presence of non-negative cycles

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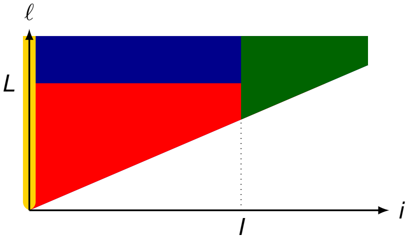
Memoryless simulate deterministic

Adam Eve



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Problem

Presence of non-negative cycles

Tool for the proof

Control the non-negative cycles with a partition of plays

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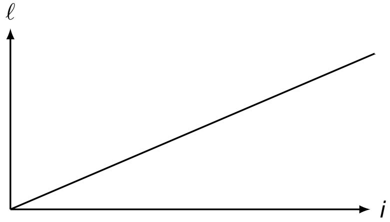
Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles



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ℓ size of play reaching the target
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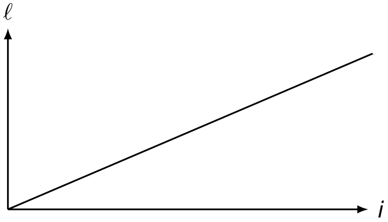
Fix a deterministic strategy for Eve



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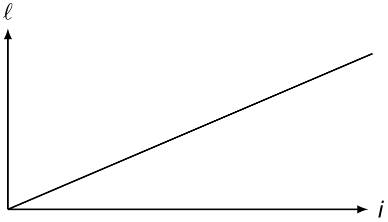
Fix a deterministic strategy for Eve



Good zones

SP $\leqslant$ dVal
$\Rightarrow \mathbb{E}($**SP**$) \leqslant$ dVal

Focus on the partition of plays

ℓ size of play reaching the target
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Fix a deterministic strategy for Eve

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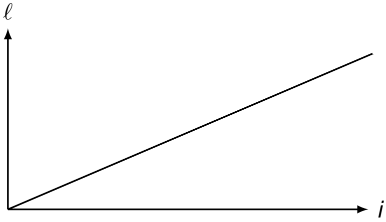


Zones to control

$\mathbb{E}($**SP**$) \leqslant \varepsilon$

Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles

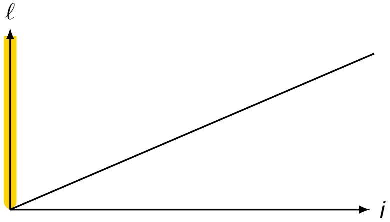
Fix a deterministic strategy for Eve

Yellow zone

All plays conforming to σ_1

Good zones

$\mathbf{SP} \leqslant \mathrm{dVal}$
$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant \mathrm{dVal}$



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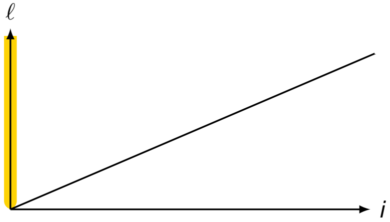

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Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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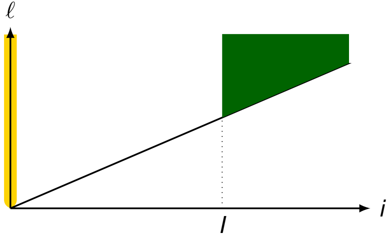
$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant \mathsf{dVal}$

Green zone

Plays contain many non-negative cycles

Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$



Focus on the partition of plays

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i number of non-negative cycles

Fix a deterministic strategy for Eve

Yellow zone

All plays conforming to σ_1
Weight of each play is $\leqslant$ dVal

Green zone

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$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

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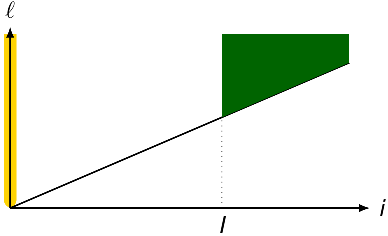

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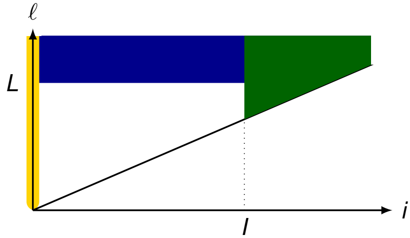

Zones to control

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Blue zone

Plays with many negative cycles and few non-negative cycles

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Fix a deterministic strategy for Eve

Yellow zone

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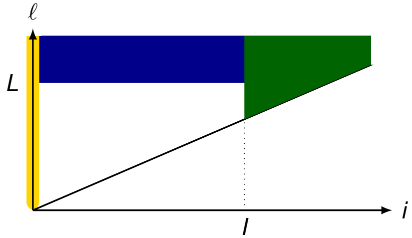

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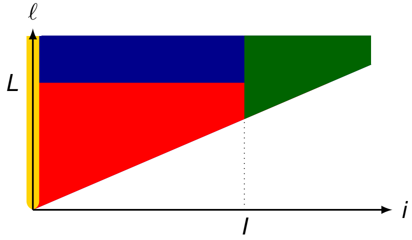

Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$

Blue zone

Plays with many negative cycles and few non-negative cycles
$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Red zone

Rest of plays

Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles

Fix a deterministic strategy for Eve

Yellow zone
All plays conforming to σ_1
Weight of each play is $\leqslant$ dVal

Green zone
Plays contain many non-negative cycles
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Good zones

$\mathbf{SP} \leqslant \mathrm{dVal}$
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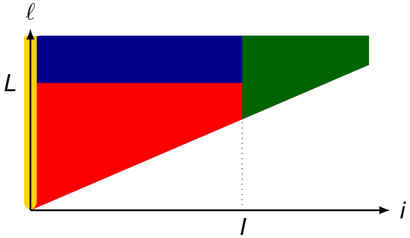

Zones to control

$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$

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Plays with many negative cycles and few non-negative cycles
$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Red zone
Rest of plays
$\exists p$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Results on shortest path games

Contributions

1. Adam has the same hope using memory or randomness.
2. Existence of an optimal memoryless strategy for Adam is testable in polynomial time.

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Perspectives

- Extension to probabilistic value (memory and randomisation)
- Extension to weighted timed games
- A polynomial-time algorithm to compute the value

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1. Adam has the same hope using memory or randomness.
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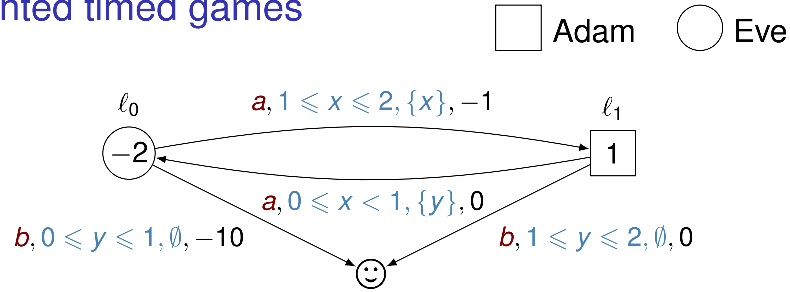
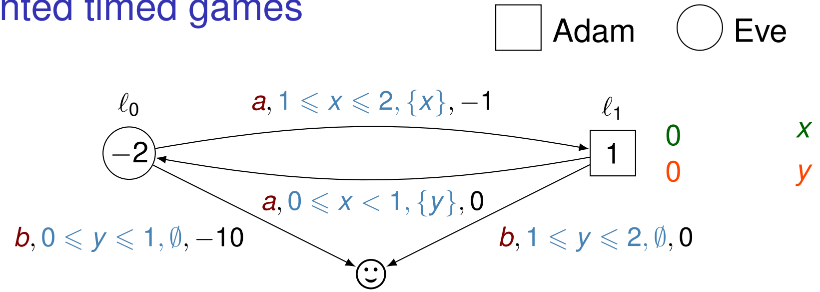
Perspectives

- Extension to probabilistic value (memory and randomisation)
- **Extension to weighted timed games**
- A polynomial-time algorithm to compute the value

Weighted timed games



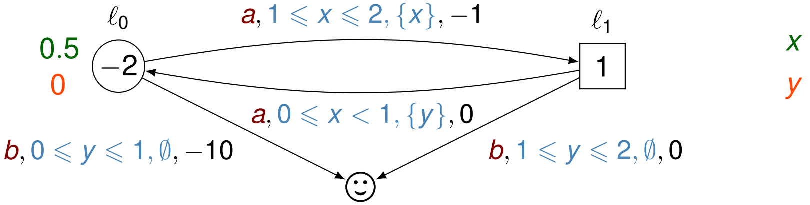
Weighted timed games



Play

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)$$

Weighted timed games

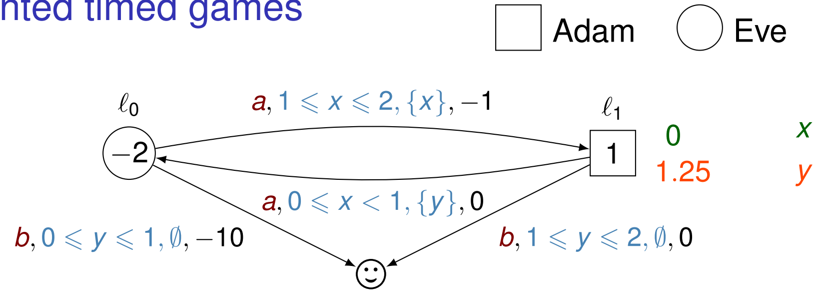


Play

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\, 0.5} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right)$$

$$1 \times 0.5 + 0$$

Weighted timed games

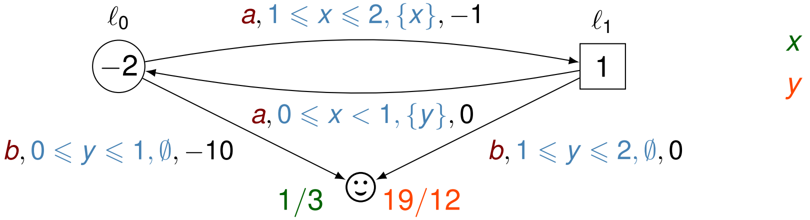


Play

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\ 0.5} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\ 1.25} \left(\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}\right)$$

$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1$

Weighted timed games

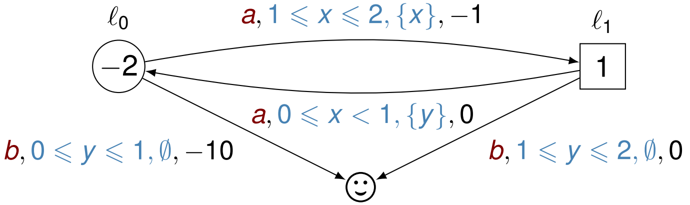


Play

$$\left(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}\right) \xrightarrow[1 \times 0.5 + 0]{a,\ 0.5} \left(\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}\right) \xrightarrow[-2 \times 1.25 - 1]{a,\ 1.25} \left(\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}\right) \xrightarrow[1 \times \frac{1}{3} + 0]{b,\ 1/3} \left(☺, \begin{pmatrix}1/3\\19/12\end{pmatrix}\right)$$

Weighted timed games

□ Adam ○ Eve



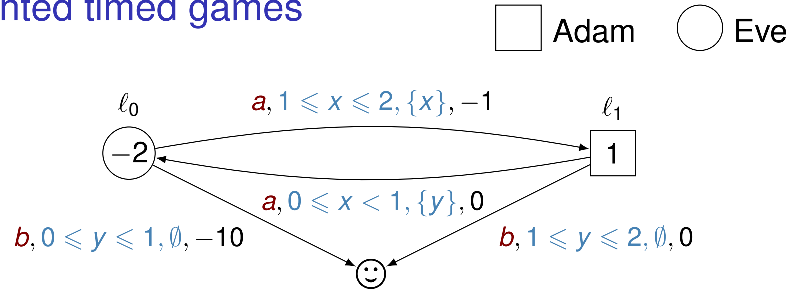
x

y

Play

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow[1 \times 0.5 + 0]{a,\ 0.5} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right) \xrightarrow[-2 \times 1.25 - 1]{a,\ 1.25} \left(\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}\right) \xrightarrow[1 \times \frac{1}{3} + 0]{b,\ 1/3} \left(☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}\right) \rightsquigarrow -\frac{8}{3}$$

Weighted timed games



Value problem

The value problem, i.e. deciding if $\mathrm{dVal}(\ell_0, \nu_0) \leqslant c$ with $c \in \mathbb{Z}$, is undecidable.

On Optimal Timed Strategies, T.Brihaye, V.Bruyère and J.-F. Raskin, 2005, FORMATS.

On the value problem in weighted timed games, P. Bouyer, S. Jaziri, and N. Markey, 2015, CONCUR.

Our objective

$$\mathsf{dVal} \stackrel{?}{=} \mathsf{mVal}$$

Our objective

Define probabilistic strategy

$$\mathrm{dVal} \stackrel{?}{=} \mathrm{mVal}$$

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Fixpoint of a
Bellman equation

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Deterministic ε-optimal
strategy for Eve

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Restriction

Divergent / Strongly non-zeno cost

Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E. Fleury and K. Larsen, 2004, FSTTCS

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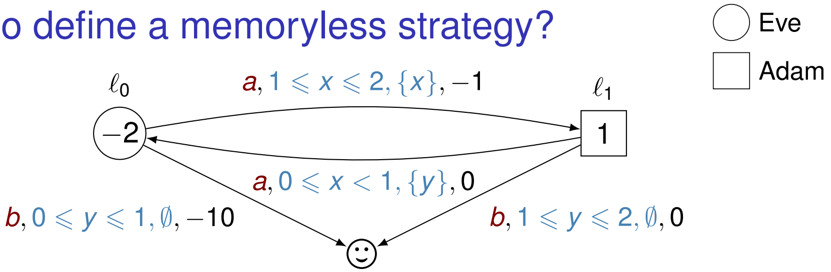
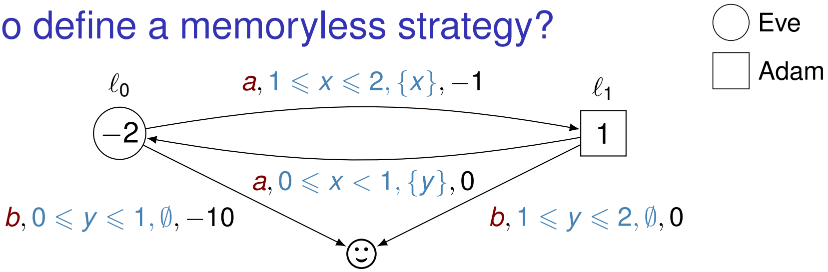
How to define a memoryless strategy?



Deterministic strategy

Choose a transition and a delay

How to define a memoryless strategy?



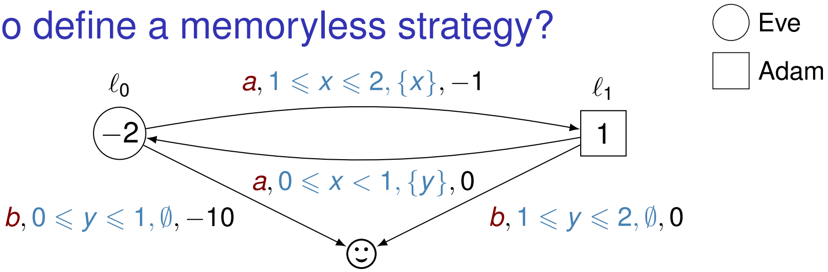
Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between *a* or *b*

How to define a memoryless strategy?



Deterministic strategy

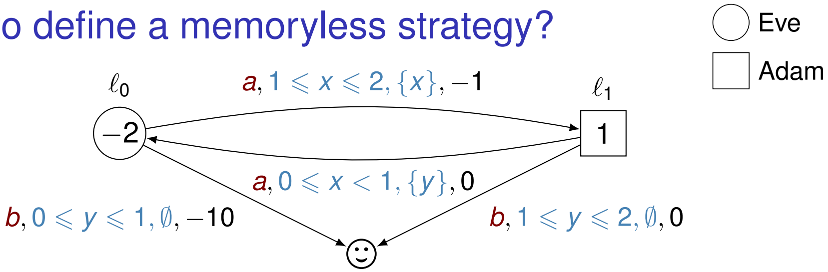
Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between a or b

- a : choose t with $t < 1$

How to define a memoryless strategy?



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Choose a transition and a delay

In $(\ell_1, (0, 0))$

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- *a* : choose *t* with $t < 1$
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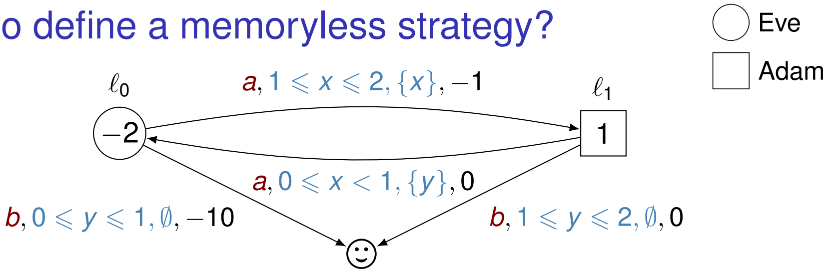

Probabilistic strategy

Distribution over possible choices

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0,0))$

Choose between a or b

- a : choose t with $t < 1$
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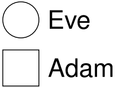
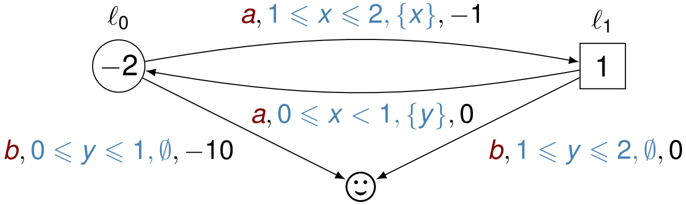




Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution

Deterministic strategy

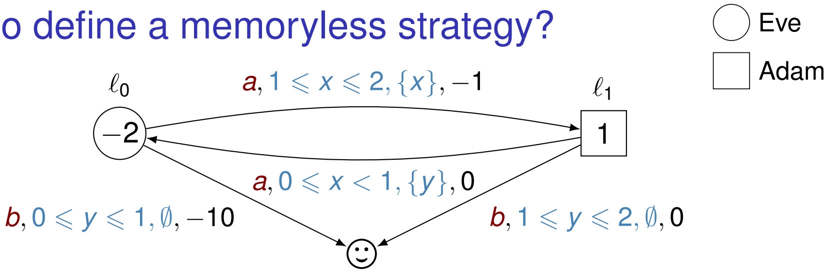
Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition *a* : finite distribution
2. Delay for *a* : infinite distribution

Deterministic strategy

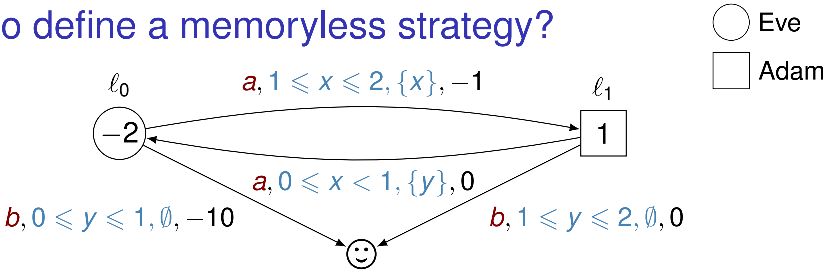
Choose a transition and a delay

In $(\ell_1, (0,0))$

Choose between *a* or *b*

- *a* : choose *t* with $t < 1$
- *b* : choose *t* with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
2. Delay for a : infinite distribution

In $(\ell_1, (0,0))$

Choose between a or b with $\mathcal{B}(p)$

Deterministic strategy

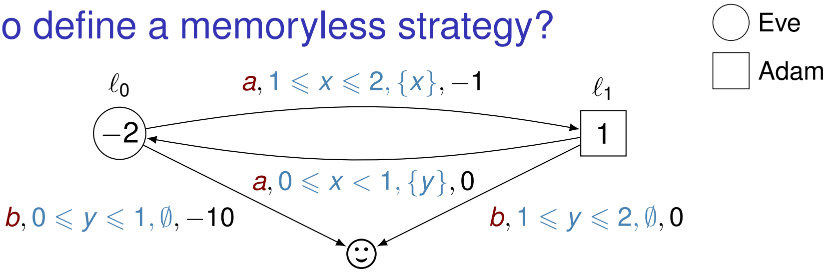
Choose a transition and a delay

In $(\ell_1, (0,0))$

Choose between a or b

- a : choose t with $t < 1$
- b : choose t with $1 \leqslant t \leqslant 2$

How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
2. Delay for a : infinite distribution

In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

- a : choose t with $\mathcal{U}([0, 1[)$

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0, 0))$

Choose between a or b

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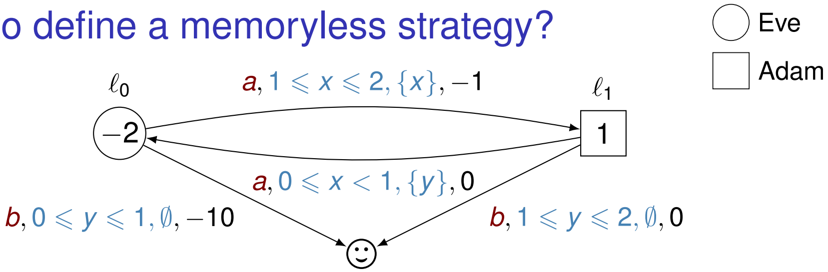
How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
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In $(\ell_1, (0,0))$

Choose between a or b with $\mathcal{B}(p)$

- a : choose t with $\mathcal{U}([0,1[)$
- b : choose t with $\delta_{1.5}$

Deterministic strategy

Choose a transition and a delay

In $(\ell_1, (0,0))$

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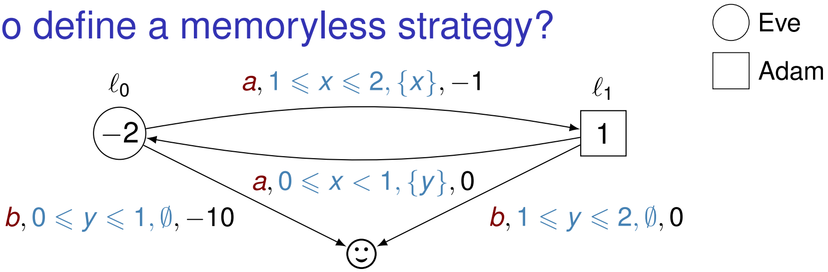
How to define a memoryless strategy?



Probabilistic strategy

Distribution over possible choices

1. Transition a : finite distribution
2. Delay for a : infinite distribution

Deterministic strategy

Choose a transition and a delay

Close related work

Stochastic timed automata

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Memoryless value

$$\overline{\mathsf{mVal}}(\ell, \nu) = \inf_{\sigma} \sup_{\tau} \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$$

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Memoryless value

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Cylinder

$\mathcal{C} = ((\ell,\nu), e_1 \ldots e_n) =$
$\{(t_1,\ldots,t_n) \mid (\ell,\nu) \xrightarrow{t_1,e_1} \ldots \xrightarrow{t_n,e_n}\}$

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Memoryless value

○ Eve
□ Adam

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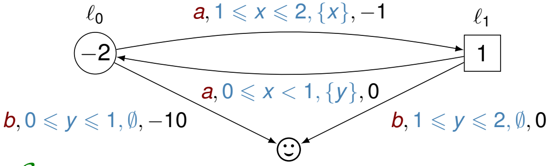

Example of cylinder $\mathcal{C}$

$$\mathcal{C} \quad = \quad ((\ell_1,(0,0)), a\ b) \quad =$$

Memoryless value

Eve

Adam

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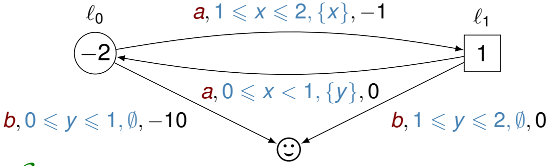

Example of cylinder $\mathcal{C}$

$$\mathcal{C} \quad = \quad ((\ell_1,(0,0)), a\ b) \quad = \quad \{(t_1,t_2) \mid (\ell_1,(0,0)) \xrightarrow{t_1,a}\xrightarrow{t_2,b}\}$$

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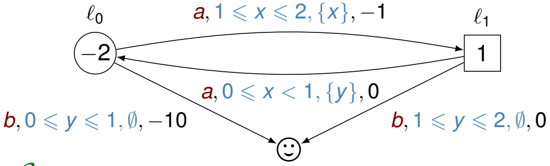

Example of cylinder $\mathcal{C}$

$$\begin{aligned}\mathcal{C} &= ((\ell_1,(0,0)), a\, b) = \{(t_1,t_2) \mid (\ell_1,(0,0)) \xrightarrow{t_1,a}\xrightarrow{t_2,b}\}\\ &= \{(\ell_1,\nu_0) \xrightarrow{t_1,a} (\ell_0,\nu_1) \xrightarrow{t_2,b} (☺,\nu_2) \mid t_1 < 1, t_2 \leqslant 1\}\end{aligned}$$

$\nu_0 \downarrow (0,0)$ $\qquad \nu_1 \downarrow (t_1,0)$ $\qquad \nu_2 \downarrow (t_1+t_2,t_2)$

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Cylinder

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$\{(t_1,\ldots,t_n) \mid (\ell,\nu) \xrightarrow{t_1,e_1} \ldots \xrightarrow{t_n,e_n}\}$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1 \in I((\ell,\nu),e_1)} p_{(\ell,\nu)}(e_1)\, \mathbb{P}(\mathcal{C}_1)\, d\mu_{(\ell,\nu),e_1}(t_1)$$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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- Interval of possible delays to pass through e_1 from (ℓ, ν)

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- Probability to choose e_1 in (ℓ, ν)
- Interval of possible delays to pass through e_1 from (ℓ, ν)
- Probability to choose a delay for e_1 from (ℓ, ν)
- $(\ell, \nu) \xrightarrow{t_1, e_1} (\ell_1, \nu_1)$
- n-dimensional integral

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Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1)\ \overbrace{\left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2) \int_{t_3} \ldots\right)}^{\mathbb{P}(\mathcal{C}_1)} d\mu_{(\ell,\nu),e_1}(t_1)$$

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Cylinder

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$$\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Conditional expectation

$$\mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1)(w(\ell)t_1 + w(e_1) + \mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}_1))d\mu_{(\ell,\nu),e_1}(t_1)$$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}_1)$$

$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1) \left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2) \int_{t_3} \dots\right) d\mu_{(\ell,\nu),e_1}(t_1)$$

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Cylinder

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Existence and integrability ?

Conditional expectation

$$\mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1)(w(\ell)t_1 + w(e_1) + \mathbb{E}^{\sigma,\tau}(\mathbf{SP}|\mathcal{C}_1))d\mu_{(\ell,\nu),e_1}(t_1)$$

Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1) \underbrace{\left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2) \int_{t_3} \dots \right)}_{\mathbb{P}(\mathcal{C}_1)} d\mu_{(\ell,\nu),e_1}(t_1)$$

Memoryless value

$$\overline{\mathsf{mVal}}(\ell,\nu) = \inf_{\sigma}\sup_{\tau} \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$$

Cylinder

$\mathcal{C} = ((\ell,\nu), e_1 \dots e_n) =$
$\{(t_1,\dots,t_n) \mid (\ell,\nu) \xrightarrow{t_1,e_1} \dots \xrightarrow{t_n,e_n}\}$

$$\sum_{\mathcal{C}} \mathbb{P}(\mathcal{C})\, \mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C})$$

Existence and integrability ?

No restriction on Eve's strategies

Conditional expectation

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Probability of a cylinder

$$\mathbb{P}(\mathcal{C}_1)$$

$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1) \left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2) \int_{t_3} \dots\right) d\mu_{(\ell,\nu),e_1}(t_1)$$

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Convergence ?

- $\mathbb{P}(\mathcal{C}) \sim \alpha^{-n}$
- $\mathbb{E}^{\sigma,\tau}(\mathbf{SP} \mid \mathcal{C}) \sim kn$

Conditional expectation

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Probability of a cylinder

$$\mathbb{P}(\mathcal{C}) = \int_{t_1} p_{(\ell,\nu)}(e_1)\ \overbrace{\left(\int_{t_2} p_{(\ell_1,\nu_1)}(e_2)\int_{t_3}\ldots\right)}^{\mathbb{P}(\mathcal{C}_1)} d\mu_{(\ell,\nu),e_1}(t_1)$$

To conclude

Define probabilistic strategy

Need an ε-optimal switching strategy

$$\mathrm{dVal} \stackrel{?}{=} \mathrm{mVal}$$

Fixpoint of a Bellman equation

Memoryless ε-optimal strategy for Eve

Deterministic ε-optimal strategy for Eve

Restriction

Divergent / Strongly non-zeno cost

Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E. Fleury and K. Larsen, 2004, FSTTCS

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Thank you! Questions?