Playing Stochastically in Weighted (Timed) Games to Emulate Memory

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GT Vérif meeting
November 18, 2021

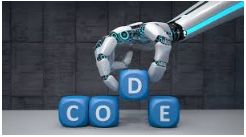
Motivation : game theory for synthesis

Classic approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents : environment and controller

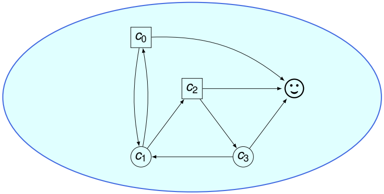


Code synthesis

Correct by construction : synthesis of controller

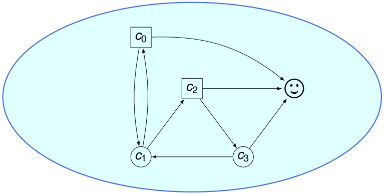
Different classes of games

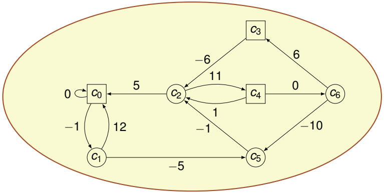
Qualitative games



Different classes of games

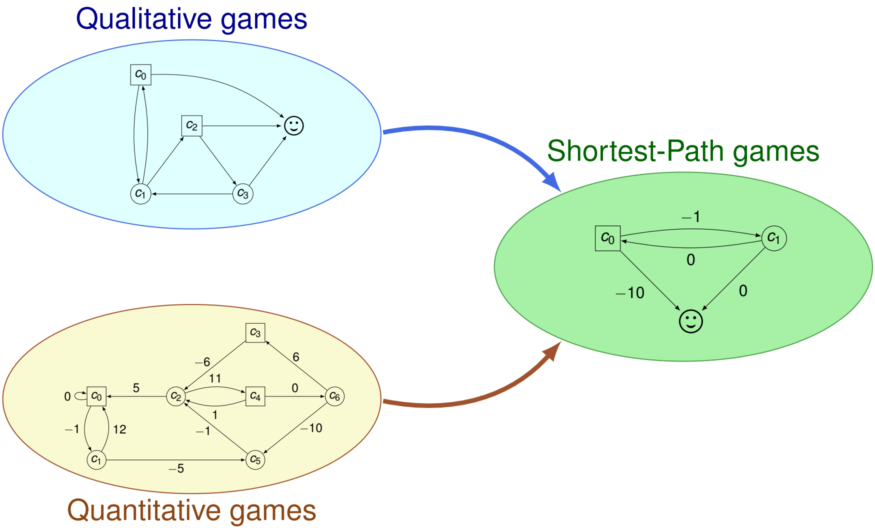
Qualitative games





Quantitative games

Different classes of games



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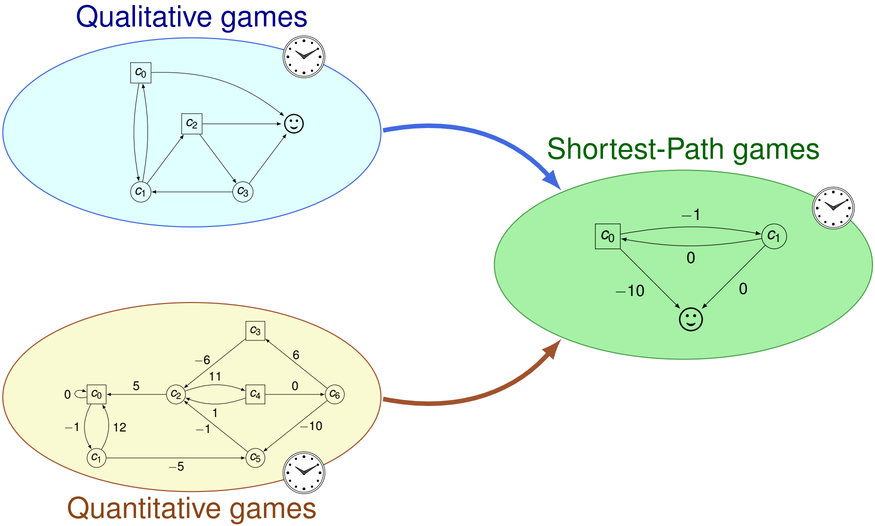
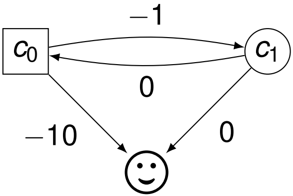


Shortest Path Game



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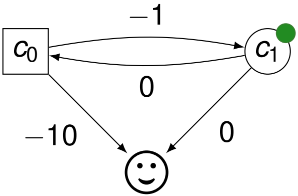
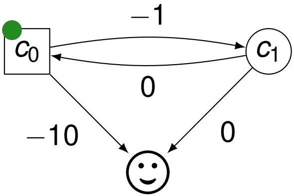


How to play?

Move a token along an edge

$$\rho = c_1$$

Shortest Path Game



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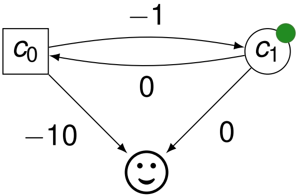

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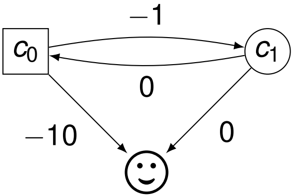
Shortest Path Game

◯ Min ☐ Max ☺ target



How to play?

Move a token along an edge

$$\rho = c_1 c_0 c_1 c_0 c_1 ☺$$

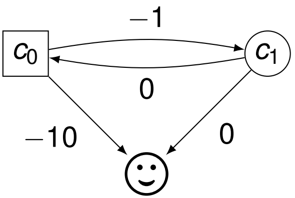
Play

Infinite path or reach the target

$\rho = (c_i)_i \in C^\omega$ $\rho = (c_i)_i ☺$

Shortest Path Game

○ Min □ Max ☺ target



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Shortest Path payoff of a play ρ

$$\mathbf{SP}(\rho) = \begin{cases} \text{wt}(\rho) & \text{if } \rho \text{ reaches } ☺ \\ +\infty & \text{if } \rho \text{ does not reach } ☺ \end{cases}$$

Shortest Path Game

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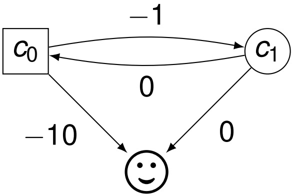

How to play?

Move a token along an edge

$$\rho = c_1 c_0 c_1 c_0 c_1 ☺$$

$$\mathbf{SP}(\rho) = 0 + (-1) + 0 + (-1) + 0 = -2$$

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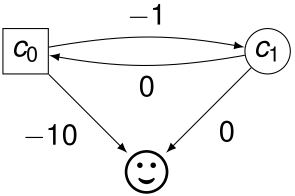

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Objectives

Max maximise the payoff

Min minimise the payoff

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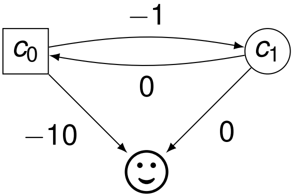

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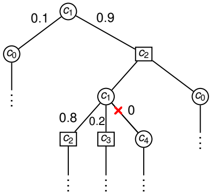
Hypothesis

Divergent games: no zero cycle

Zoology of strategies

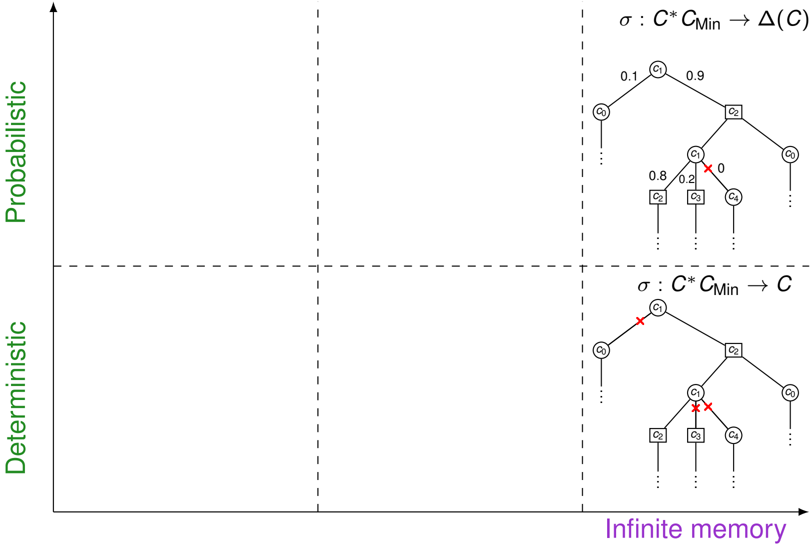
$\sigma : C^* C_{\mathrm{Min}} \to \Delta(C)$



Probabilistic

Infinite memory

Zoology of strategies



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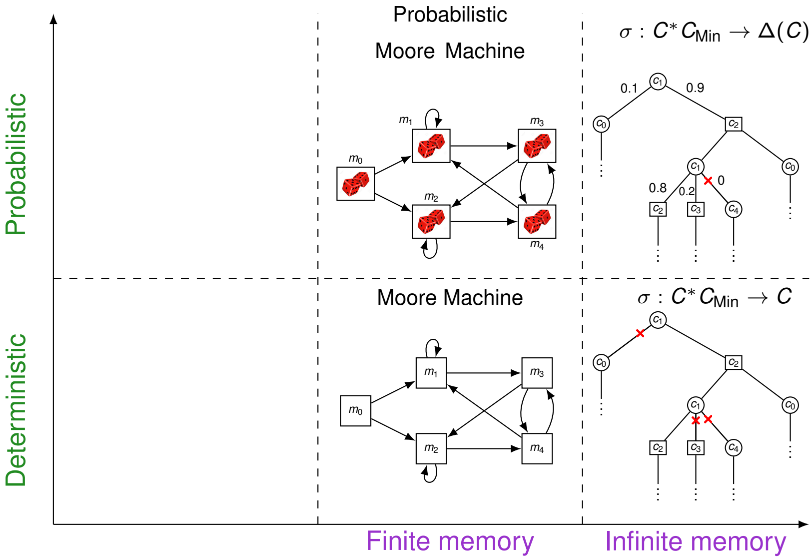

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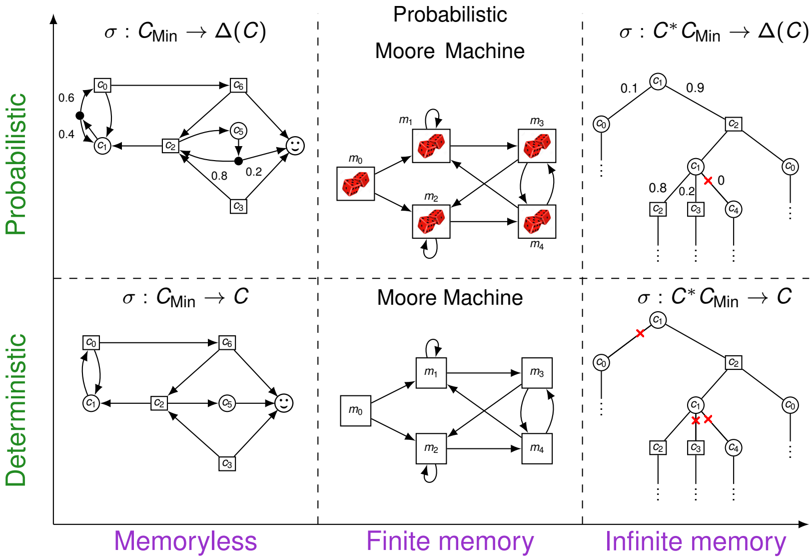
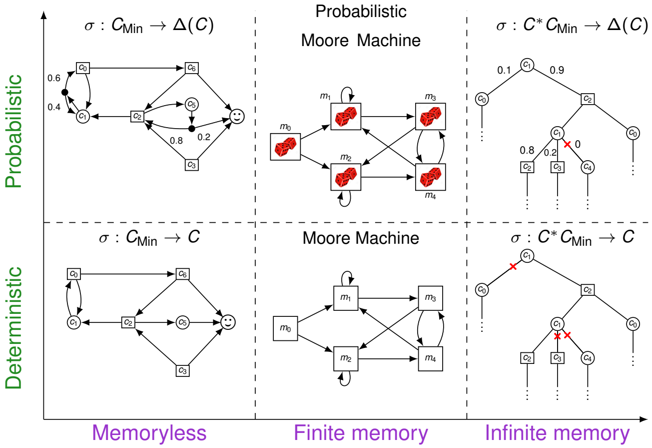
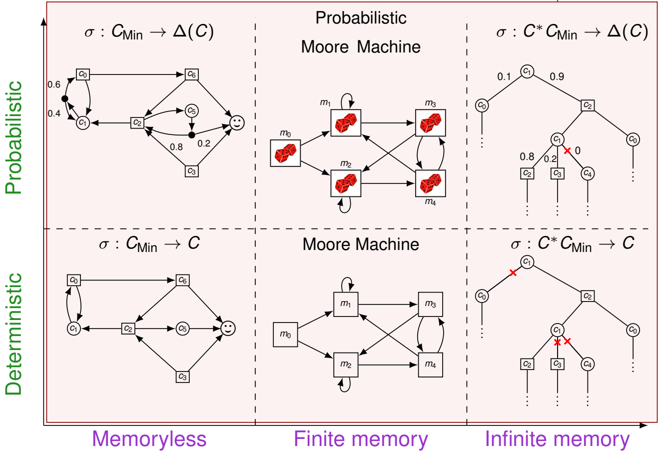


Stochastic values



Stochastic values

$\mathrm{Val} = \inf_\sigma \sup_\tau \mathbb{E}_c^{\sigma,\tau}(\mathbf{SP})$



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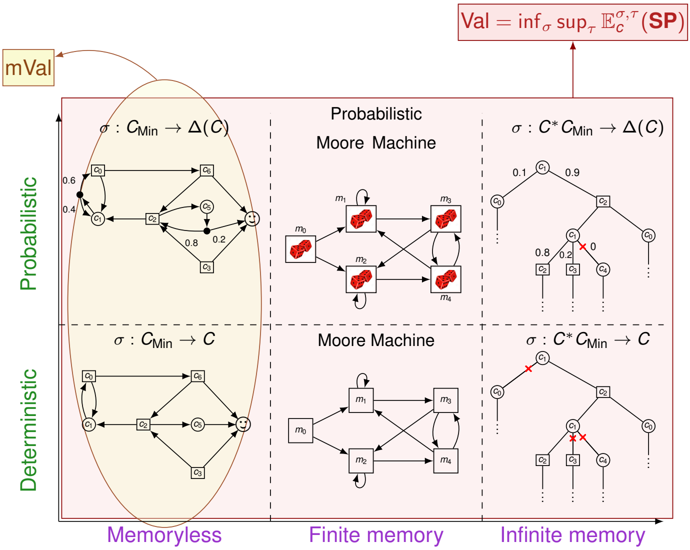

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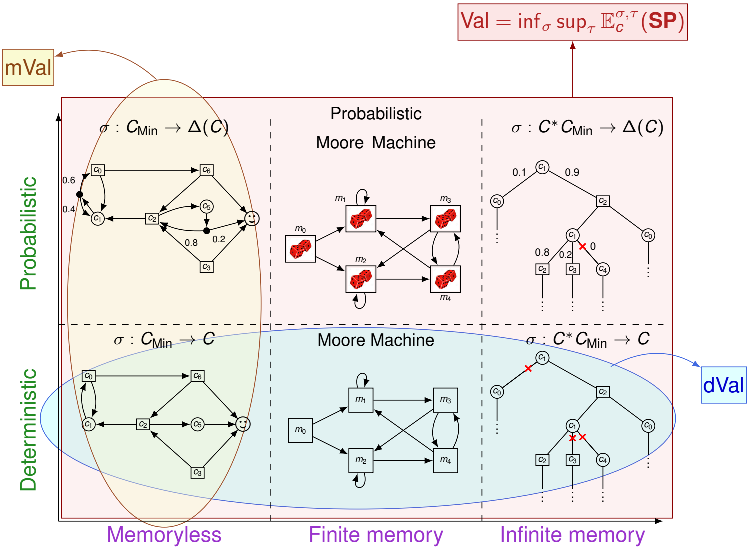

Contribution

$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

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Trade-off between memory and randomness

- Stochastic games with qualitative objectives

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Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives
- Reachability Timed Games

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Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

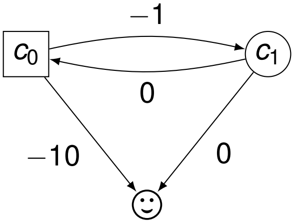
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathrm{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(c, \sigma, \tau))$$



Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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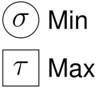

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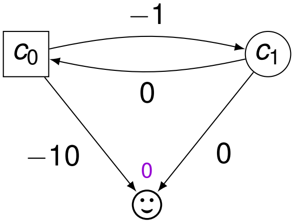

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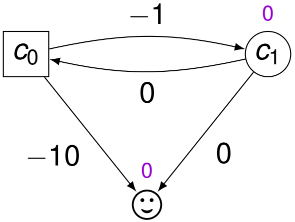

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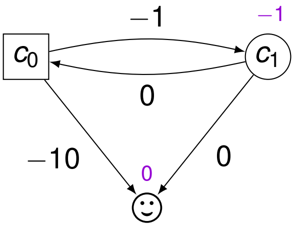

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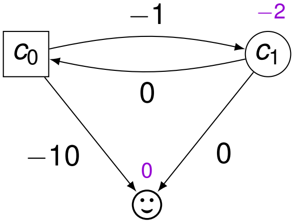

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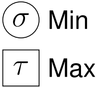

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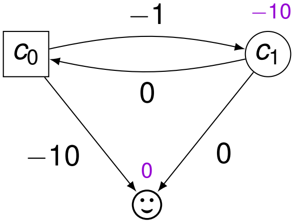

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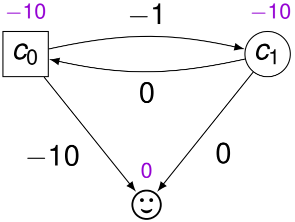

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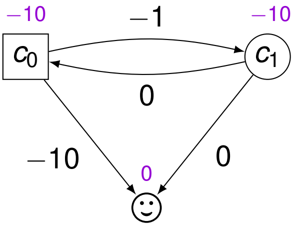

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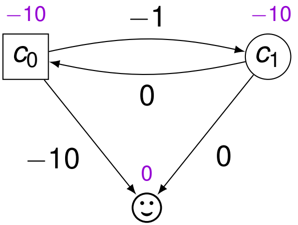

Optimal strategy for Min

An optimal strategy for Min may require finite memory.

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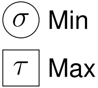

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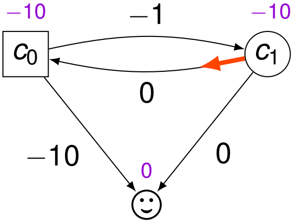

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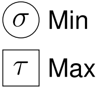

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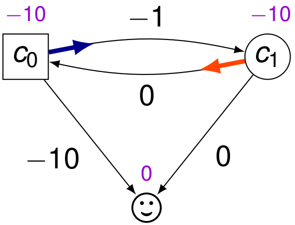

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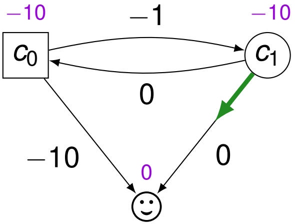

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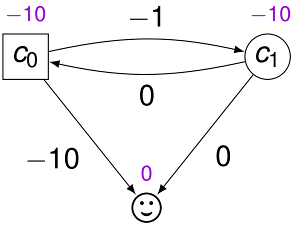

Optimal strategy for Min

Switching strategy:

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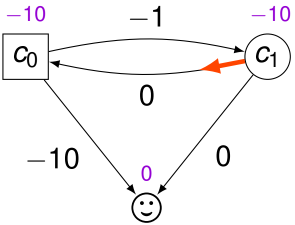

Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle

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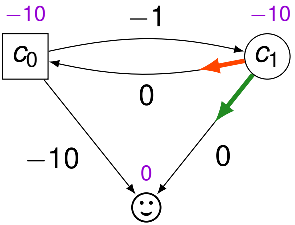
$\mathsf{dVal}^{\sigma^*}(c) \leqslant \mathsf{dVal}(c)$



Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle
- σ_2: reach ☺

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Optimal strategy

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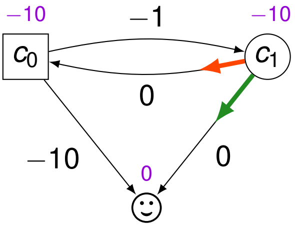


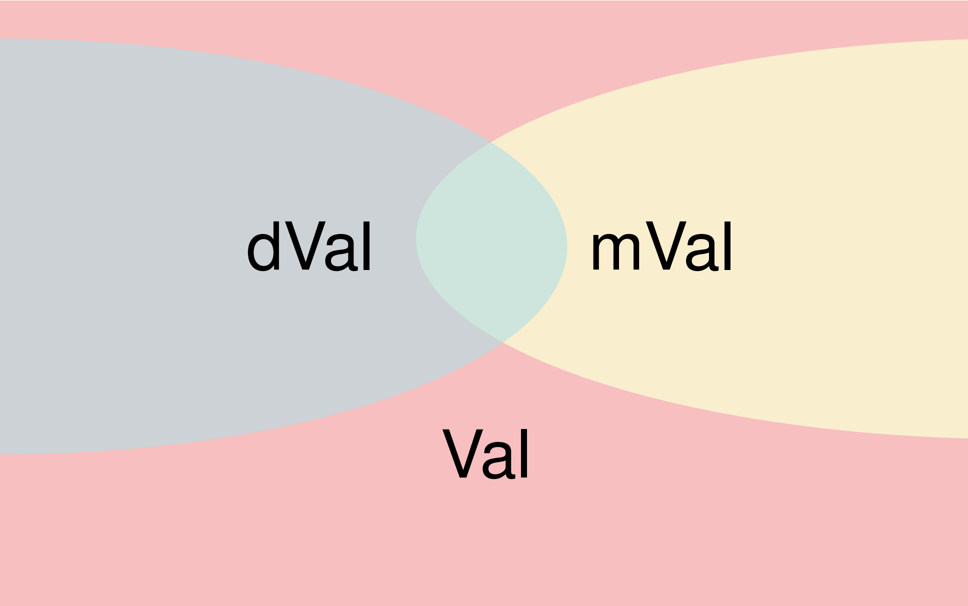
Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Contribution



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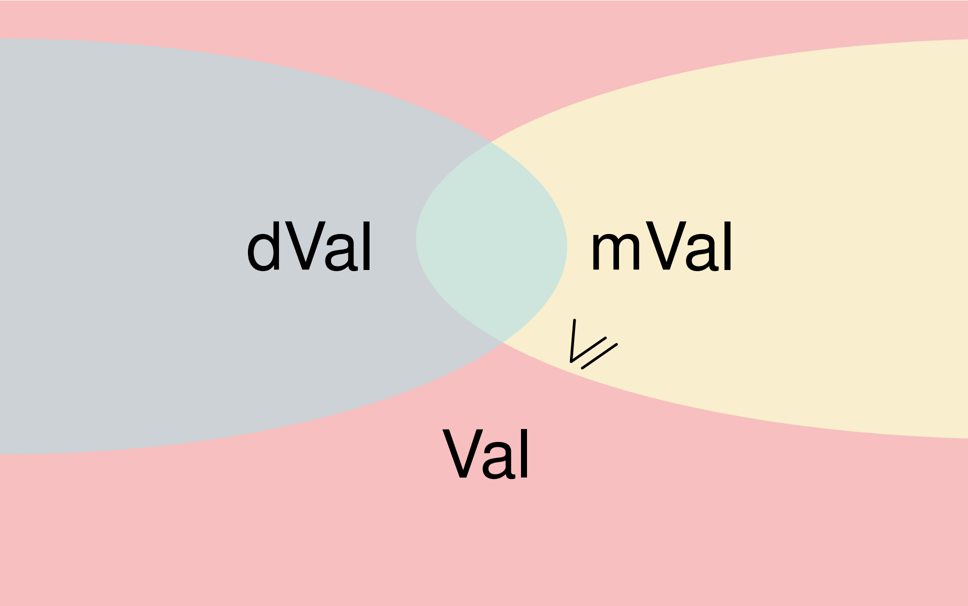

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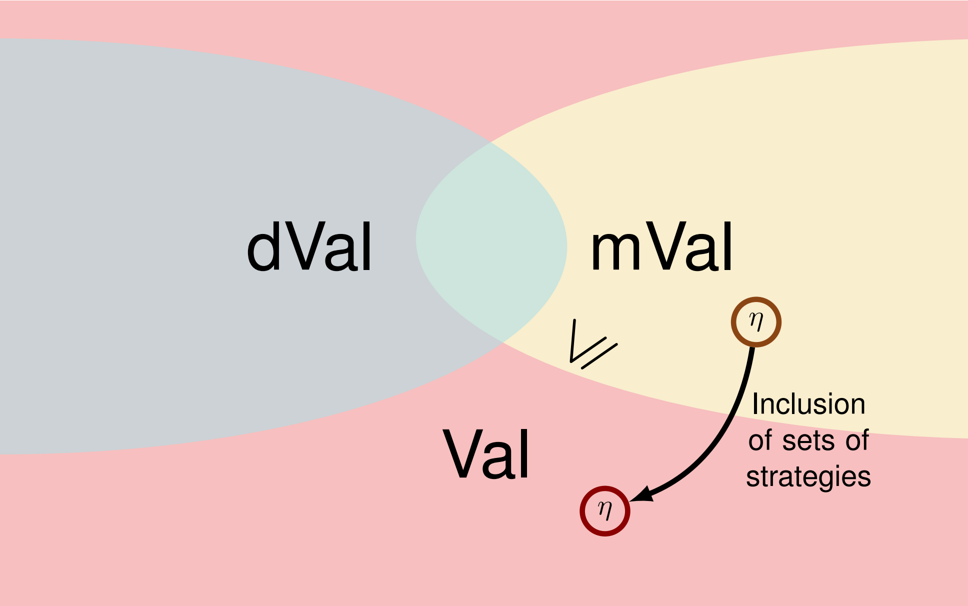

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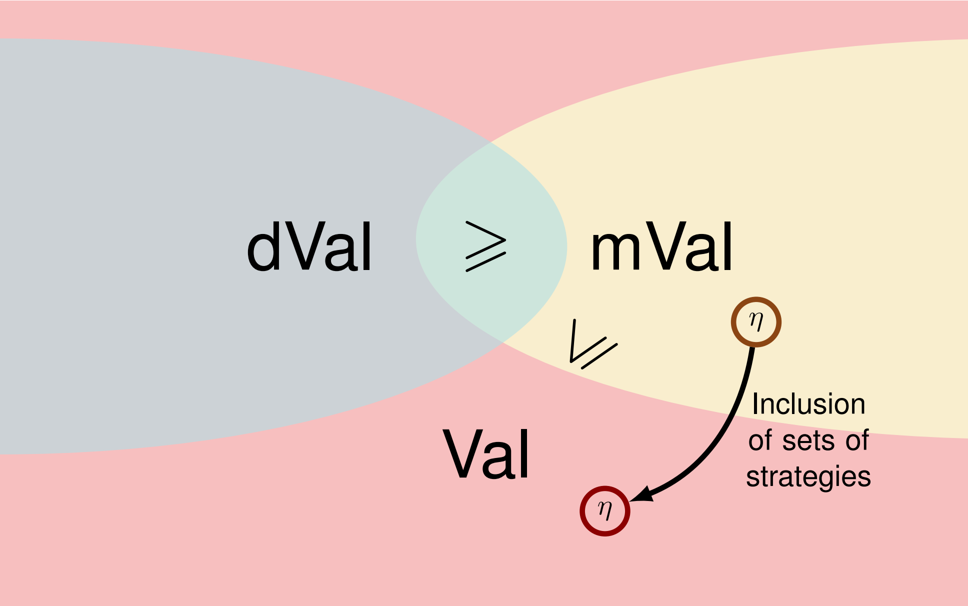

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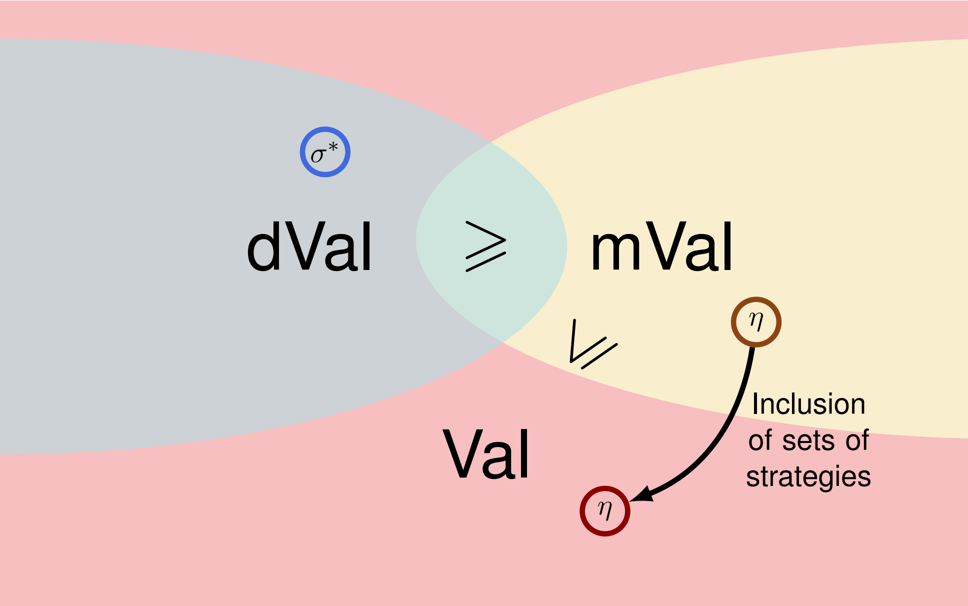

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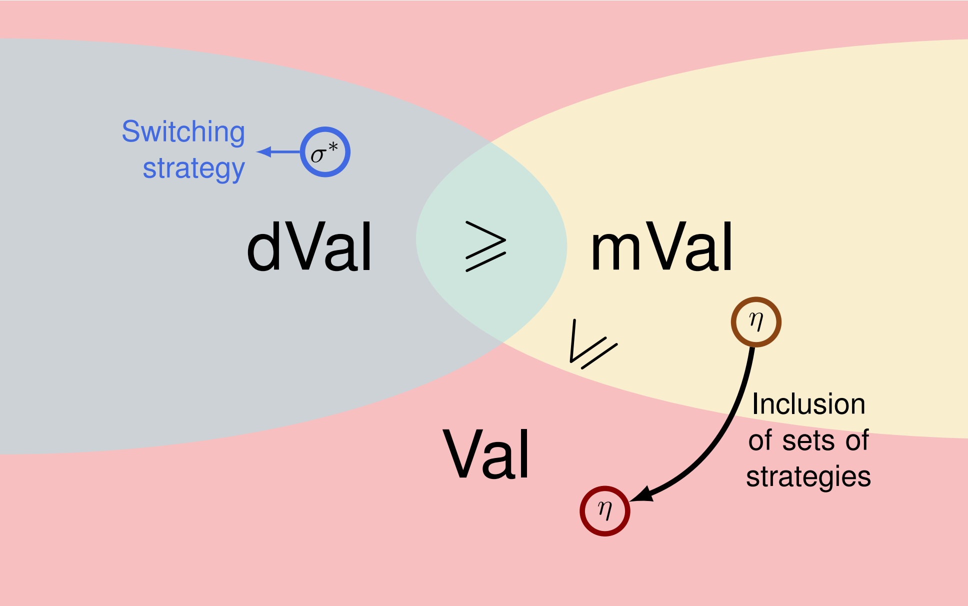

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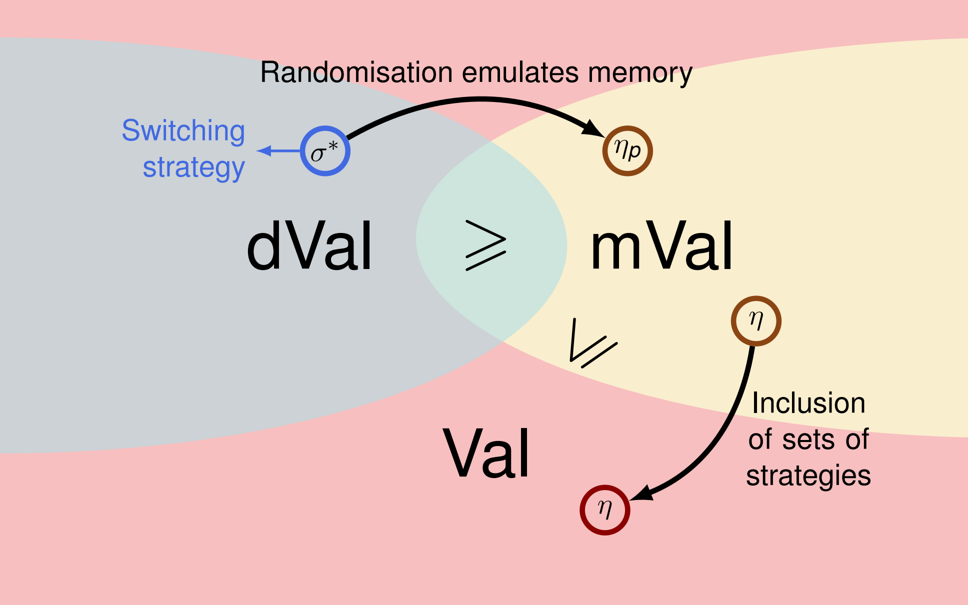

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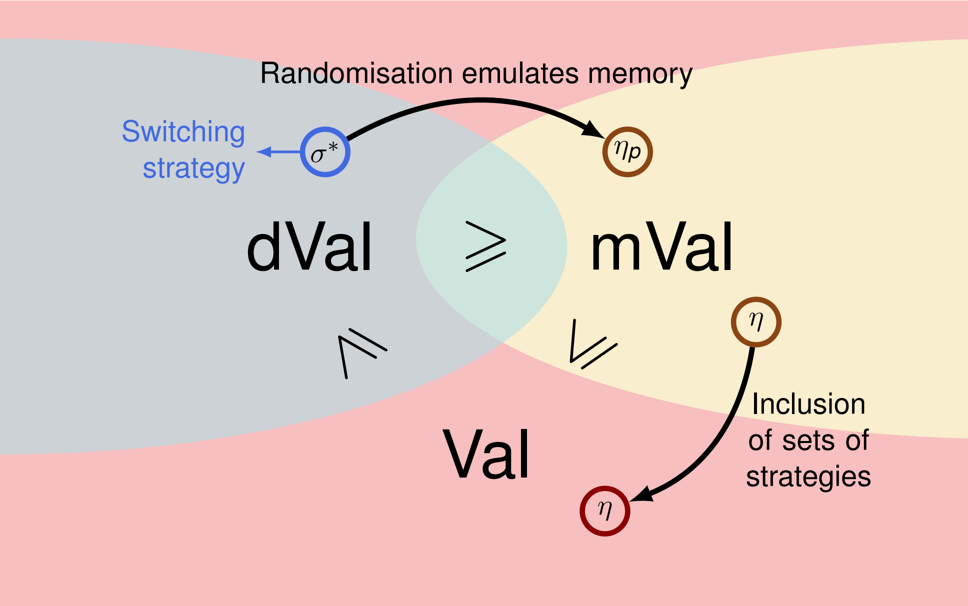

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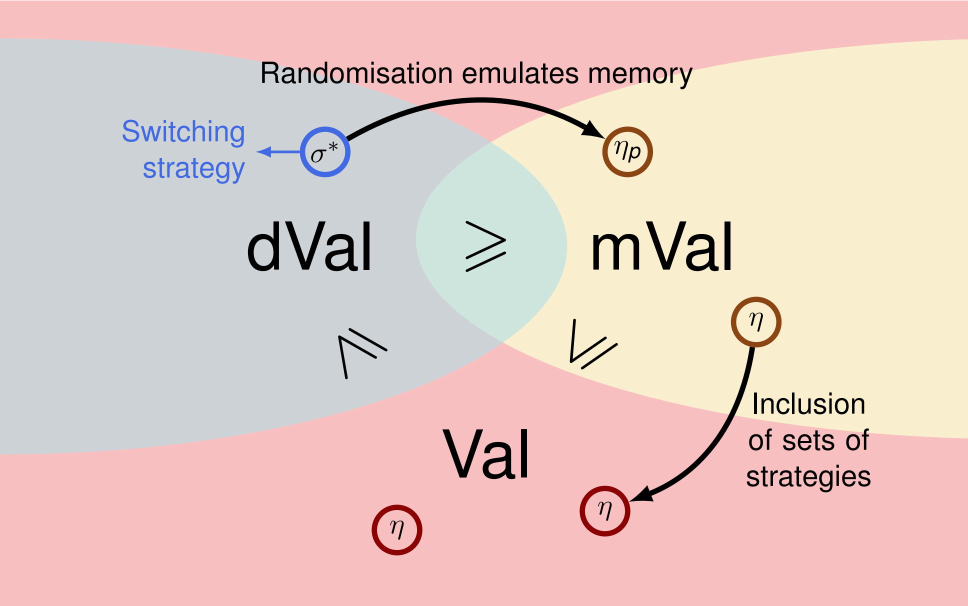

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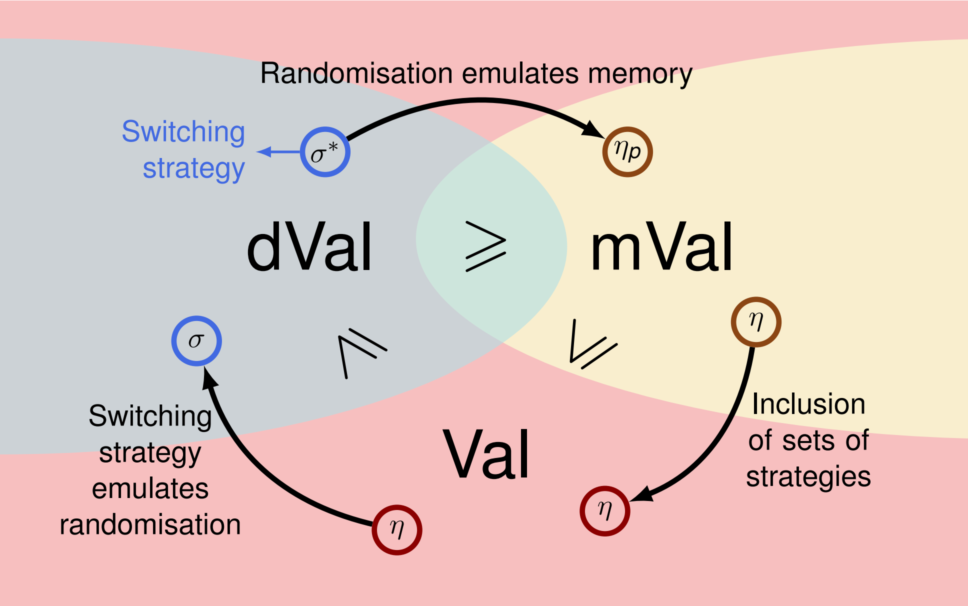

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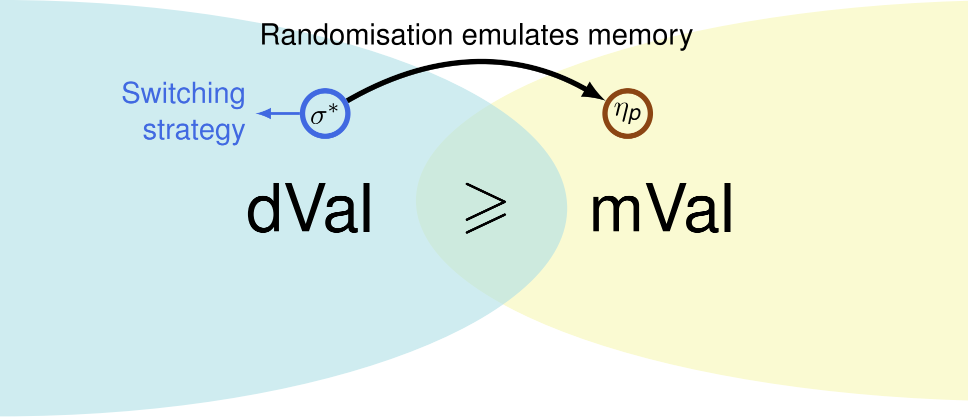

Randomisation emulates memory

Claim

$$\forall c, \lim_{\substack{p \to 1 \\ p < 1}} \mathrm{mVal}^{\eta_p}(c) \leqslant \mathrm{dVal}(c)$$

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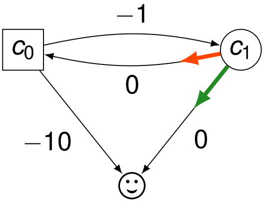

Strategy η_p

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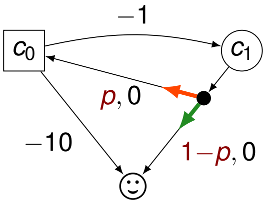

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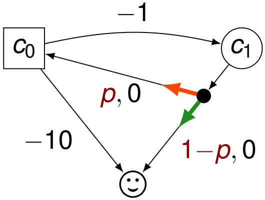

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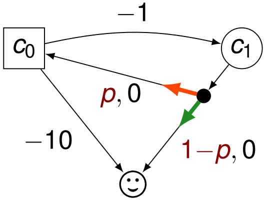

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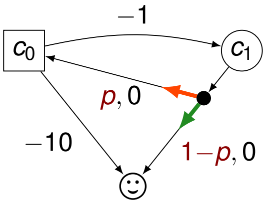

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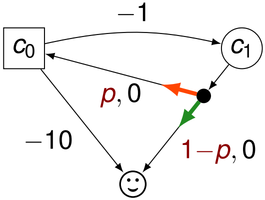

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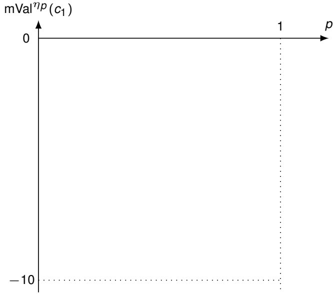

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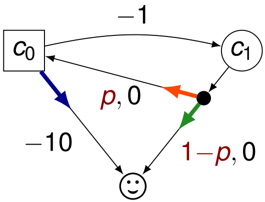

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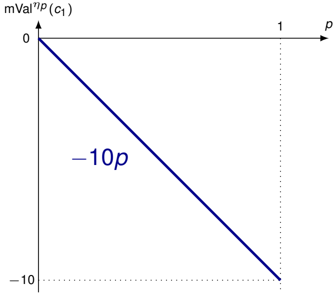

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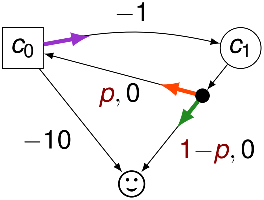

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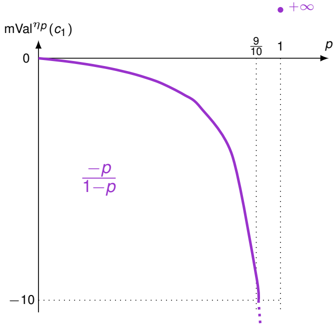

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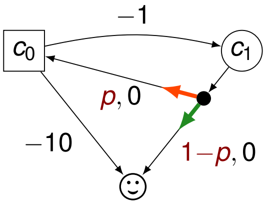

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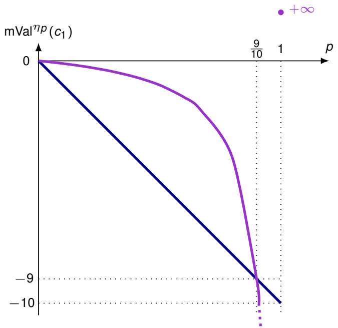

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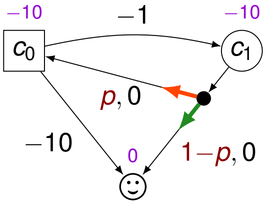

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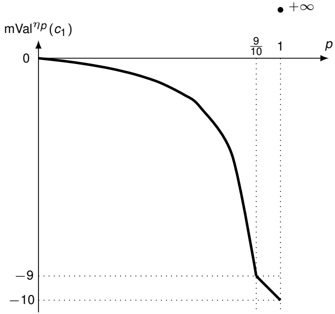

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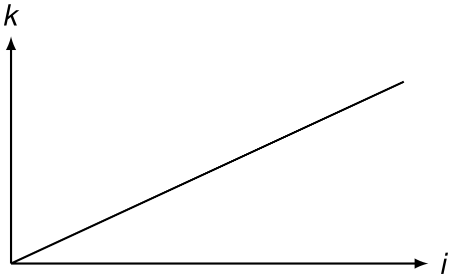

k size of play reaching the target

i number of choices given by σ_2

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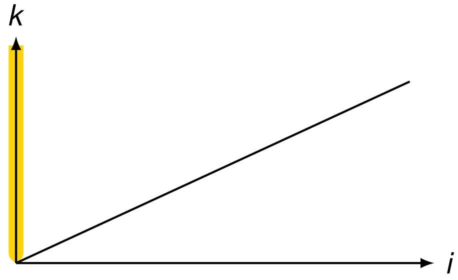

Yellow zone

All plays conforming to σ_1

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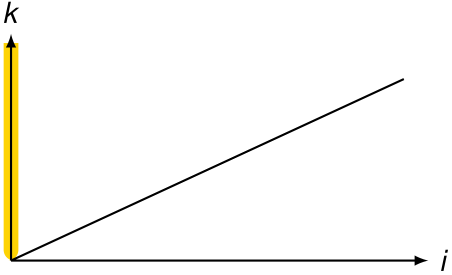

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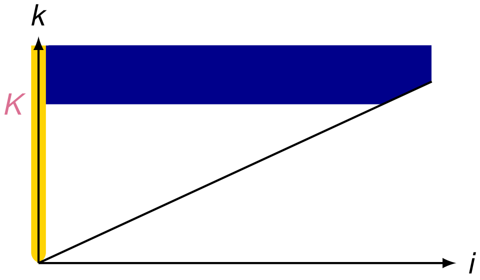

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Plays with many negative cycles

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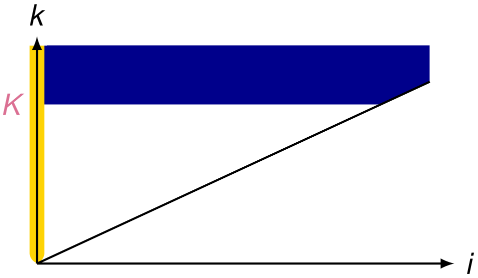

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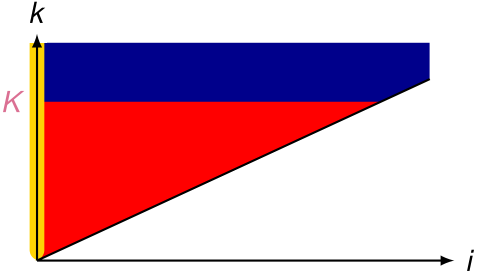
Rest of plays



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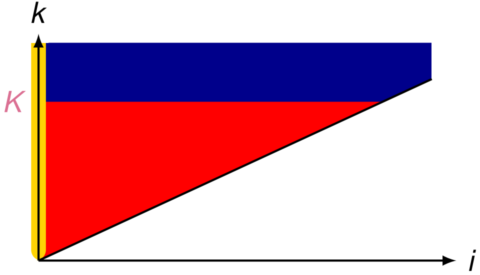
Red zone

Rest of plays $\quad \mathbb{E} \xrightarrow[\substack{p\to 1\\ p<1}]{} 0$



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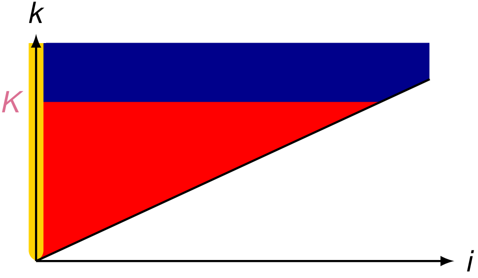

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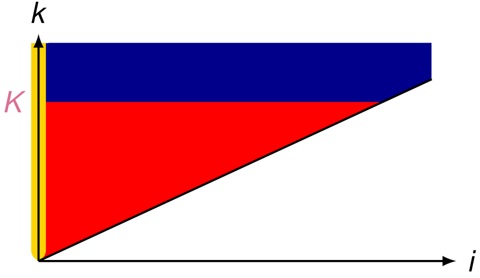

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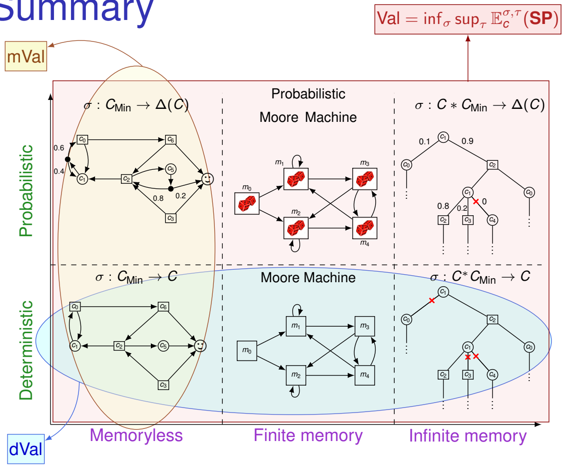
Summary



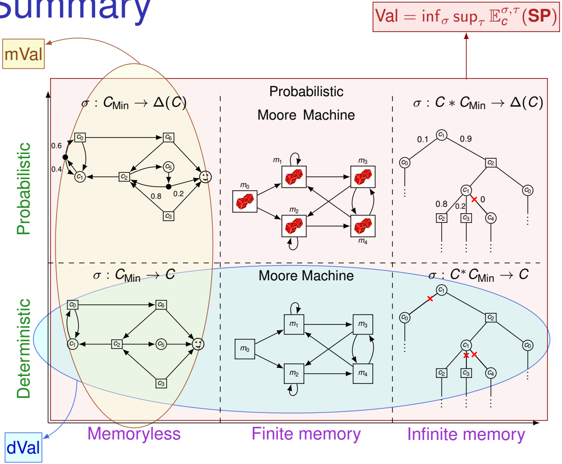
For which classes of games?

- Divergent finite shortest path games

Theorem

$$\text{Val} = \text{dVal} = \text{mVal}$$

Summary



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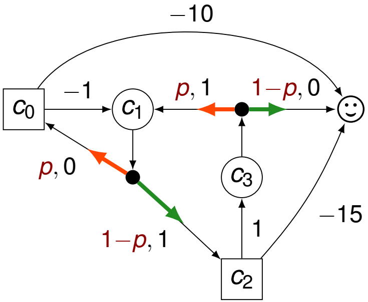
Extension to shortest path games



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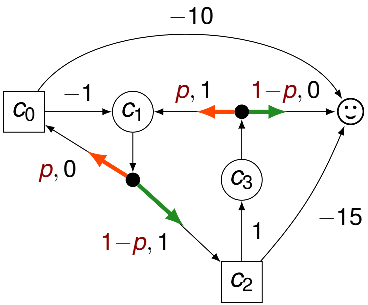
Extension to shortest path games

◯ Min ☐ Max



Claim

$$\forall c, \lim_{\substack{p \to 1 \\ p < 1}} \mathsf{mVal}^{\eta_p}(c) \leqslant \mathsf{dVal}(c)$$

Problem

Presence of non-negative cycles

Strategy η_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an optimal switching strategy, for all $p \in (0, 1)$,

$$\eta_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Extension to shortest path games

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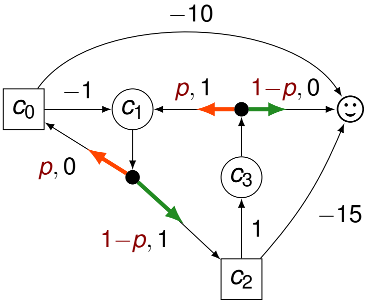
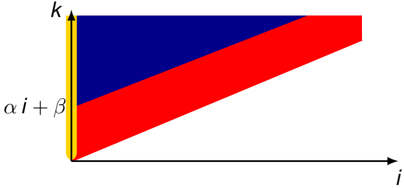


Claim

$$\forall c, \lim_{\substack{p \to 1 \\ p < 1}} \mathrm{mVal}^{\eta_p}(c) \leqslant \mathrm{dVal}(c)$$

Need a new partition



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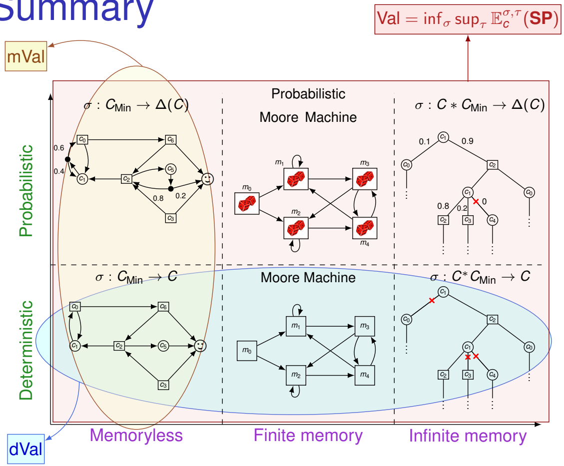
Summary



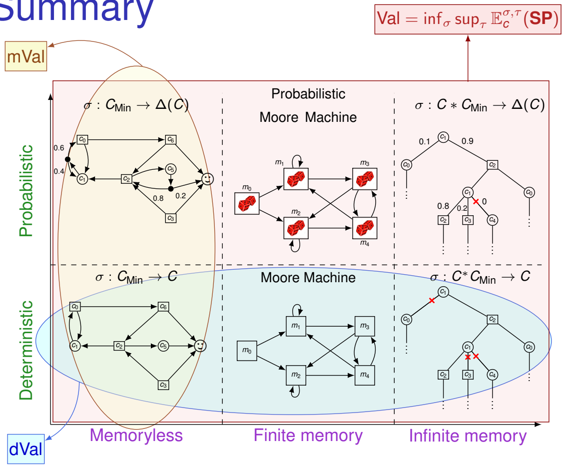
For which classes of games?

- Finite shortest path games

Theorem

$\mathsf{Val} = \mathsf{dVal} = \mathsf{mVal}$

Summary



For which classes of games?

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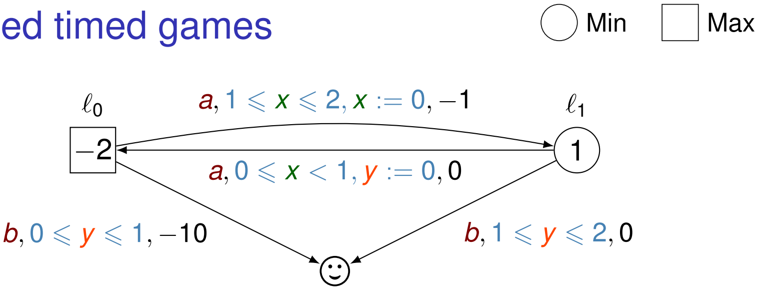
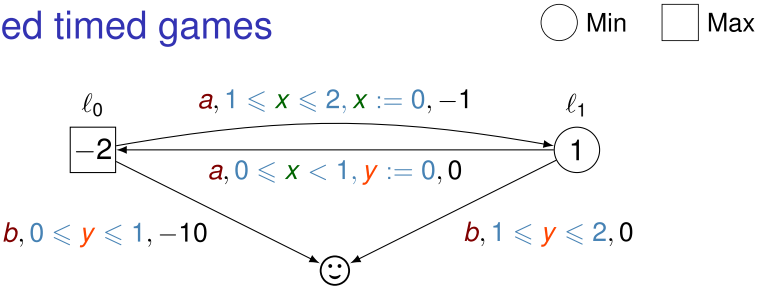
Weighted timed games



Weighted timed games

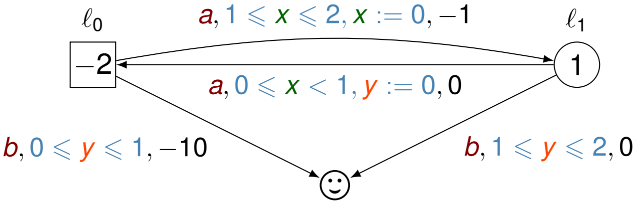


Play ρ

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\ 0.5} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\ 1.25} \left(\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}\right) \xrightarrow{b,\ 1/3} \left(☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}\right)$$

Weighted timed games

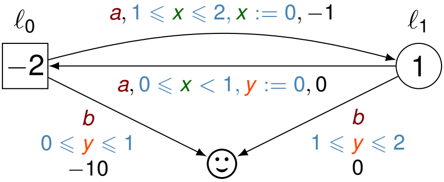
◯ Min ☐ Max



Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow[1 \times 0.5 + 0]{a,\ 0.5} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow[-2 \times 1.25 - 1]{a,\ 1.25} (\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow[1 \times \frac{1}{3} + 0]{b,\ 1/3} (☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}) \rightsquigarrow -\frac{8}{3}$$

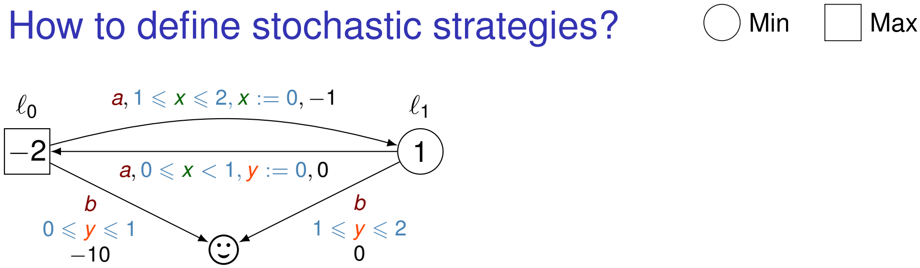
How to define stochastic strategies?



Deterministic strategy

Choose an edge and a delay

How to define stochastic strategies?



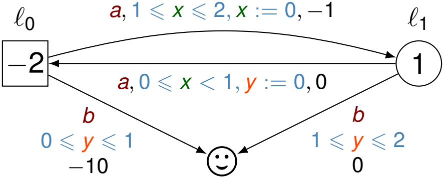
Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

How to define stochastic strategies?



Probabilistic strategy

Distribution over possible choices

Deterministic strategy

Choose an edge and a delay

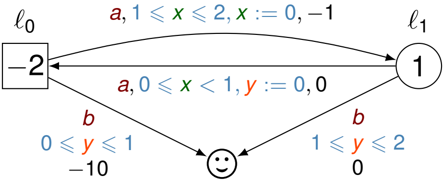
In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

How to define stochastic strategies?

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge *a*: finite distribution

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose *a* with $t = \frac{1}{3}$

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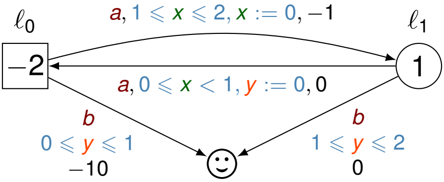
How to define stochastic strategies?

◯ Min   □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Deterministic strategy

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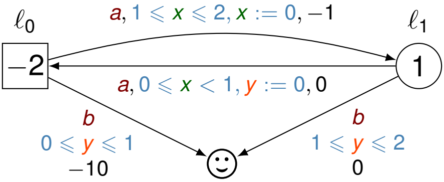

Deterministic strategy

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Choose *a* with $t = \frac{1}{3}$

Probabilistic strategy

Distribution over possible choices

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2. Delay for *a*: infinite distribution

In $(\ell_1, (0, 0))$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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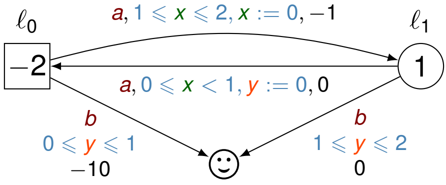

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Probabilistic strategy

Distribution over possible choices

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In $(\ell_1, (0,0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- a: choose t with $\mathcal{U}([0,1[)$

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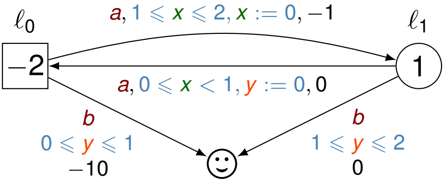

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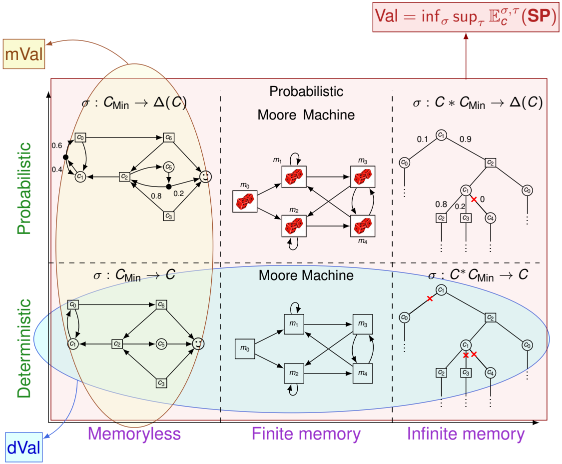
In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- a: choose t with $\mathcal{U}([0, 1[)$
- b: choose t with $\delta_{1.5}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Summary



For which classes of games?

- Finite shortest path games
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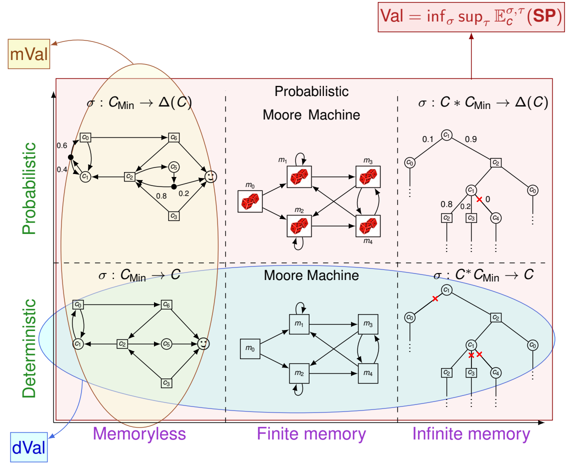
Theorem

Val = dVal = mVal

Summary

For divergent weighted timed games

- Definition of $\mathbb{P}_c^{\sigma,\tau}$



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Summary

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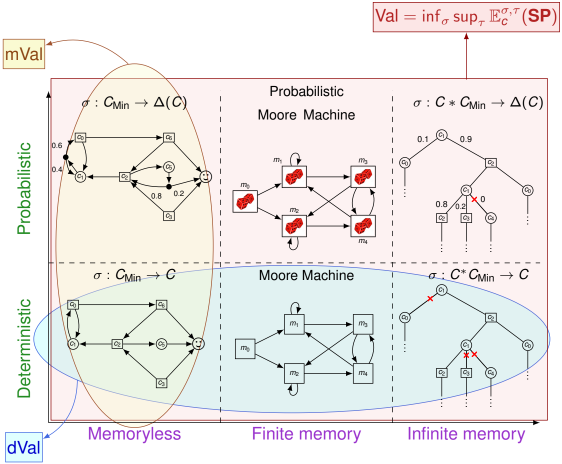
- Definition of $\mathbb{P}_c^{\sigma,\tau}$
- Definition of $\mathbb{E}_c^{\sigma,\tau}(\mathbf{SP})$



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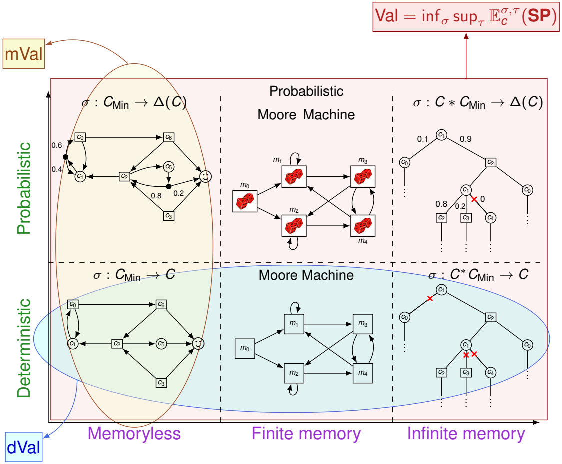

For divergent weighted timed games

- Definition of $\mathbb{P}_c^{\sigma,\tau}$
- Definition of $\mathbb{E}_c^{\sigma,\tau}(\mathbf{SP})$
- Safety conditions on strategies

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Summary: perspectives

For divergent weighted timed games

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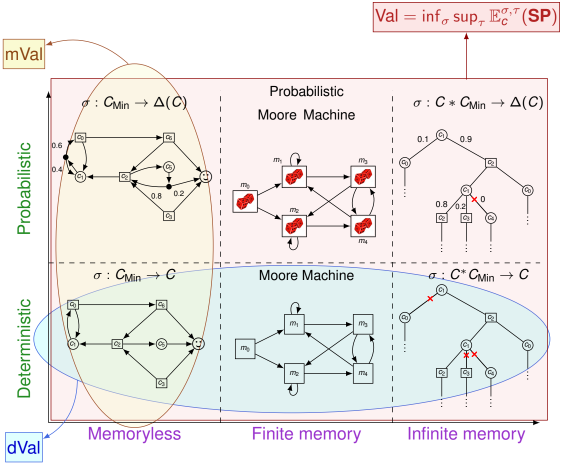

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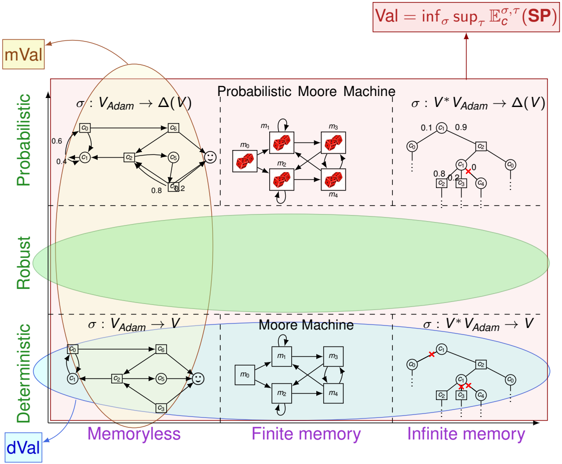
Summary: perspectives



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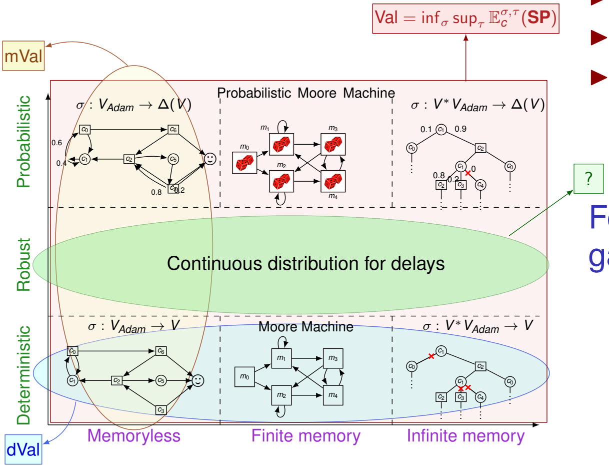
Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Summary: perspectives



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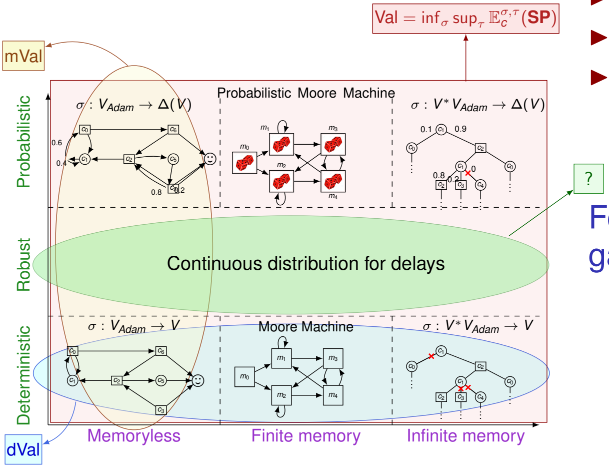

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Thank you! Questions?