Playing Stochastically in Weighted Timed Games to Emulate Memory

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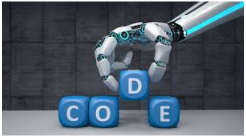
Motivation: game theory for synthesis

Classical approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents: environment and controller

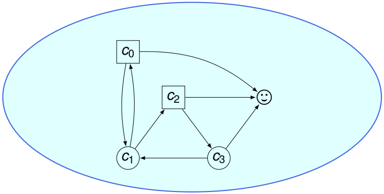


Code synthesis

Correct by construction: synthesis of controller

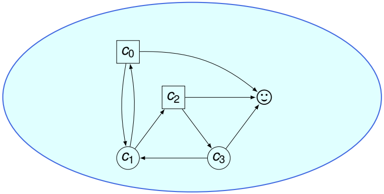
Different classes of games

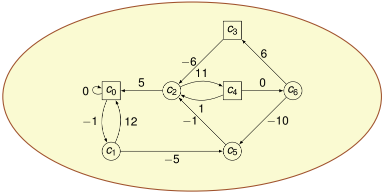
Qualitative games



Different classes of games

Qualitative games

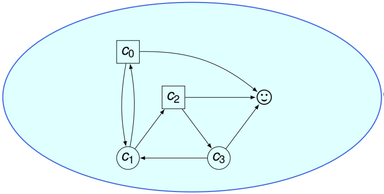




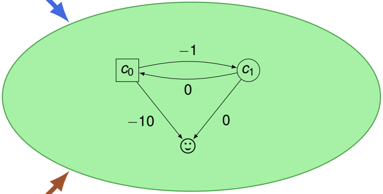
Quantitative games

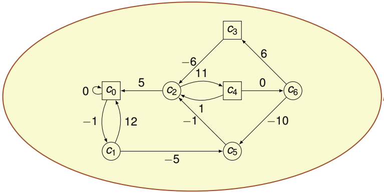
Different classes of games

Qualitative games



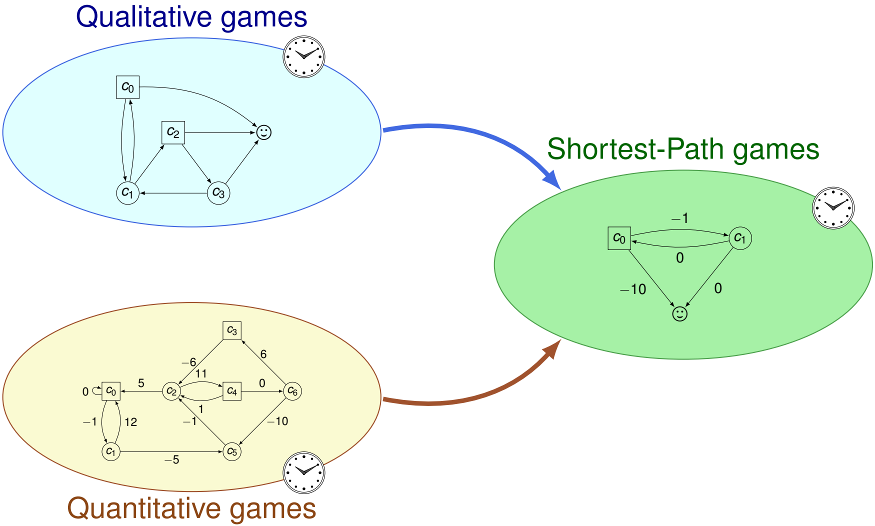
Shortest-Path games



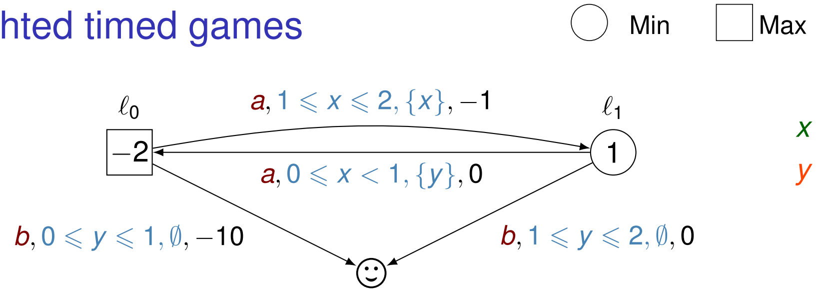


Quantitative games

Different classes of games



Weighted timed games

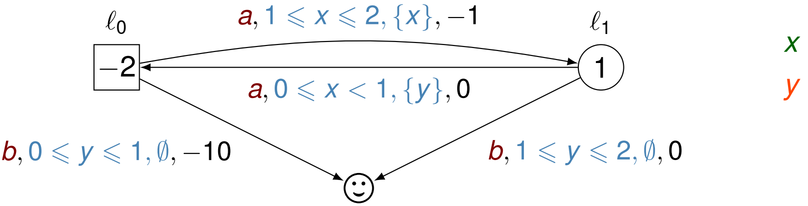


Play ρ

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{b,\ 1/3} (\smile, \begin{pmatrix}1/3\\19/12\end{pmatrix})$$

Weighted timed games

○ Min □ Max



Play ρ

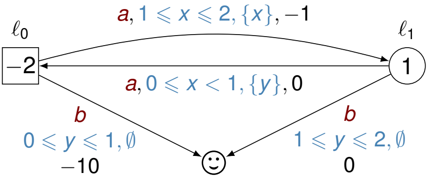
$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{b,\ 1/3} (☺, \begin{pmatrix}1/3\\19/12\end{pmatrix}) \rightsquigarrow -\frac{8}{3}$$

$$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1 \qquad 1 \times \frac{1}{3} + 0$$

Shortest-path payoff

$$\mathbf{SP}(\rho) = \begin{cases} \mathrm{wt}(\rho) & \text{if } \rho \text{ reaches ☺} \\ +\infty & \text{otherwise} \end{cases}$$

Strategies

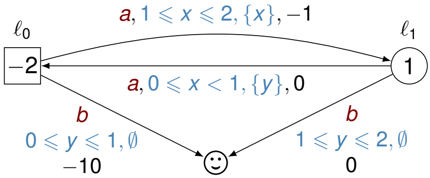


Deterministic strategy

Choose an edge and a delay

Strategies

◯ Min ▢ Max



Deterministic strategy

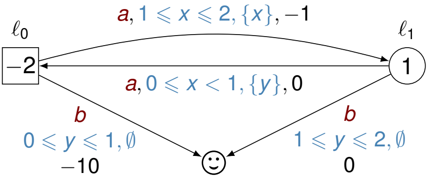
Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose *a* with $t = \frac{1}{3}$

Strategies

Min Max



Probabilistic strategy

Distribution over possible choices

Deterministic strategy

Choose an edge and a delay

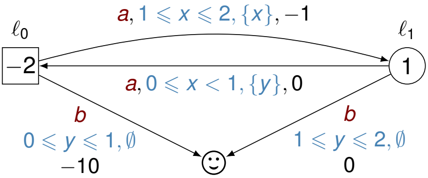
In $(\ell_1, (0,0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

○ Min ☐ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

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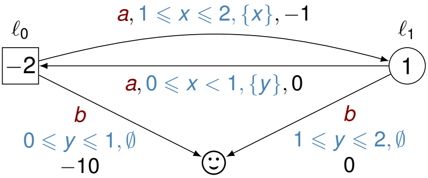
Strategies

Min Max



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

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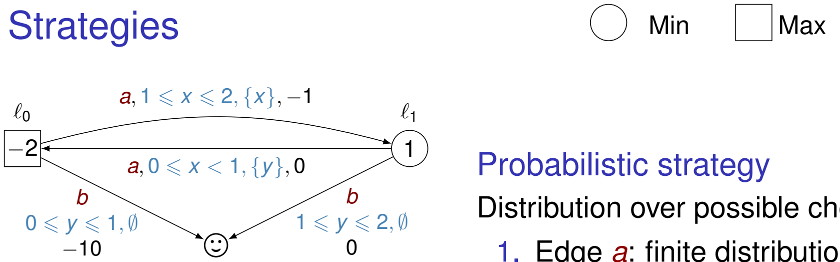
Strategies



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
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Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

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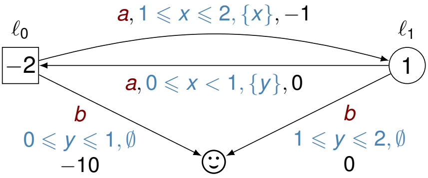
Strategies

Min Max



Probabilistic strategy

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In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

- a: choose t with $\mathcal{U}([0, 1[)$

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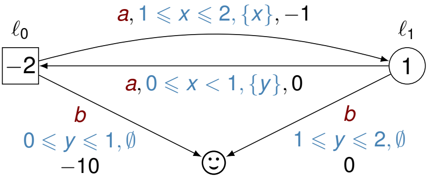
Strategies

○ Min ☐ Max



Probabilistic strategy

Distribution over possible choices

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Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

In $(\ell_1, (0, 0))$

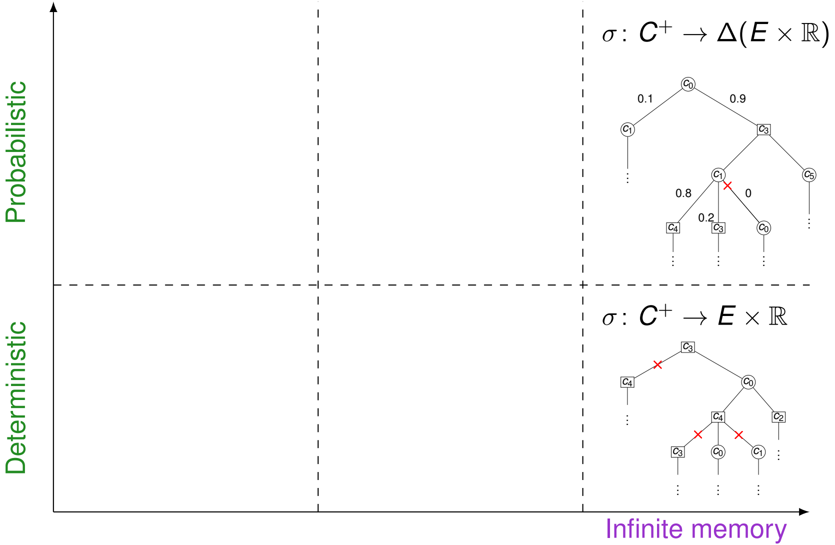
Choose between a or b with $\mathcal{B}(p)$

- a: choose t with $\mathcal{U}([0, 1[)$
- b: choose t with $\delta_{1.5}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



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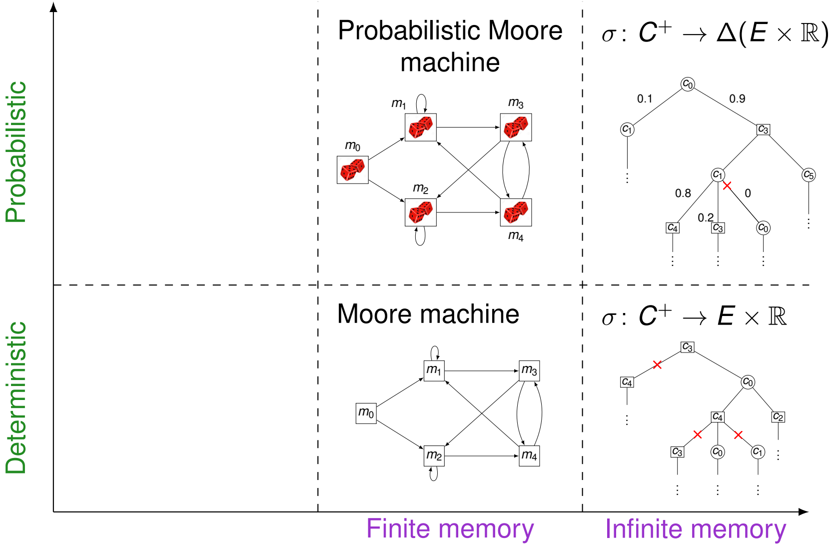

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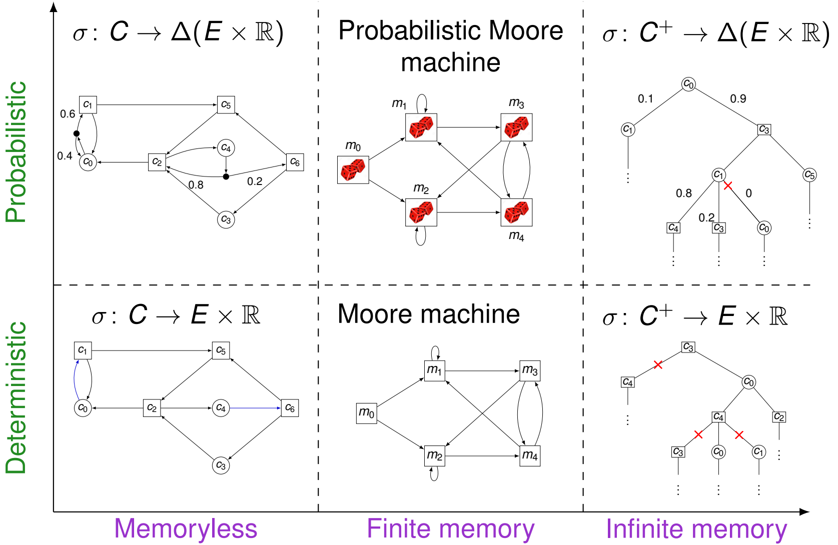
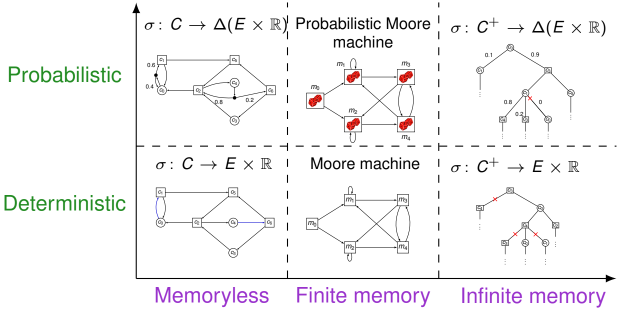


Contributions

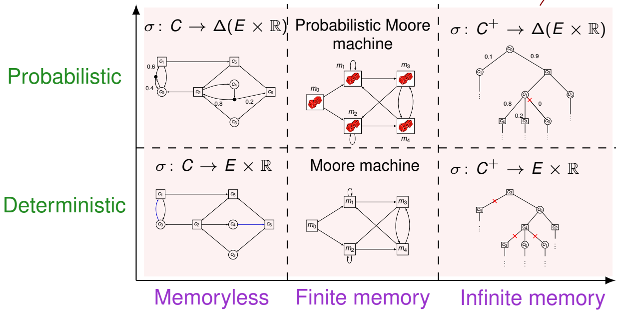
$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}_{\ell,\nu}^{\sigma,\tau}(\textbf{SP})$



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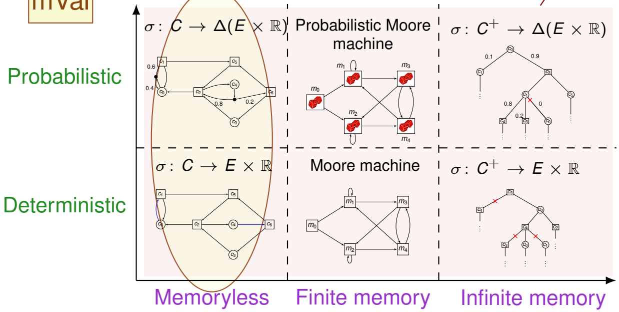
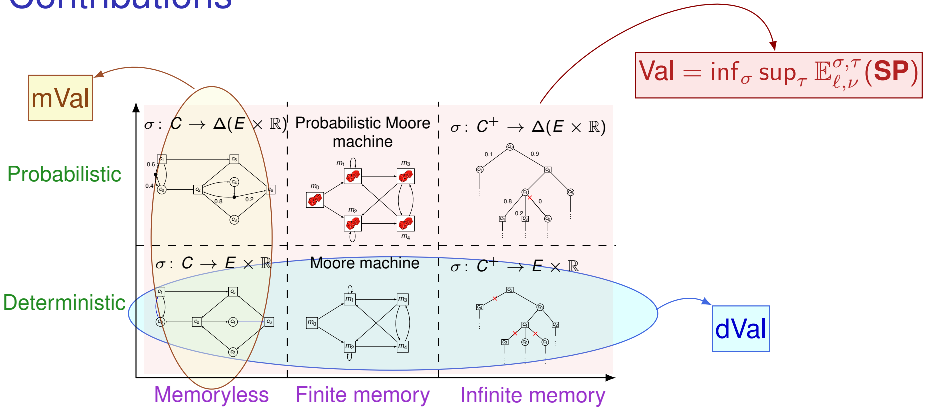
mVal



Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



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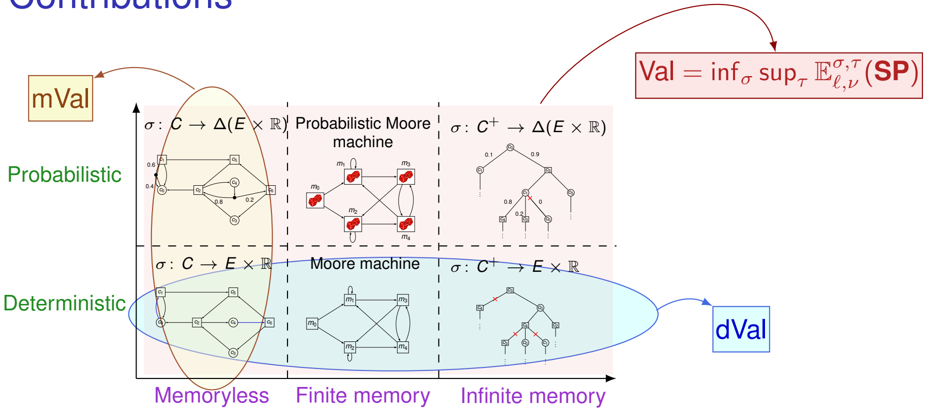


Theorem

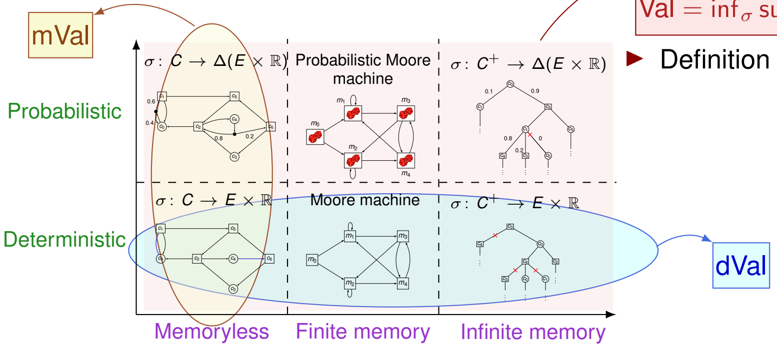
$\mathrm{dVal} = \mathrm{Val} = \mathrm{mVal}$

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\mathsf{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$



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Contributions

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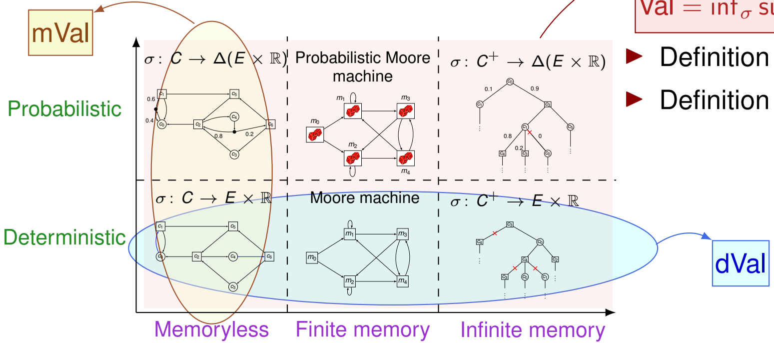
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$



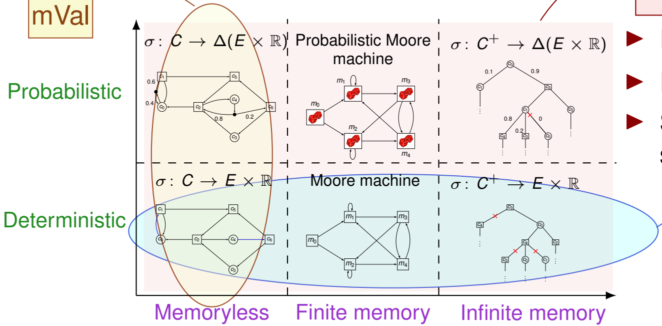
Theorem

$\text{dVal} = \text{Val} = \text{mVal}$

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

mVal



dVal

$\mathrm{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies

Theorem

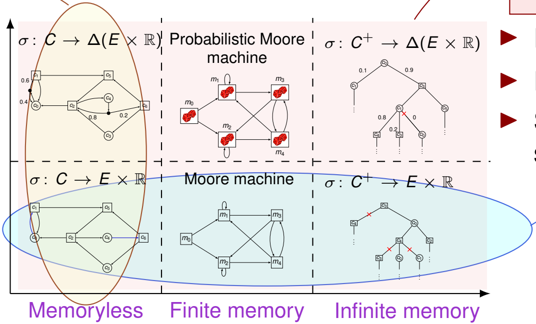
$\mathrm{dVal} = \mathrm{Val} = \mathrm{mVal}$

Contributions

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

mVal

$\text{Val} = \inf_\sigma \sup_\tau \mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$



- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$
- Safety conditions on strategies

dVal

Theorem

dVal = Val = mVal

Thank you! Any questions?