Playing Stochastically in Weighted Timed Games to Emulate Memory

Julie Parreaux
Benjamin Monmege   Pierre-Alain Reynier

Séminaire MOVE
October 7, 2021

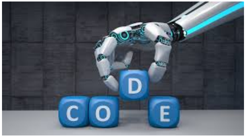
Motivation: game theory for synthesis

Classical approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents: environment and controller

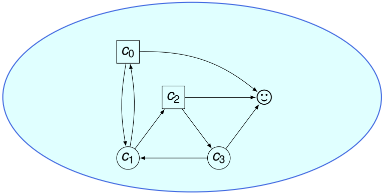


Code synthesis

Correct by construction: synthesis of controller

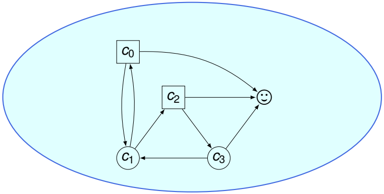
Different classes of games

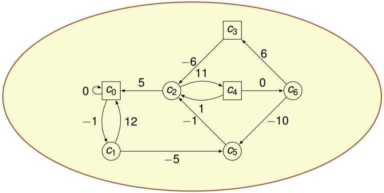
Qualitative games



Different classes of games

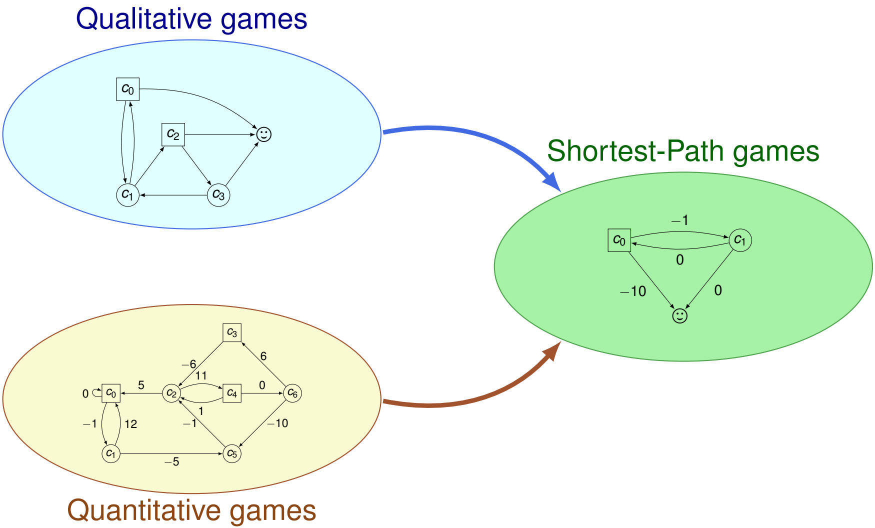
Qualitative games



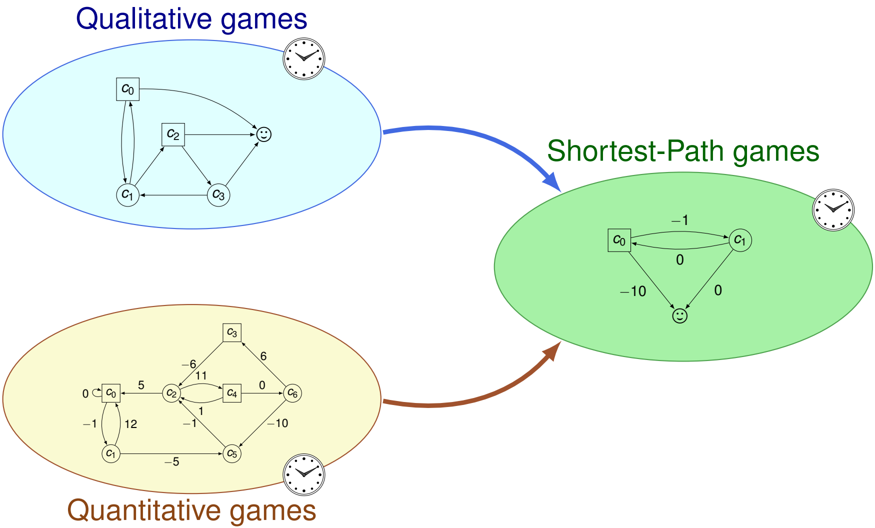


Quantitative games

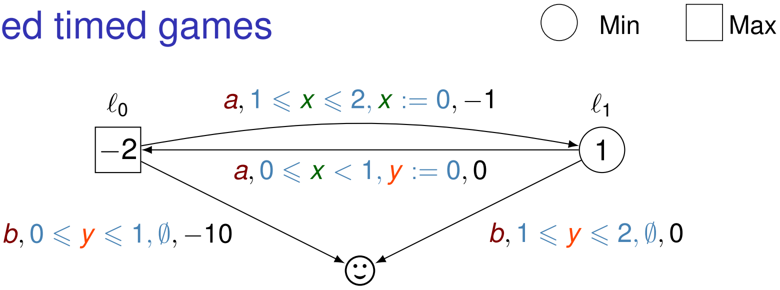
Different classes of games



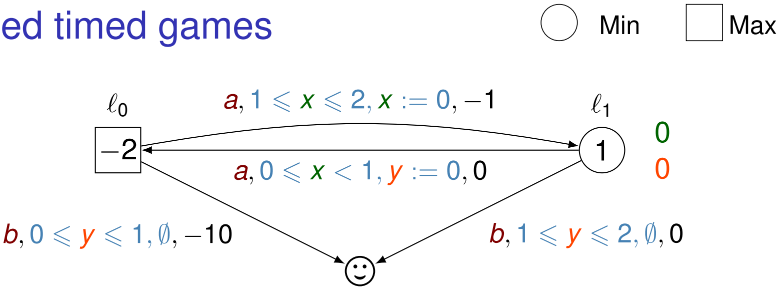
Different classes of games



Weighted timed games



Weighted timed games

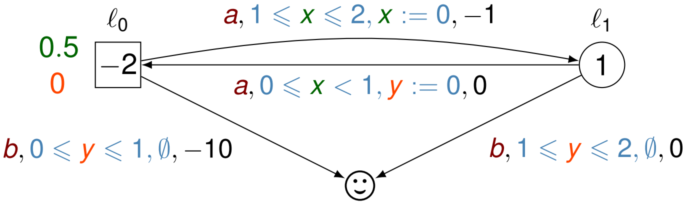


Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix})$$

Weighted timed games

Min Max

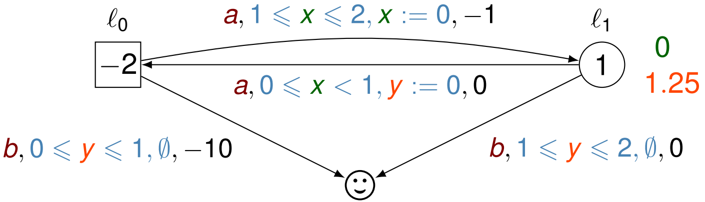


Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix})$$

Weighted timed games

Min Max

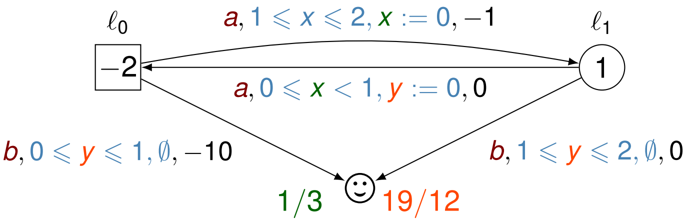


Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix})$$

Weighted timed games

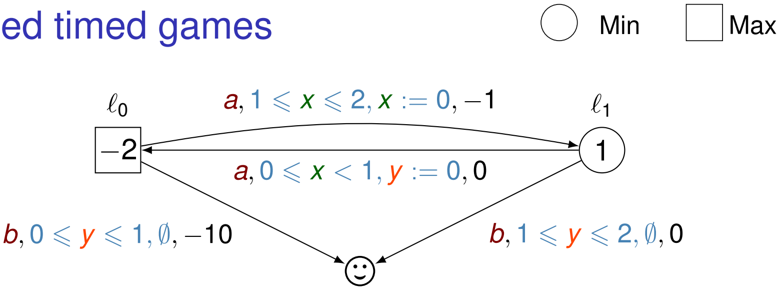
○ Min ☐ Max



Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow{b,\ 1/3} (☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix})$$

Weighted timed games



Play ρ

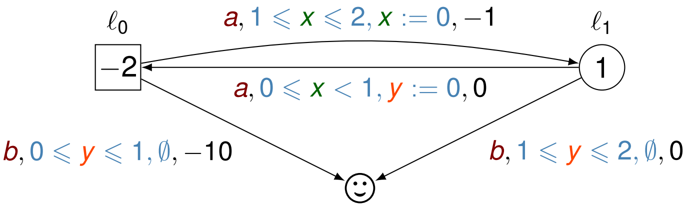
$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{b,\ 1/3} (☺, \begin{pmatrix}1/3\\19/12\end{pmatrix})$$

Shortest-path payoff

$$\mathbf{SP}(\rho) = \begin{cases} \sum_{i=0}^{n-1} (\mathrm{wt}(e_i) + t\,\mathrm{wt}(\ell_i)) & \text{if } n \text{ is the smallest index s.t. } \pi_n = ☺ \\ +\infty & \text{if } \pi \text{ does not reach ☺} \end{cases}$$

Weighted timed games

Min Max



Play ρ

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\ 0.5} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right) \xrightarrow{a,\ 1.25} \left(\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}\right) \xrightarrow{b,\ 1/3} \left(☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}\right)$$

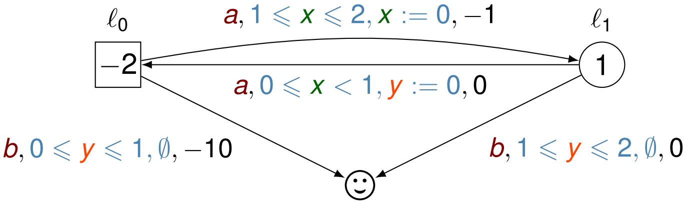
$1 \times 0.5 + 0$

Shortest-path payoff

$$\mathbf{SP}(\rho) = \begin{cases} \sum_{i=0}^{n-1} (\mathrm{wt}(e_i) + t\,\mathrm{wt}(\ell_i)) & \text{if } n \text{ is the smallest index s.t. } \pi_n = ☺ \\ +\infty & \text{if } \pi \text{ does not reach } ☺ \end{cases}$$

Weighted timed games

Min Max



Play ρ

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{a,\ 0.5} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{a,\ 1.25} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{b,\ 1/3} (☺, \begin{pmatrix}1/3\\19/12\end{pmatrix})$$

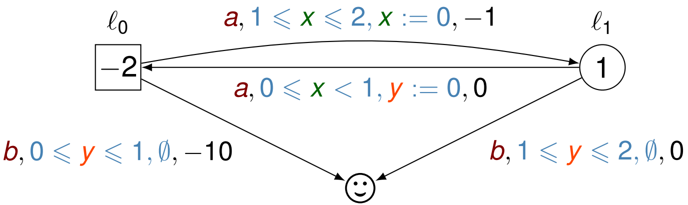
$$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1 \qquad 1 \times \frac{1}{3} + 0$$

Shortest-path payoff

$$\mathbf{SP}(\rho) = \begin{cases} \sum_{i=0}^{n-1} (\mathrm{wt}(e_i) + t\,\mathrm{wt}(\ell_i)) & \text{if } n \text{ is the smallest index s.t. } \pi_n = ☺ \\ +\infty & \text{if } \pi \text{ does not reach } ☺ \end{cases}$$

Weighted timed games

○ Min □ Max



Play ρ

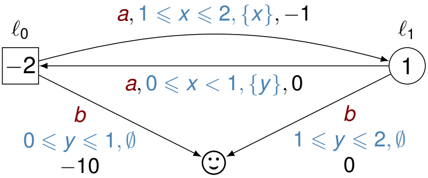
$$\left(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}\right) \xrightarrow{a,\ 0.5} \left(\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}\right) \xrightarrow{a,\ 1.25} \left(\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}\right) \xrightarrow{b,\ 1/3} \left(☺, \begin{pmatrix}1/3\\19/12\end{pmatrix}\right) \rightsquigarrow -\frac{8}{3}$$

$$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1 \qquad 1 \times \tfrac{1}{3} + 0$$

Shortest-path payoff

$$\mathbf{SP}(\rho) = \begin{cases} \sum_{i=0}^{n-1} (\mathrm{wt}(e_i) + t\,\mathrm{wt}(\ell_i)) & \text{if } n \text{ is the smallest index s.t. } \pi_n = ☺ \\ +\infty & \text{if } \pi \text{ does not reach } ☺ \end{cases}$$

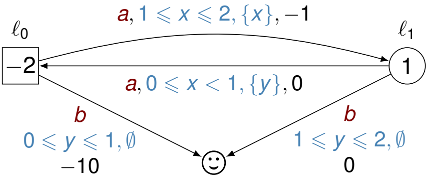
Strategies



Deterministic strategy

Choose an edge and a delay

Strategies



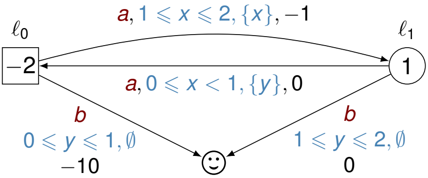
Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Strategies



Probabilistic strategy

Distribution over possible choices

Deterministic strategy

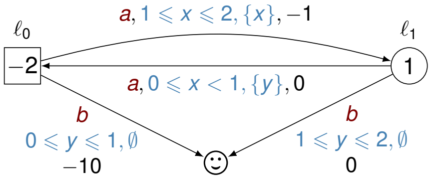
Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution

Deterministic strategy

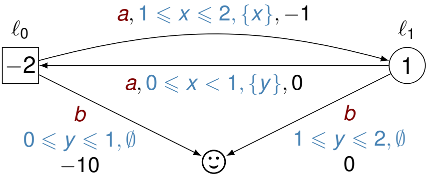
Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies



Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Probabilistic strategy

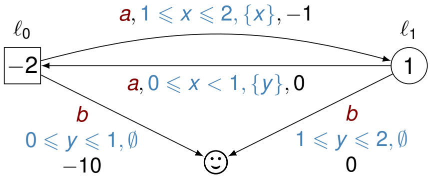
Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

Min Max



Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

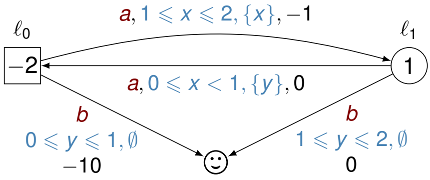
In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(p)$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

◯ Min ☐ Max



Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

In $(\ell_1, (0, 0))$

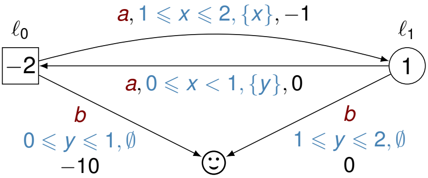
Choose between a or b with $\mathcal{B}(p)$

- a: choose t with $\mathcal{U}([0, 1[)$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Strategies

Min Max



Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

In $(\ell_1, (0, 0))$

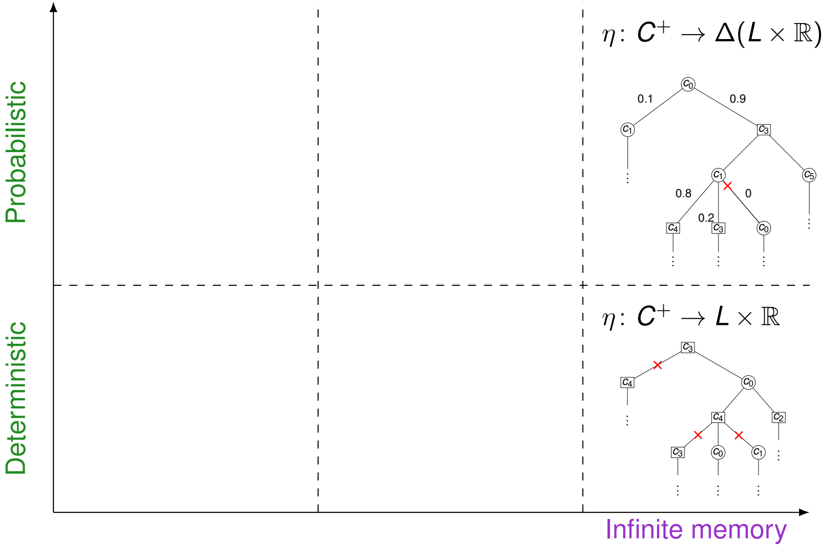
Choose between a or b with $\mathcal{B}(p)$

- a: choose t with $\mathcal{U}([0, 1[)$
- b: choose t with $\delta_{1.5}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

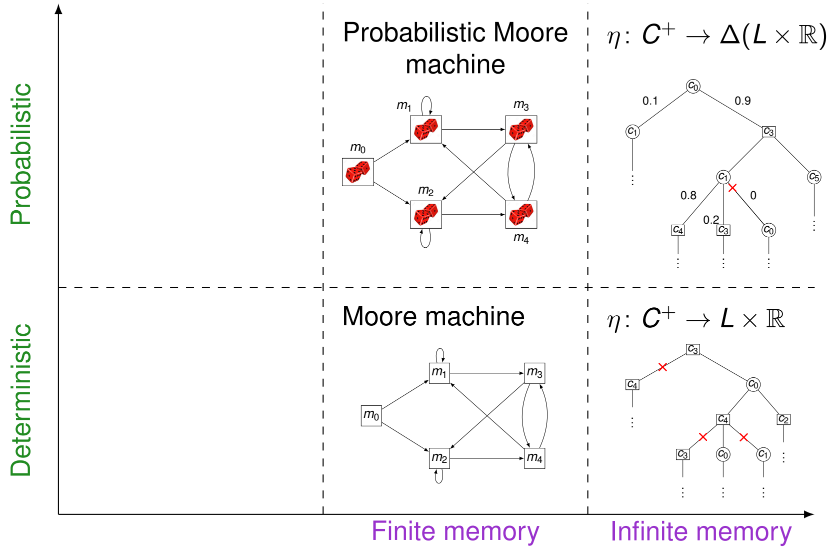
Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



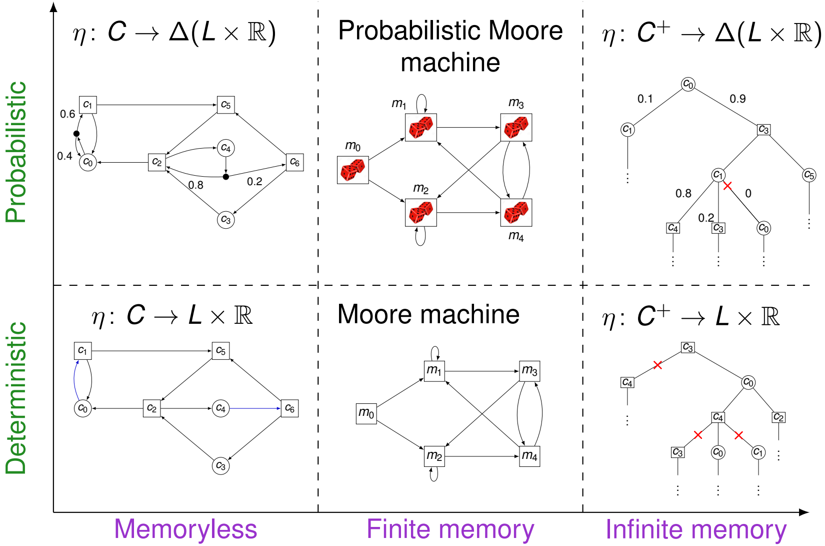
Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



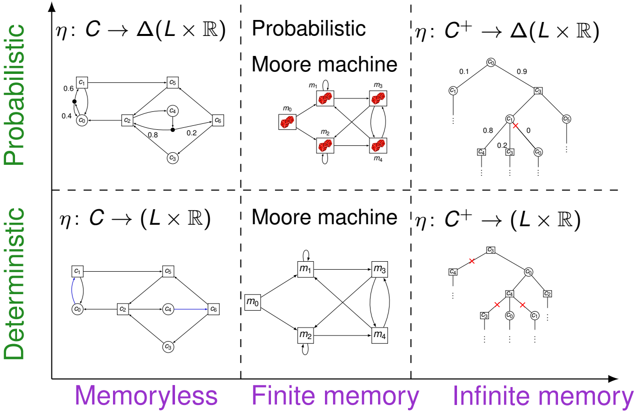
Zoology of strategies

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Stochastic value

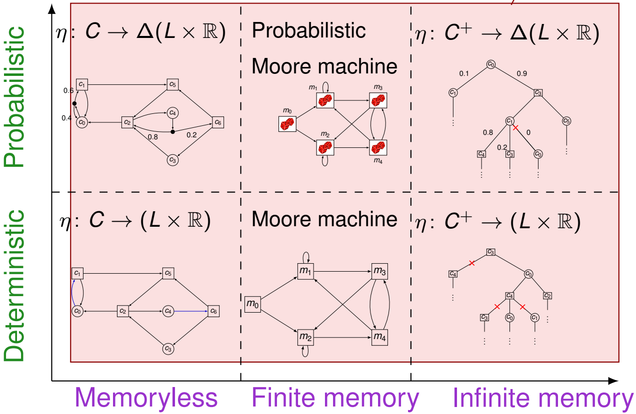
$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Stochastic value

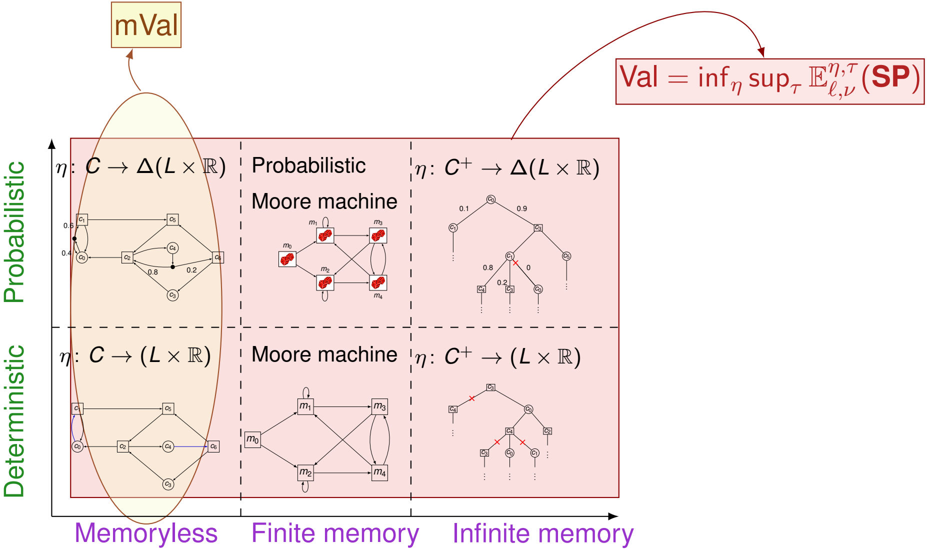
$C = L \times \mathbb{R}^{|\mathcal{C}|}$

$\text{Val} = \inf_\eta \sup_\tau \mathbb{E}_{\ell,\nu}^{\eta,\tau}(\mathbf{SP})$



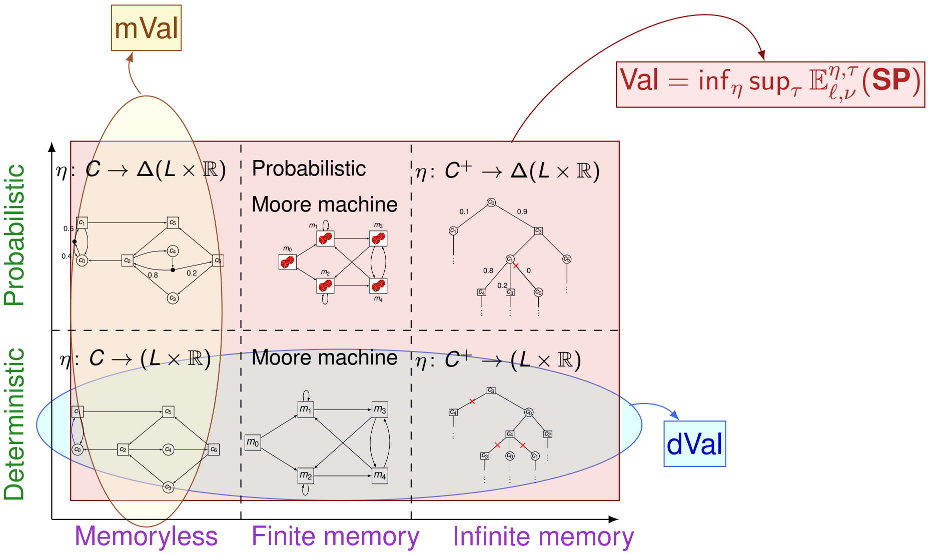
Stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Stochastic value

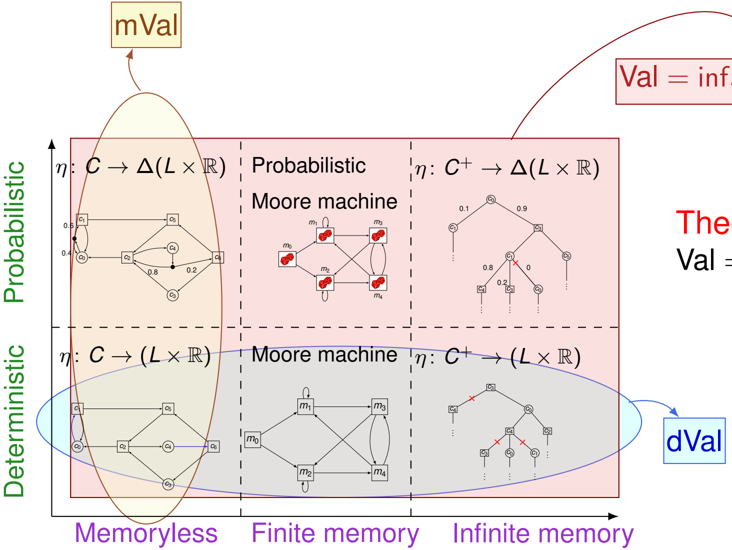
$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

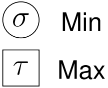
$\mathsf{Val} = \inf_\eta \sup_\tau \mathbb{E}^{\eta,\tau}_{\ell,\nu}(\mathbf{SP})$



Theorem

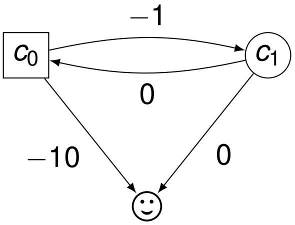
$\mathsf{Val} = \mathsf{dVal} = \mathsf{mVal}$

Deterministic strategies: Min needs memory



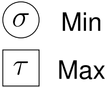
Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))$$



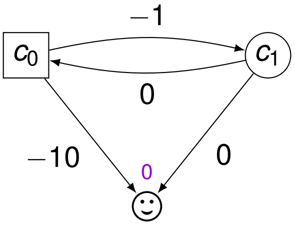
Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic strategies: Min needs memory



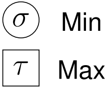
Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))$$



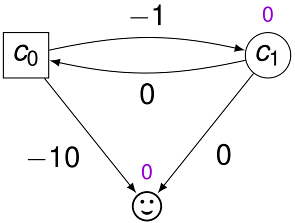
Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic strategies: Min needs memory



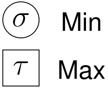
Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))$$



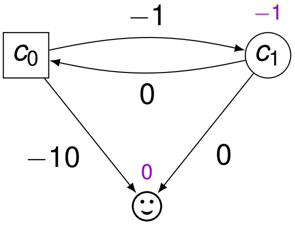
Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic strategies: Min needs memory



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))$$



Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

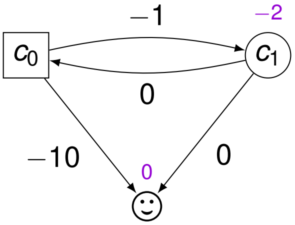
Deterministic strategies: Min needs memory

σ Min

τ Max

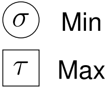
Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))$$



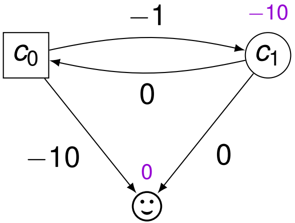
Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic strategies: Min needs memory



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))$$



Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

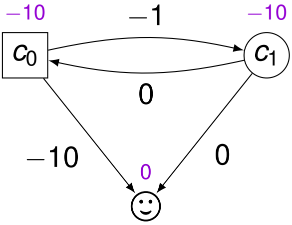
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))$$



Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

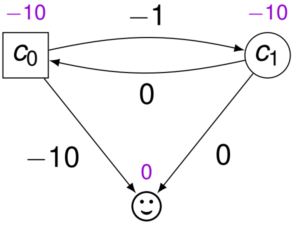
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v,\sigma,\tau))}_{\mathsf{dVal}^{\sigma}(v)}$$

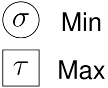


Optimal strategy

$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

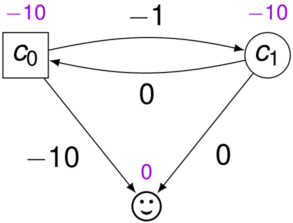
Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic strategies: Min needs memory



Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v,\sigma,\tau))}_{\mathrm{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

An optimal strategy for Min may require finite memory.

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

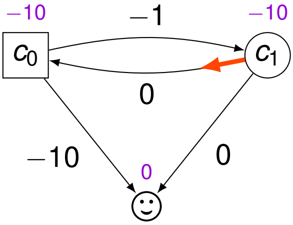
Deterministic strategies: Min needs memory

$\bigcirc\!\!\!\sigma$ Min

$\boxed{\tau}$ Max

Value

$$\mathrm{dVal}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbf{SP}(\mathrm{Play}(v,\sigma,\tau))}_{\mathrm{dVal}^\sigma(v)}$$



Optimal strategy for Min

An optimal strategy for Min may require finite memory.

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

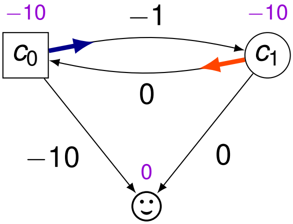
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v,\sigma,\tau))}_{\mathsf{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

An optimal strategy for Min may require finite memory.

Optimal strategy

$$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

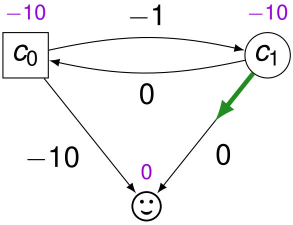
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

An optimal strategy for Min may require finite memory.

Optimal strategy

$$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

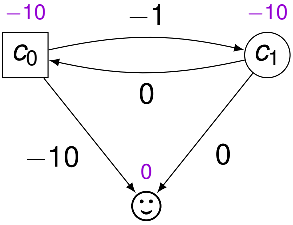
Deterministic strategies: Min needs memory

$\bigcirc\!\!\!\!\sigma$ Min

$\boxed{\tau}$ Max

Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v,\sigma,\tau))}_{\mathrm{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

Switching strategy:

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

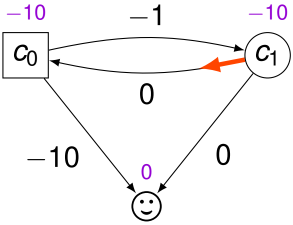
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle

Optimal strategy

$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

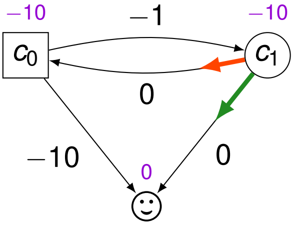
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathrm{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathrm{Play}(v, \sigma, \tau))}_{\mathrm{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle
- σ_2: reach ☺

Optimal strategy

$\mathrm{dVal}^{\sigma^*}(v) \leqslant \mathrm{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

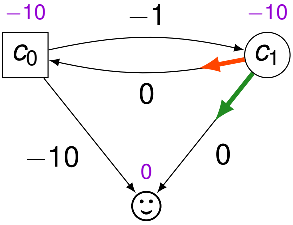
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$



Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

Optimal strategy

$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

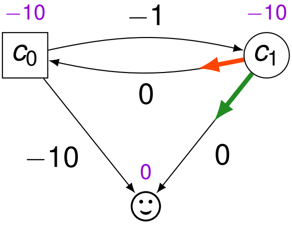
Deterministic strategies: Min needs memory

σ Min

τ Max

Value

$$\mathsf{dVal}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbf{SP}(\mathsf{Play}(v,\sigma,\tau))}_{\mathsf{dVal}^\sigma(v)}$$



Optimal strategy for Min

Switching strategy:

- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

Optimal strategy

$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

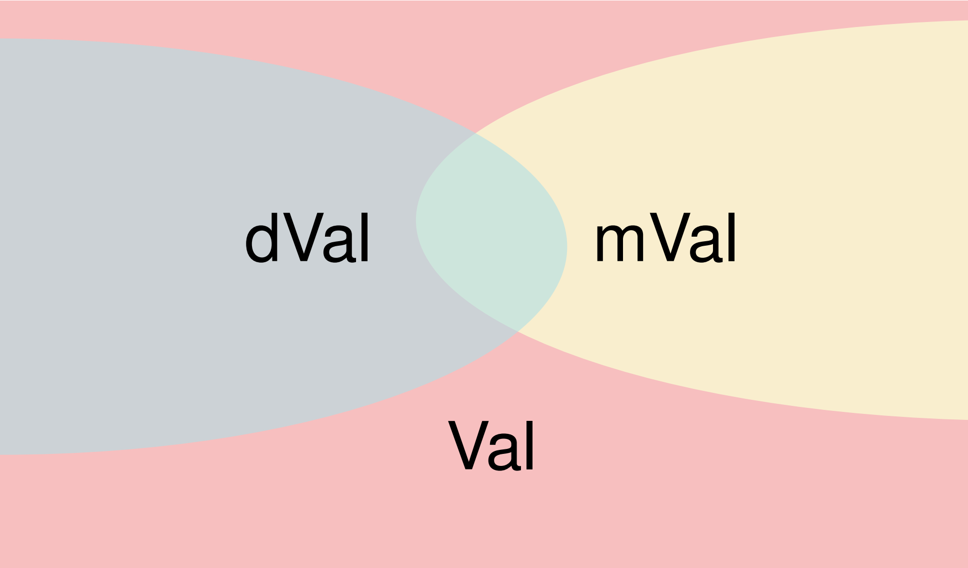
ε-optimal strategy

$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v) + \varepsilon$

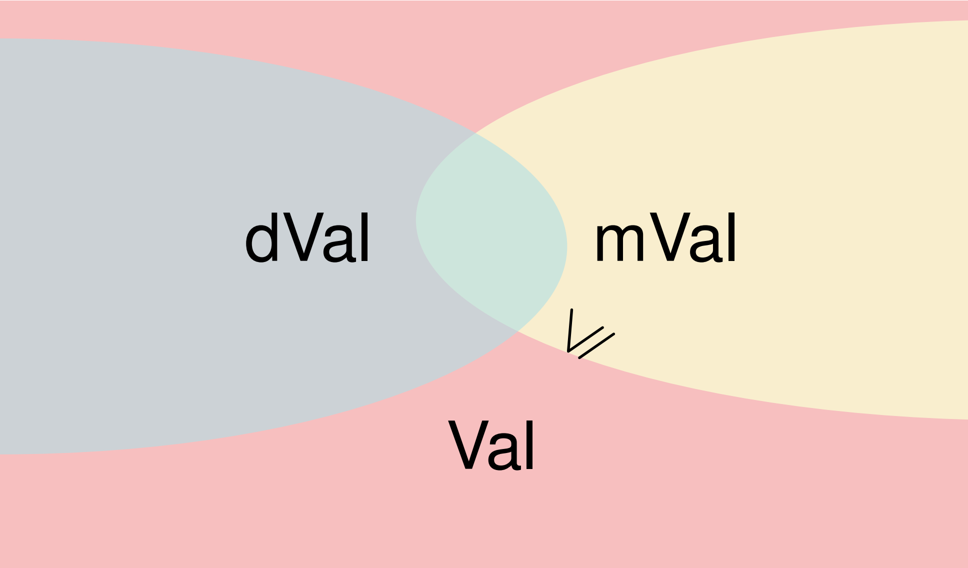
Contribution

$$\text{dVal} = \text{Val} = \text{mVal}$$

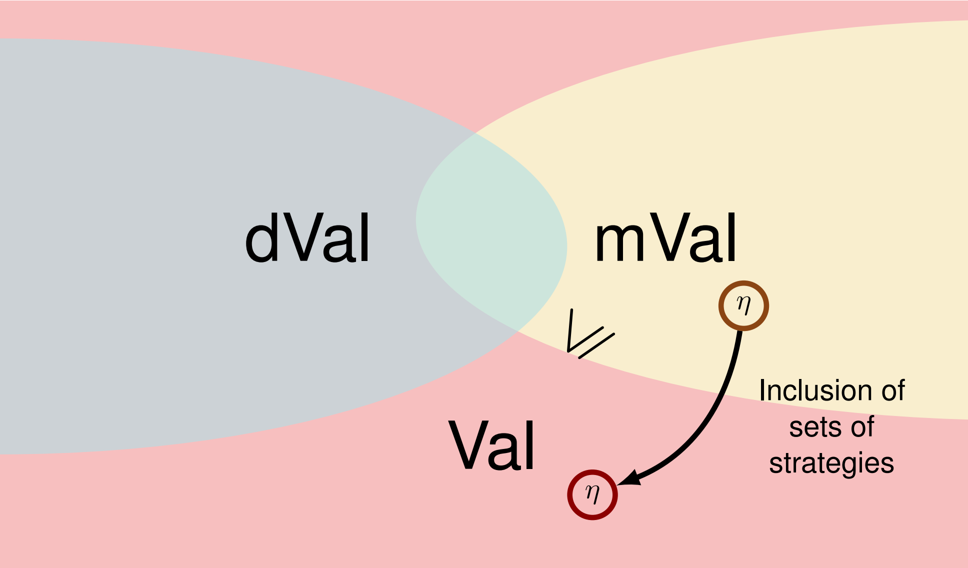
Contribution



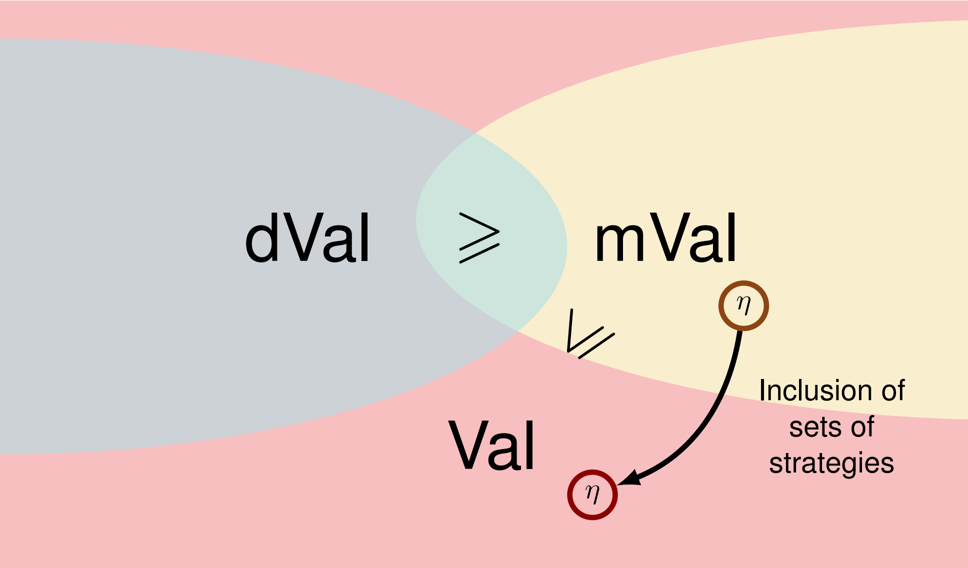
Contribution



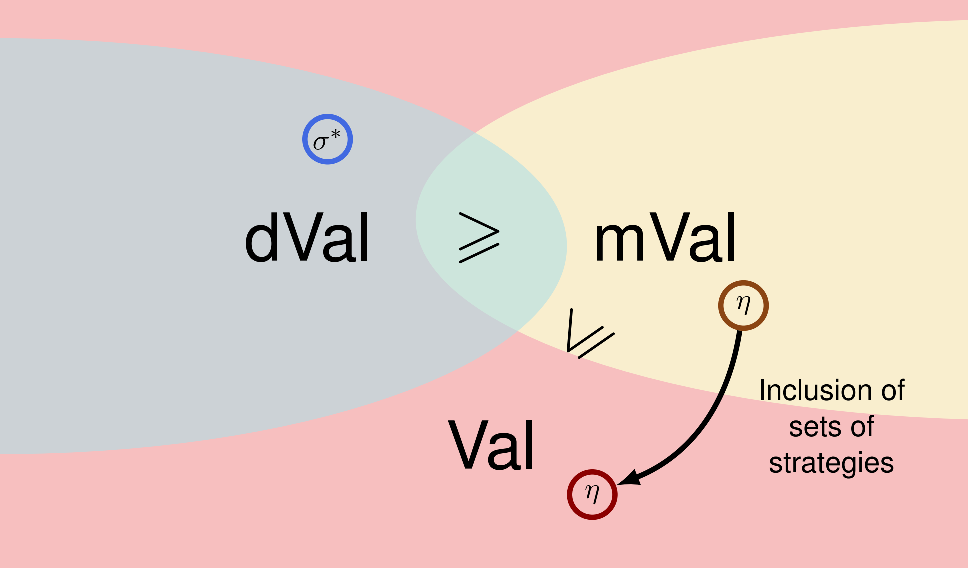
Contribution



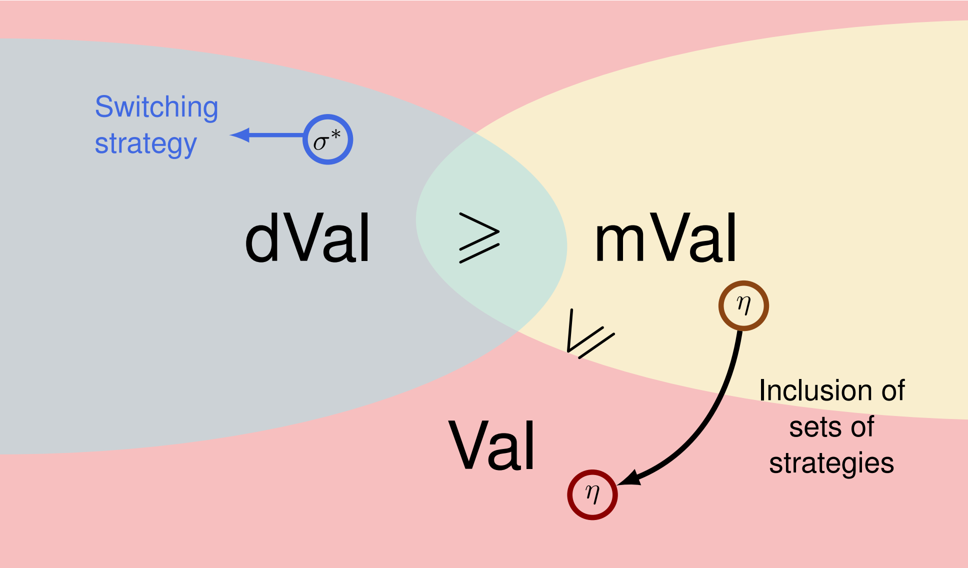
Contribution



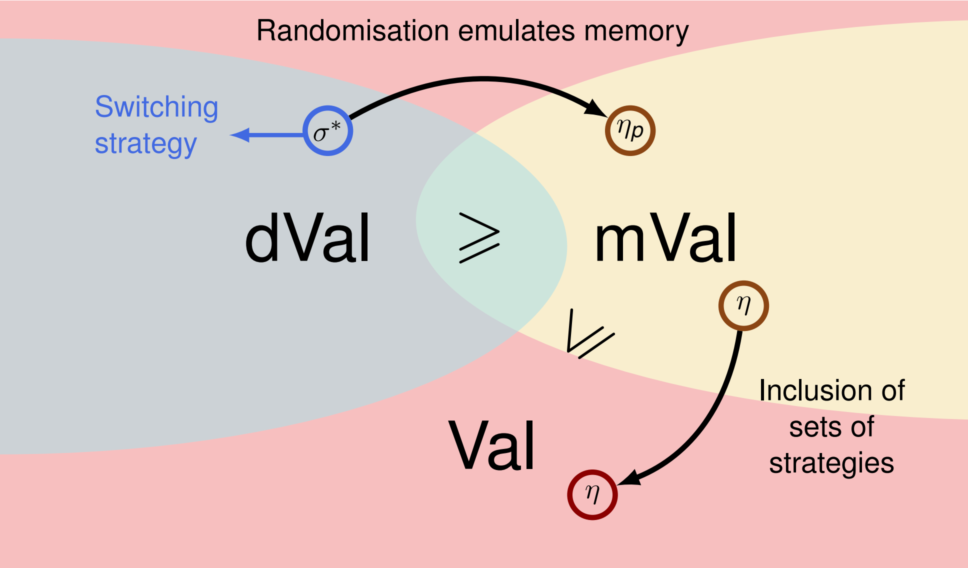
Contribution



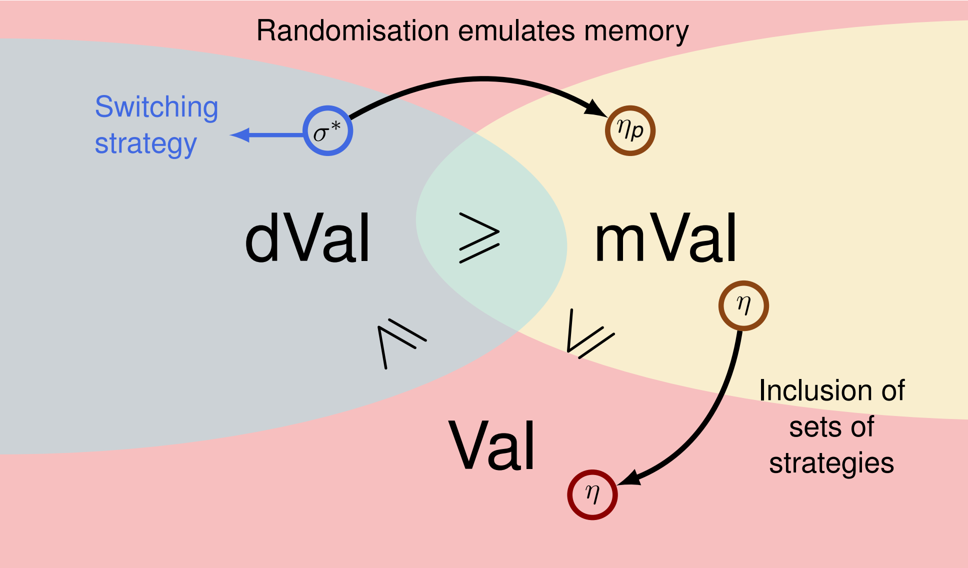
Contribution



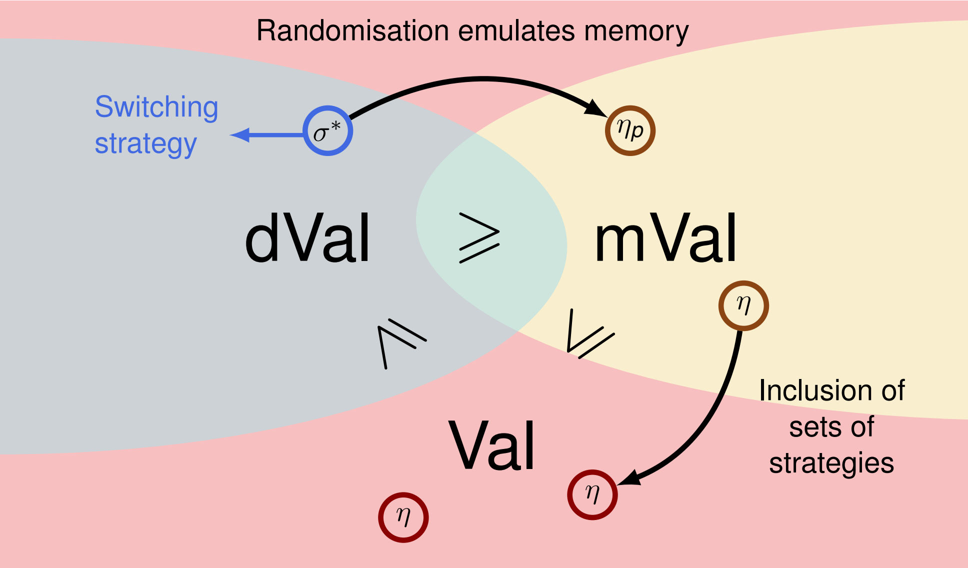
Contribution



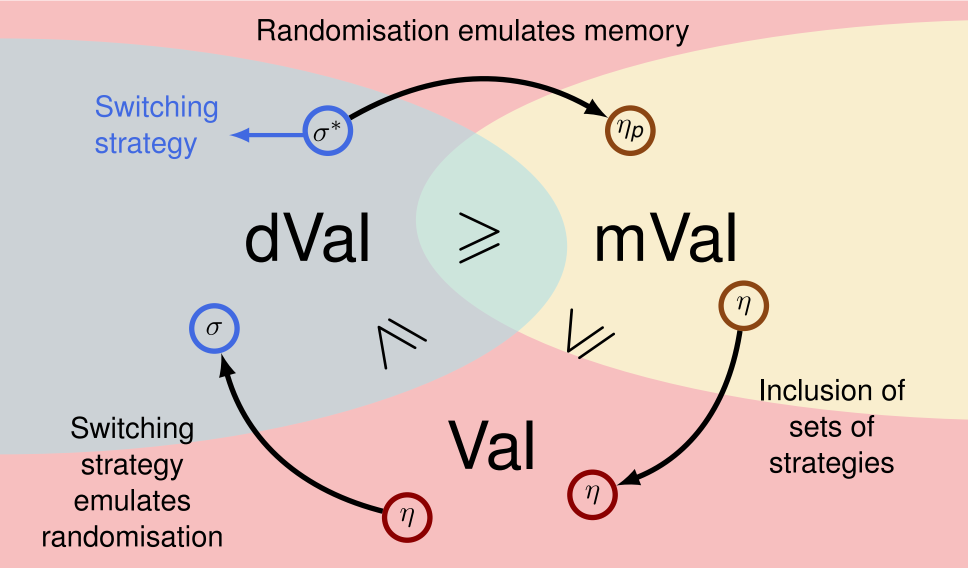
Contribution



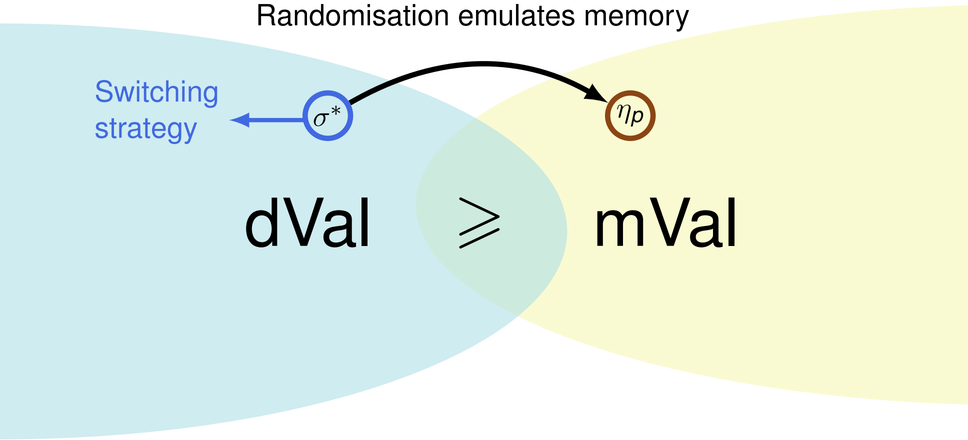
Contribution



Contribution



Contribution

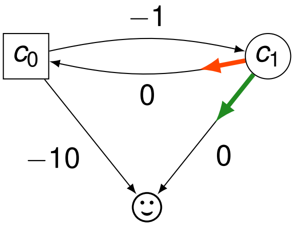


Randomisation emulates memory

Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p\to 1\\ p<1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Randomisation emulates memory



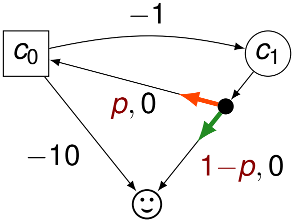
Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1 \\ p < 1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy,

Randomisation emulates memory



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p\to 1\\ p<1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

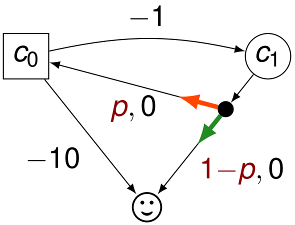
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K\rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory

Min Max



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1 \\ p < 1}} \mathrm{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathrm{dVal}(\ell,\nu)$$

Properties of η_p

- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\diamond☺) = 1$

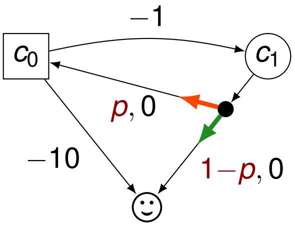
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory

○ Min □ Max



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1 \\ p < 1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Properties of η_p

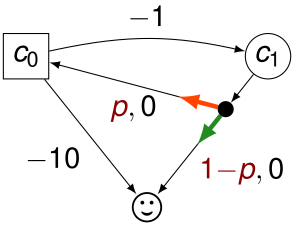
- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\diamond☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p\to 1\\ p<1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Properties of η_p

- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\diamond ☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

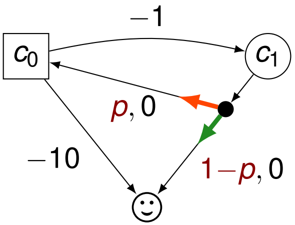
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K\rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory

○ Min □ Max



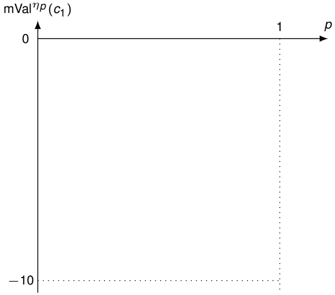
Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1 \\ p < 1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Properties of η_p

- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\diamond ☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

Computation of $\mathsf{mVal}^{\eta_p}(c_1)$



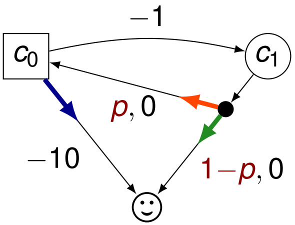
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory

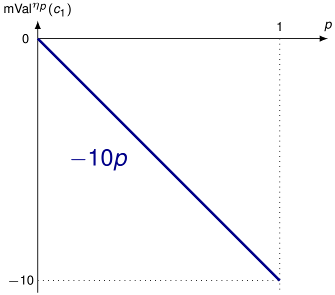
Min Max



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1 \\ p < 1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Computation of $\mathsf{mVal}^{\eta_p}(c_1)$



Properties of η_p

- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\diamond ☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

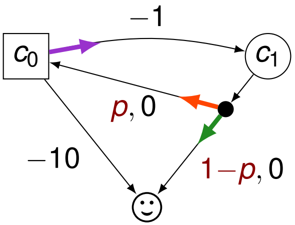
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory

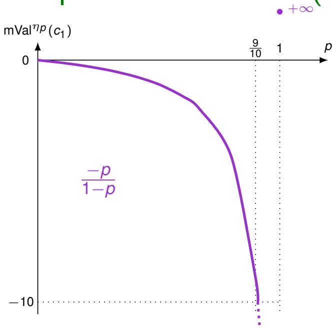
○ Min □ Max



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1 \\ p<1}} \mathrm{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathrm{dVal}(\ell,\nu)$$

Computation of $\mathrm{mVal}^{\eta_p}(c_1)$



Properties of η_p

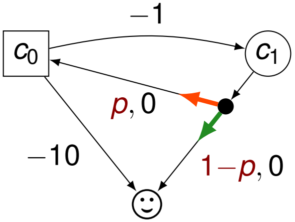
- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\diamond☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

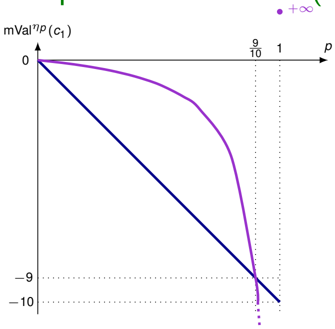
Randomisation emulates memory



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p \to 1\\ p<1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Computation of $\mathsf{mVal}^{\eta_p}(c_1)$



Properties of η_p

- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\Diamond ☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

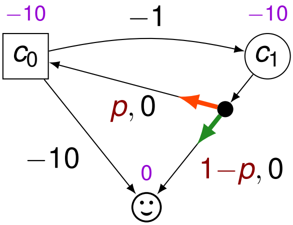
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Randomisation emulates memory

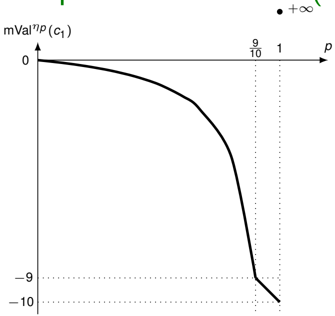
Min Max



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p\to 1\\ p<1}} \mathsf{mVal}^{\eta_p}(\ell,\nu) \leqslant \mathsf{dVal}(\ell,\nu)$$

Computation of $\mathsf{mVal}^{\eta_p}(c_1)$



Properties of η_p

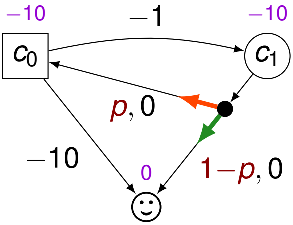
- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\Diamond☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K\rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

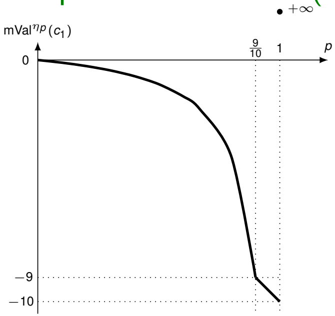
Randomisation emulates memory



Claim

$$\forall(\ell,\nu),\ \lim_{\substack{p\to 1\\ p<1}} \mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) = \mathrm{dVal}(\ell,\nu)$$

Computation of $\mathrm{mVal}^{\eta_p}(c_1)$



Properties of η_p

- For all τ, $\mathbb{P}^{\eta_p,\tau}_{\ell,\nu}(\Diamond ☺) = 1$
- For all τ, $\mathbb{E}^{\eta_p,\tau}_{\ell,\nu}(\mathbf{SP}) < \infty$
- Max has an ε-best response deterministic strategy: τ

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2, K\rangle$ be an ε-optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Computation of the expectation

Claim

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

Computation of the expectation

Claim

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

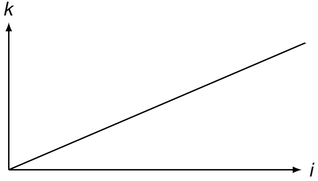
$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho)$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p \to 1 \\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \quad + \quad +$$

Computation of the expectation

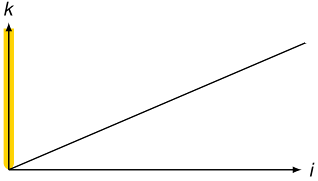
i number of choices given by σ_2
k size of play reaching the target

Claim

$\lim_{\substack{p \to 1 \\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$

Yellow zone

All plays conforming to σ_1



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \mathbb{E} + \quad +$$

Computation of the expectation

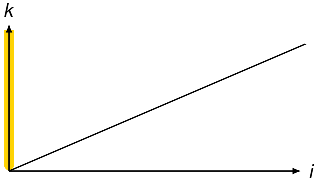
i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p \to 1 \\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \quad +$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

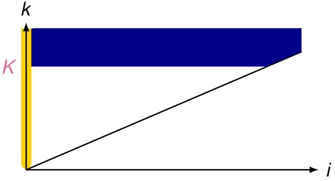
$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$

Blue zone

Plays with many negative cycles



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} +$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

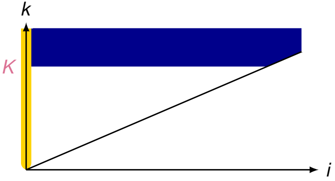
$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} +$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p \to 1 \\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

Yellow zone

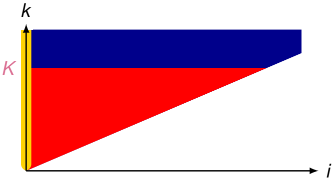
All plays conforming to σ_1

$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$

Blue zone

Plays with many negative cycles

$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p \to 1 \\ p<1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

Yellow zone

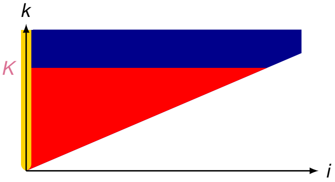
All plays conforming to σ_1

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$

Blue zone

Plays with many negative cycles

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho)$$

Red zone

Rest of plays

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

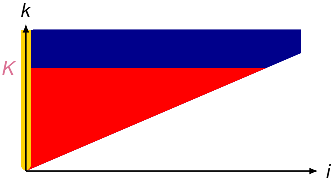
$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) \leqslant \sum_{i=1}^{K} iW2^{K}(1-p)^{i}$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

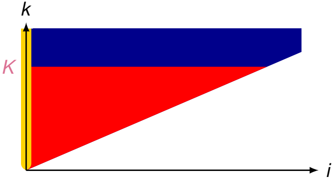
$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$$

Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$

Blue zone

Plays with many negative cycles
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) \leqslant \sum_{i=1}^{K} i W\, 2^{K} (1-p)^{i} \xrightarrow[\substack{p \to 1 \\ p < 1}]{} 0$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

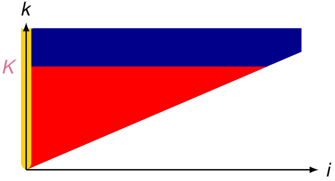
Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

$$\mathbb{E} + \mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) + \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho)$$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

$$\mathbb{E} \xrightarrow[\substack{p\to 1\\ p<1}]{} 0$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

Yellow zone

All plays conforming to σ_1

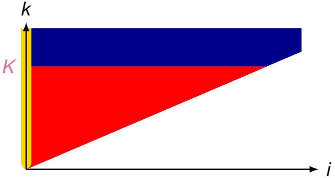
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

$$\mathbb{E} + \mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) + \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle} \times (\mathbb{P} + \mathbb{P})$$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

$$\mathbb{E} \underset{\substack{p\to 1\\ p<1}}{\longrightarrow} 0$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

Yellow zone

All plays conforming to σ_1

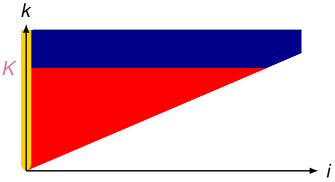
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles

$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

$$\mathbb{E} + \mathbb{E} \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle} \times (\mathbb{P} + \mathbb{P}) = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle} \times (1 - \mathbb{P})$$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

$$\mathbb{E} \underset{\substack{p\to 1\\ p<1}}{\longrightarrow} 0$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

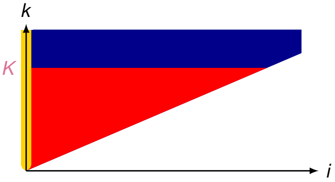
Yellow zone

All plays conforming to σ_1
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles
$\mathbf{SP}(\rho) \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

$$\mathbb{E} + \mathbb{E} \leqslant \mathsf{dVal}^{\langle \sigma_1, \sigma_2, K\rangle} \times (1 - \mathbb{P}) \xrightarrow[\substack{p\to 1\\ p<1}]{} \mathsf{dVal}^{\langle \sigma_1, \sigma_2 K\rangle}$$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

Red zone

Rest of plays

$$\mathbb{E} \xrightarrow[\substack{p\to 1\\ p<1}]{} 0$$

Computation of the expectation

i number of choices given by σ_2
k size of play reaching the target

Claim

$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$$

Yellow zone

All plays conforming to σ_1

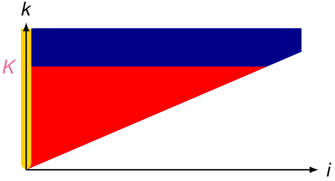
$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

Blue zone

Plays with many negative cycles

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K\rangle}$

$$\lim_{\substack{p\to 1\\ p<1}} \mathbb{E} + \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2 K\rangle}$$



$$\mathbb{E} = \sum_{\rho} \mathbf{SP}(\rho)\, \mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E}$$

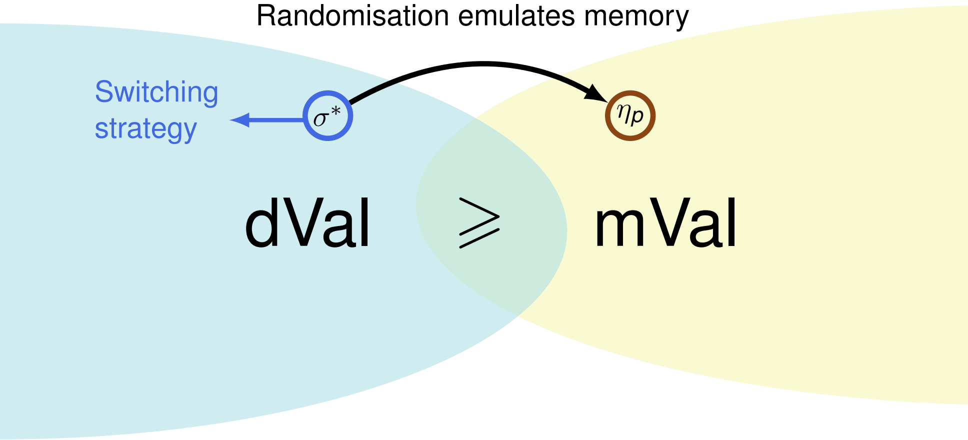
$$\Rightarrow \lim_{\substack{p\to 1\\ p<1}} \mathbb{E} = \mathrm{dVal}^{\langle \sigma_1, \sigma_2 K\rangle}$$

Red zone

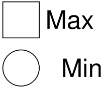
Rest of plays

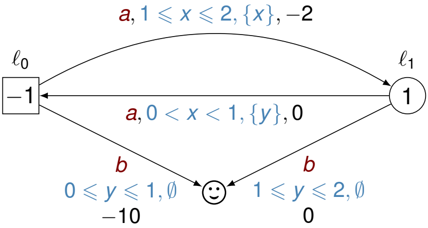
$$\mathbb{E} \xrightarrow[\substack{p\to 1\\ p<1}]{} 0$$

Contribution



Existence of an ε-optimal switching strategy

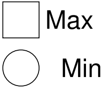


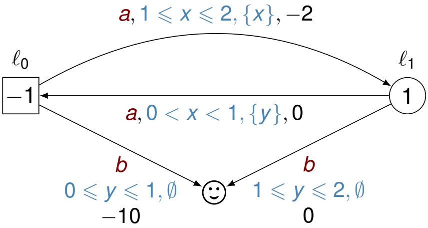


Switching strategy

- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

Existence of an ε-optimal switching strategy





Switching strategy

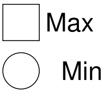
- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

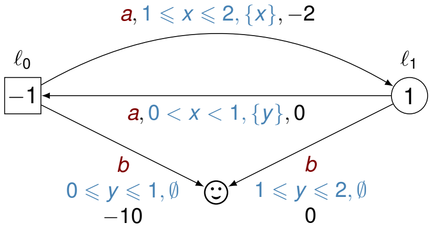
Divergent weighted timed game

All SCCs contain only negative cycles or positive cycles

Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Existence of an ε-optimal switching strategy





Switching strategy

- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

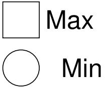
Divergent weighted timed game

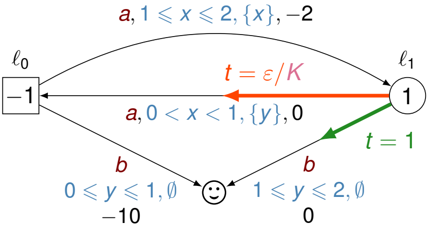
All SCCs contain only negative cycles or positive cycles

Theorem

Min has an ε-optimal switching strategy

Existence of an ε-optimal switching strategy





Switching strategy

- σ_1: reach negative cycle
- σ_2: reach ☺
- K: number of turns before switch

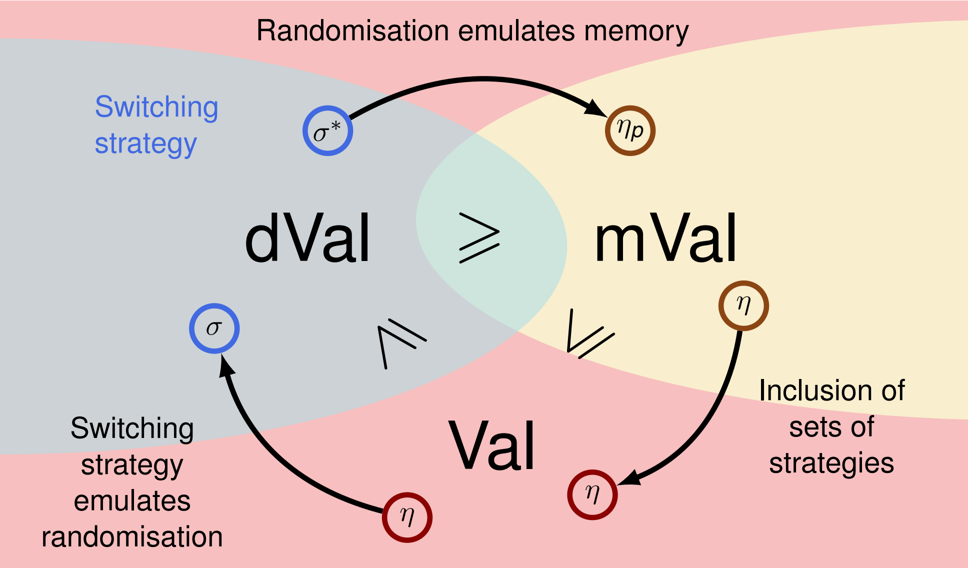
Divergent weighted timed game

All SCCs contain only negative cycles or positive cycles

Theorem

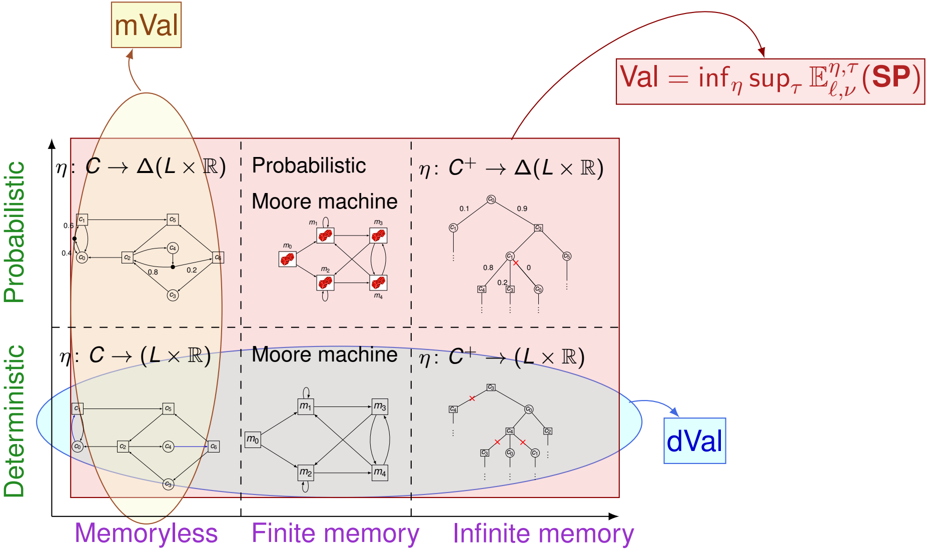
Min has an ε-optimal switching strategy : $\langle \sigma_1, \sigma_2, K \rangle$

Contribution



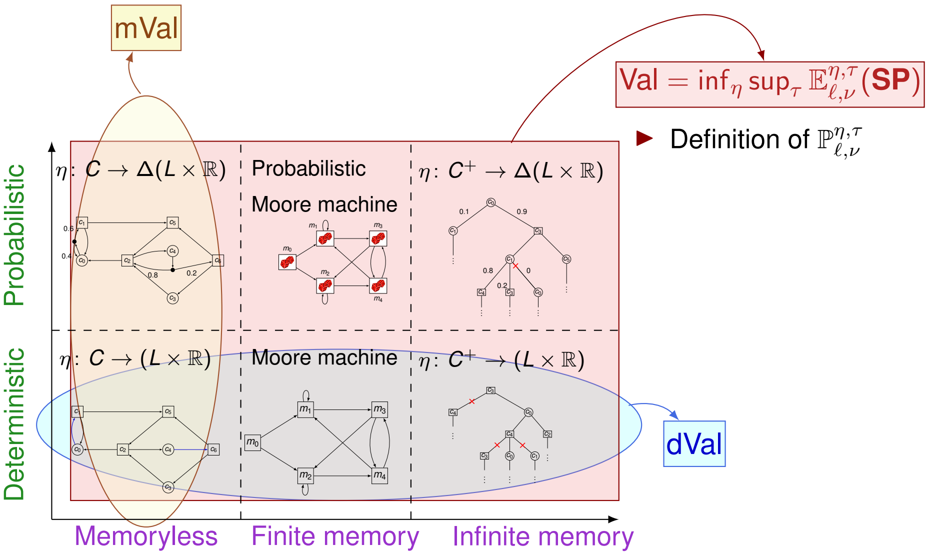
Existence of the stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



Existence of the stochastic value

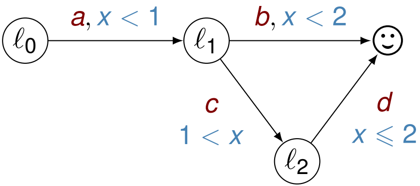
$C = L \times \mathbb{R}^{|\mathcal{C}|}$



- Definition of $\mathbb{P}^{\eta,\tau}_{\ell,\nu}$

Probability of a path

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



Probabilistic strategy

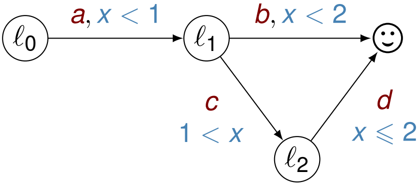
Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Probability of a path

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



Probabilistic strategy

Distribution over possible choices

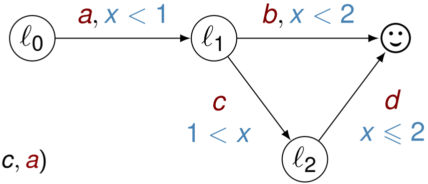
1. Edge a: finite distribution
2. Delay for a: infinite distribution

Path: $\pi = (c, e_1 \dots e_n) = \{t_1, \dots, t_n \mid c \xrightarrow{t_1, e_1} \dots \xrightarrow{t_n, e_n}\}$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Probability of a path

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution : $\eta_E(c)$
2. Delay for a: infinite distribution : $\eta_{\mathbb{R}^+}(c, a)$

Path: $\pi = (c, e_1 \dots e_n) = \{t_1, \dots, t_n \mid c \xrightarrow{t_1, e_1} \dots \xrightarrow{t_n, e_n}\}$

$$\mathbb{P}_c(e\,\pi) = \int_{t \in I(c,e)} \eta_E(c)(e)\, \mathbb{P}_{c_1}(\pi)\, \mathrm{d}\eta_{\mathbb{R}^+}(c, e)(t)$$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

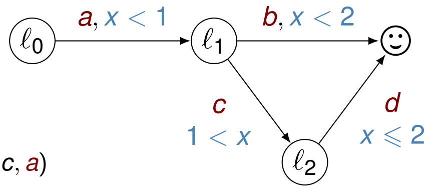
Probability of a path

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$

Probabilistic strategy

Distribution over possible choices

1. Edge *a*: finite distribution : $\eta_E(c)$
2. Delay for *a*: infinite distribution : $\eta_{\mathbb{R}^+}(c, a)$



Path: $\pi = (c, e_1 \dots e_n) = \{t_1, \dots, t_n \mid c \xrightarrow{t_1, e_1} \dots \xrightarrow{t_n, e_n}\}$

$$\mathbb{P}_c(e\,\pi) = \int_{t \in I(c,e)} \eta_E(c)(e)\, \mathbb{P}_{c_1}(\pi)\, \mathrm{d}\eta_{\mathbb{R}^+}(c, e)(t)$$

$$\int_{t_2 \in I(c_1, e_2)} \eta_E(c_1)(e_2)\, \mathbb{P}_{c_2}(\pi_2)\, \mathrm{d}\eta_{\mathbb{R}^+}(c_1, e_2)(t_2)$$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

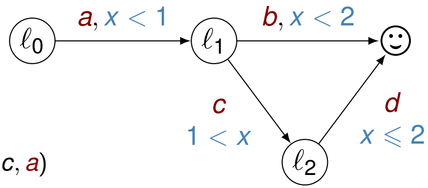
Probability of a path

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$

Probabilistic strategy

Distribution over possible choices

1. Edge a: finite distribution : $\eta_E(c)$
2. Delay for a: infinite distribution : $\eta_{\mathbb{R}^+}(c, a)$



Path: $\pi = (c, e_1 \dots e_n) = \{t_1, \dots, t_n \mid c \xrightarrow{t_1, e_1} \dots \xrightarrow{t_n, e_n}\}$

$$\mathbb{P}_c(e\,\pi) = \int_{t \in I(c,e)} \eta_E(c)(e)\,\mathbb{P}_{c_1}(\pi)\,\mathrm{d}\eta_{\mathbb{R}^+}(c, e)(t)$$

$$\int_{t_2 \in I(c_1, e_2)} \eta_E(c_1)(e_2)\,\mathbb{P}_{c_2}(\pi_2)\,\mathrm{d}\eta_{\mathbb{R}^+}(c_1, e_2)(t_2)$$

$$\dots$$

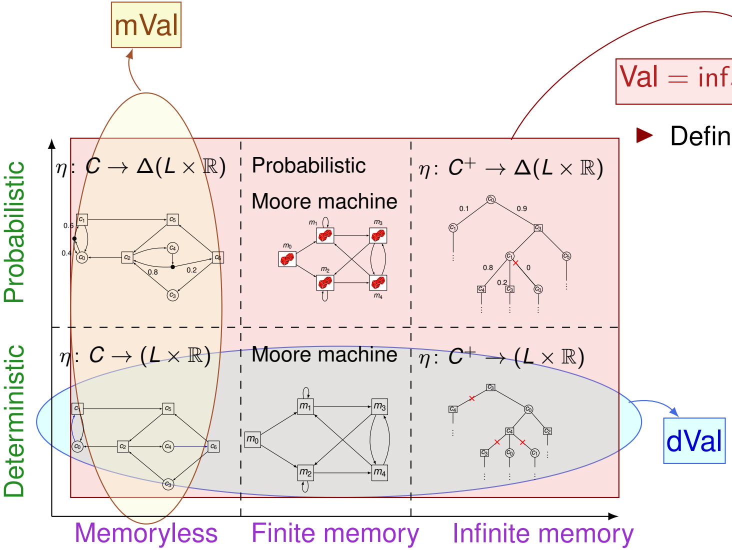
Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Existence of the stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

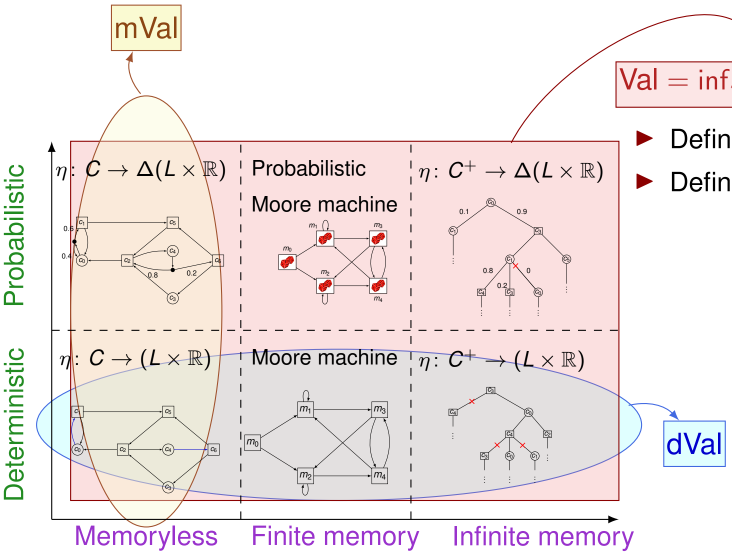
$\text{Val} = \inf_\eta \sup_\tau \mathbb{E}^{\eta,\tau}_{\ell,\nu}(\textbf{SP})$

- Definition of $\mathbb{P}^{\eta,\tau}_{\ell,\nu}$



Existence of the stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

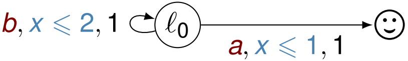


$\mathsf{Val} = \inf_\eta \sup_\tau \mathbb{E}^{\eta,\tau}_{\ell,\nu}(\mathbf{SP})$

- Definition of $\mathbb{P}^{\eta,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\eta,\tau}_{\ell,\nu}(\mathbf{SP})$

Expectation of SP

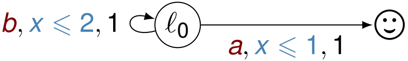
$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



$$\mathbb{E}_c(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c(\pi)$$

Expectation of SP

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$

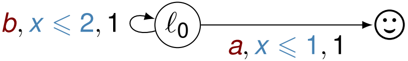


$$\mathbb{E}_c(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c(\pi)$$

$$\int_{t \in I(c,e)} \eta_E(c)(e) \left[(t\,\mathrm{wt}(c) + \mathrm{wt}(e))\, \mathbb{P}_{c_1}(\pi) + \mathbb{E}_{c_1}(\pi) \right] \, \mathrm{d}\eta_{\mathbb{R}^+}(c,e)(t)$$

Expectation of SP

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



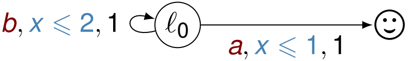
$$\mathbb{E}_c(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c(\pi)$$

$$\int_{t\in I(c,e)} \eta_E(c)(e)\left[(t\,\mathrm{wt}(c) + \mathrm{wt}(e))\,\mathbb{P}_{c_1}(\pi) + \mathbb{E}_{c_1}(\pi)\right]\,\mathrm{d}\eta_{\mathbb{R}^+}(c,e)(t)$$

Convergence ?

Expectation of SP

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



$$\mathbb{E}_c(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c(\pi)$$

$$\int_{t \in I(c,e)} \eta_E(c)(e) \left[(t\,\mathrm{wt}(c) + \mathrm{wt}(e))\, \mathbb{P}_{c_1}(\pi) + \mathbb{E}_{c_1}(\pi) \right] \, \mathrm{d}\eta_{\mathbb{R}^+}(c,e)(t)$$

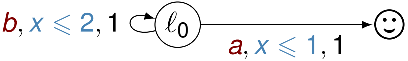
Convergence ?

Restriction on strategies for Min

- $\mathbb{P}_c(\diamond ☺) = 1$

Expectation of SP

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



$$\mathbb{E}_c(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c(\pi)$$

$$\int_{t \in I(c,e)} \eta_E(c)(e)\left[(t\,\mathrm{wt}(c) + \mathrm{wt}(e))\ \mathbb{P}_{c_1}(\pi) + \mathbb{E}_{c_1}(\pi)\right]\ \mathrm{d}\eta_{\mathbb{R}^+}(c,e)(t)$$

Convergence ?

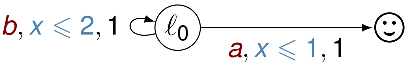
- $\mathbf{SP}(\pi) \sim k\,|\pi|$
- $\mathbb{P}_c(\pi) \sim \alpha^{-|\pi|}$

Restriction on strategies for Min

- $\mathbb{P}_c(\diamond ☺) = 1$

Expectation of SP

$c \in C = L \times \mathbb{R}^{|\mathcal{C}|}$



$$\mathbb{E}_c(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c(\pi)$$

$$\int_{t \in I(c,e)} \eta_E(c)(e)\left[(t\,\mathrm{wt}(c) + \mathrm{wt}(e))\,\mathbb{P}_{c_1}(\pi) + \mathbb{E}_{c_1}(\pi)\right]\,\mathrm{d}\eta_{\mathbb{R}^+}(c,e)(t)$$

Convergence ?

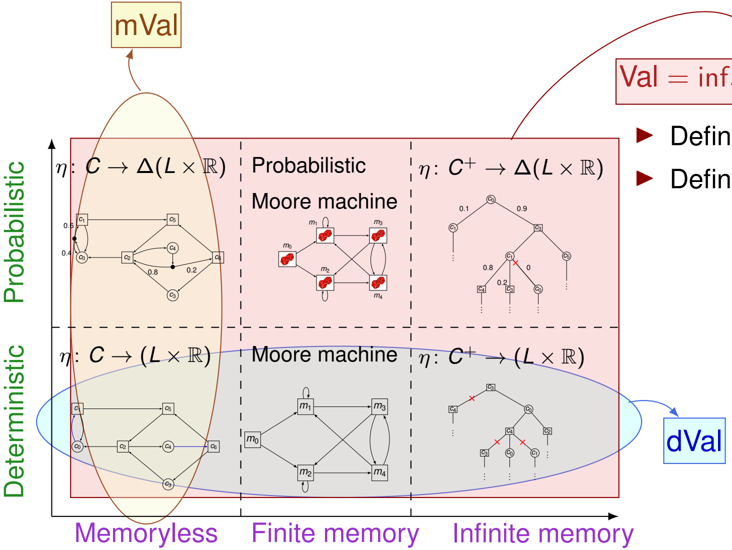
- $\mathbf{SP}(\pi) \sim k\,|\pi|$
- $\mathbb{P}_c(\pi) \sim \alpha^{-|\pi|}$

Restriction on strategies for Min

- $\mathbb{P}_c(\diamond ☺) = 1$
- ☺ must be reached quickly enough

Existence of the stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

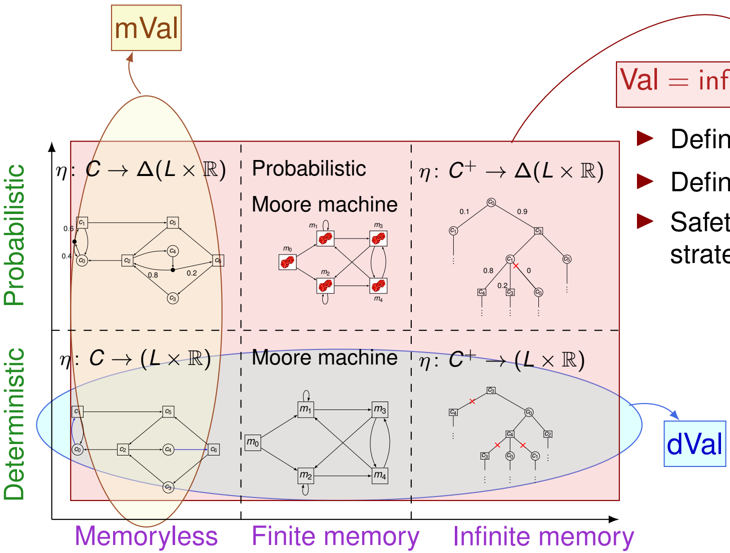


$\text{Val} = \inf_\eta \sup_\tau \mathbb{E}^{\eta,\tau}_{\ell,\nu}(\textbf{SP})$

- Definition of $\mathbb{P}^{\eta,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\eta,\tau}_{\ell,\nu}(\textbf{SP})$

Existence of the stochastic value

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

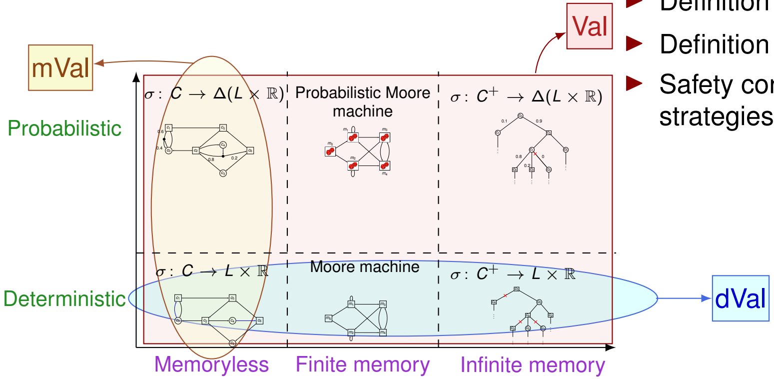


$\mathsf{Val} = \inf_\eta \sup_\tau \mathbb{E}^{\eta,\tau}_{\ell,\nu}(\mathbf{SP})$

- Definition of $\mathbb{P}^{\eta,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\eta,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies

To conclude

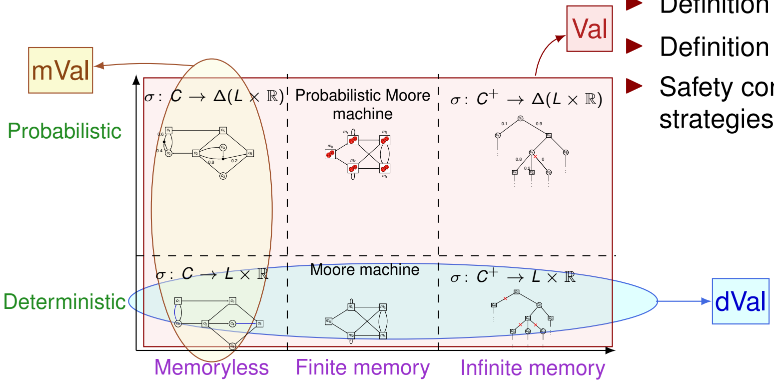
$C = L \times \mathbb{R}^{|\mathcal{C}|}$



- Definition of $\mathbb{P}_{\ell,\nu}^{\sigma,\tau}$
- Definition of $\mathbb{E}_{\ell,\nu}^{\sigma,\tau}(\mathbf{SP})$
- Safety conditions on strategies

To conclude

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



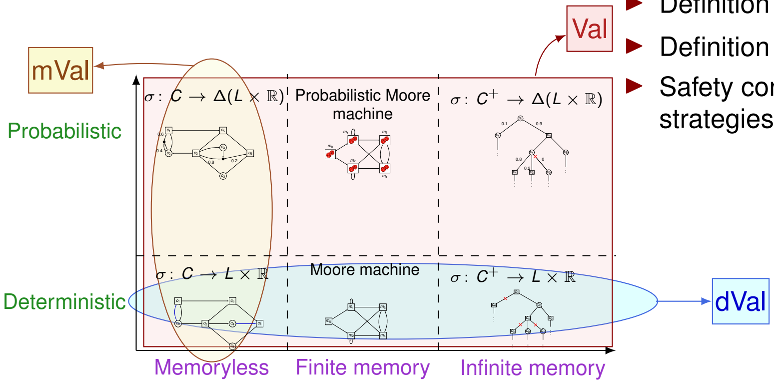
- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies

Theorem

In divergent weighted timed games, dVal $=$ Val $=$ mVal

To conclude: perspectives

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



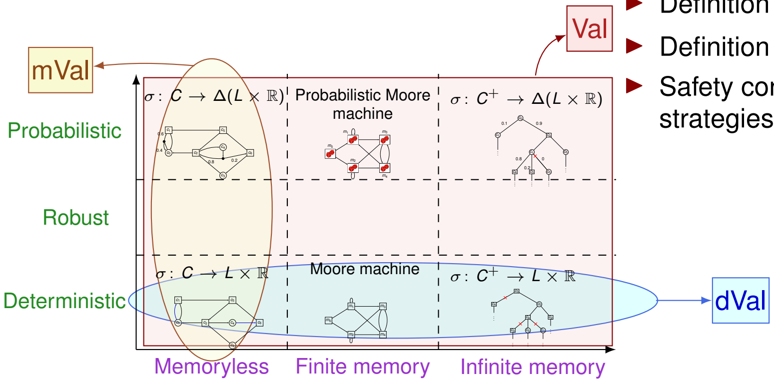
- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies

Theorem

In divergent weighted timed games, dVal $=$ Val $=$ mVal

To conclude: perspectives

$C = L \times \mathbb{R}^{|\mathcal{C}|}$



- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\textbf{SP})$
- Safety conditions on strategies

Theorem

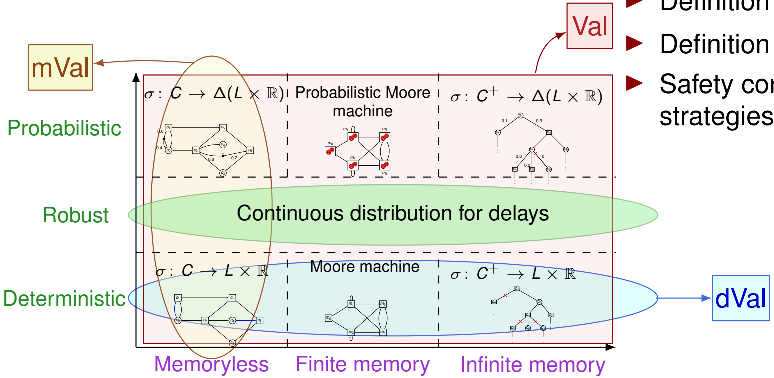
In divergent weighted timed games, dVal $=$ Val $=$ mVal

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

To conclude: perspectives

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}$(**SP**)
- Safety conditions on strategies



Theorem

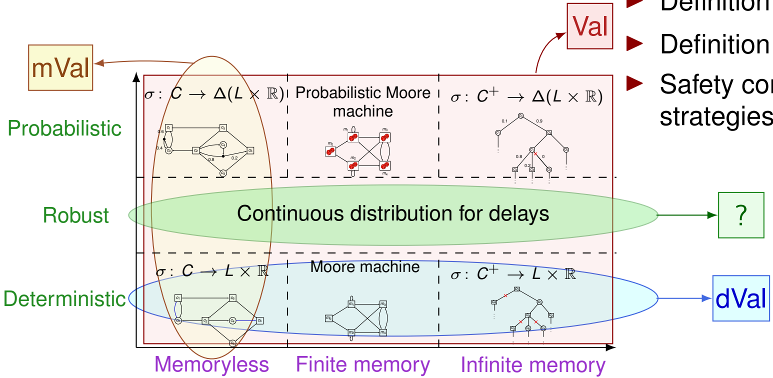
In divergent weighted timed games, dVal $=$ Val $=$ mVal

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

To conclude: perspectives

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}$(**SP**)
- Safety conditions on strategies



Theorem

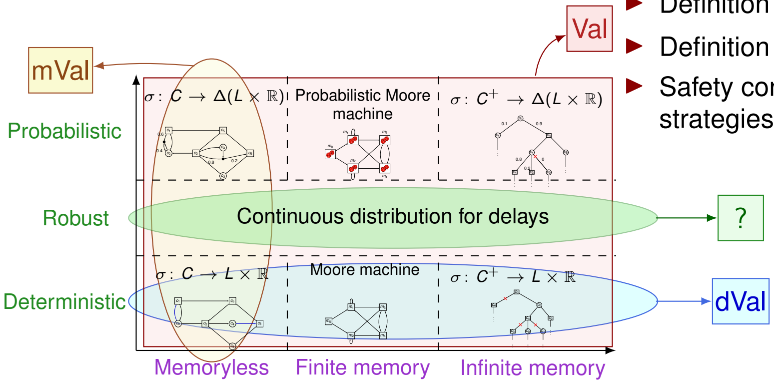
In divergent weighted timed games, dVal $=$ Val $=$ mVal

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

To conclude: perspectives

$C = L \times \mathbb{R}^{|\mathcal{C}|}$

- Definition of $\mathbb{P}^{\sigma,\tau}_{\ell,\nu}$
- Definition of $\mathbb{E}^{\sigma,\tau}_{\ell,\nu}(\mathbf{SP})$
- Safety conditions on strategies



Theorem

In divergent weighted timed games, dVal $=$ Val $=$ mVal

Thank you! Questions ?