Playing Stochastically in Weighted Timed Games to Emulate Memory

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TU Dresden
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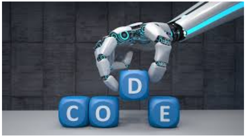
Motivation : game theory for synthesis

Game theory

Interaction between two antagonistic agents : environment and controller



Classical approach

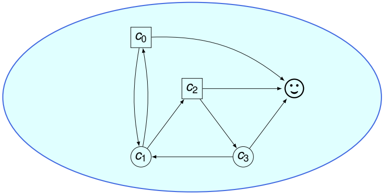
Check the correctness of a system

Code synthesis

Correct by construction : synthesis of controller

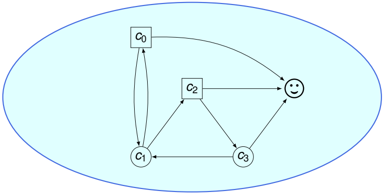
Different classes of games

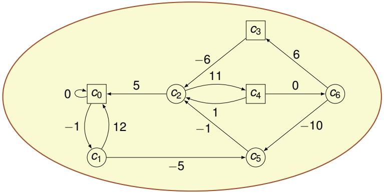
Qualitative games



Different classes of games

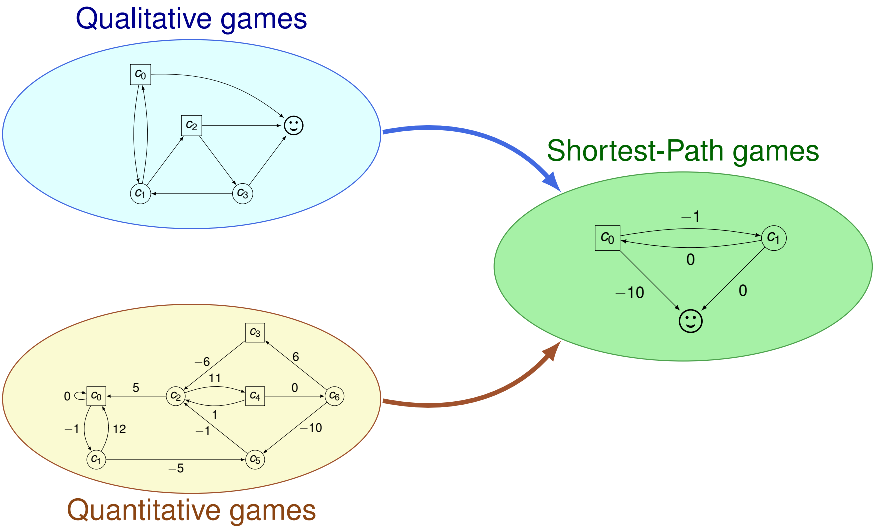
Qualitative games



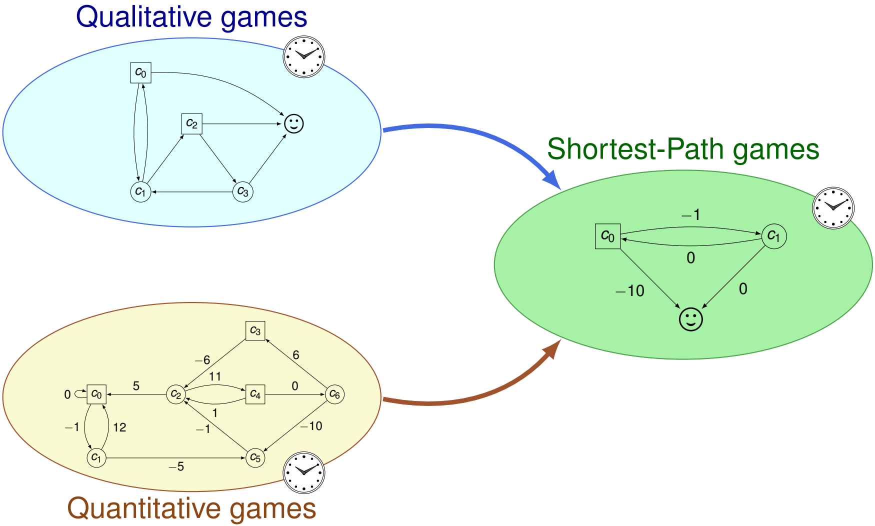


Quantitative games

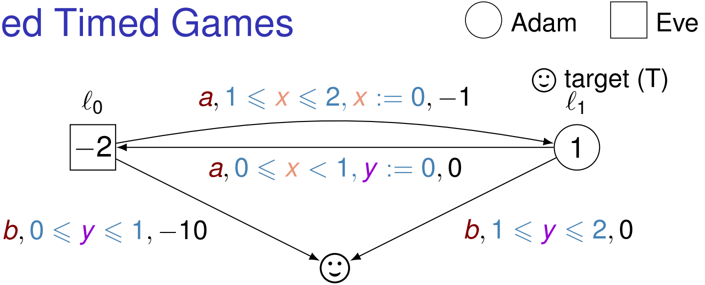
Different classes of games



Different classes of games



Weighted Timed Games



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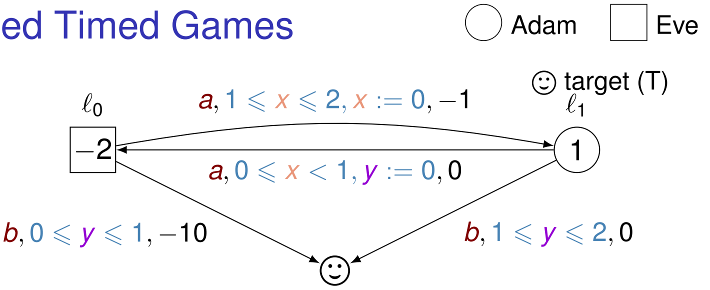
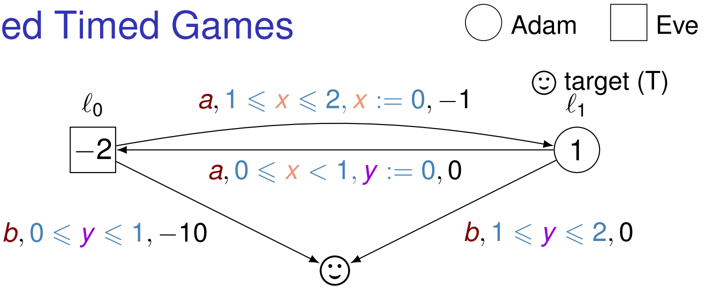


Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix})$$

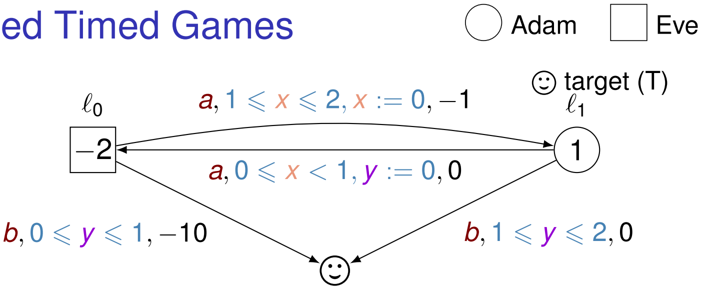
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5,\ a}$$

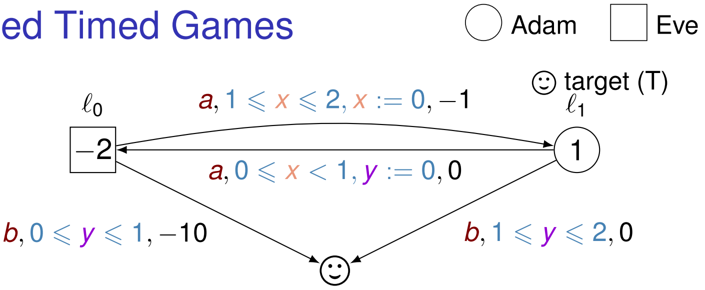
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5,\, a} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix})$$

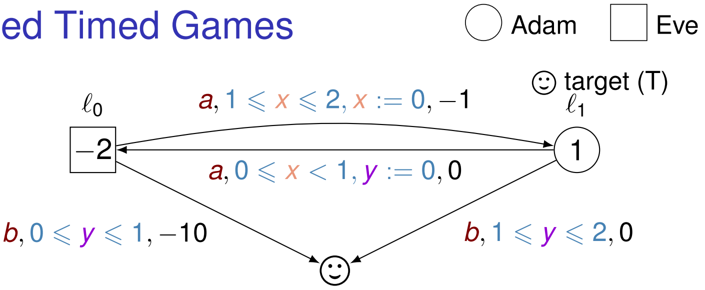
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5,\, a} (\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow{1.25,\, a} (\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow{1/3,\, b} (☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix})$$

Weighted Timed Games



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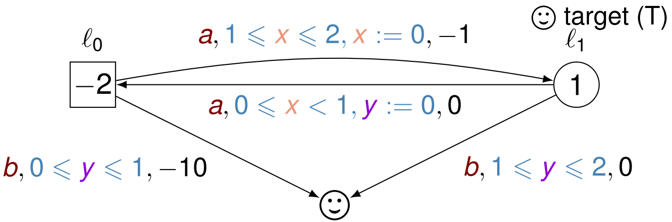
$$1 \times 0.5 + 0$$

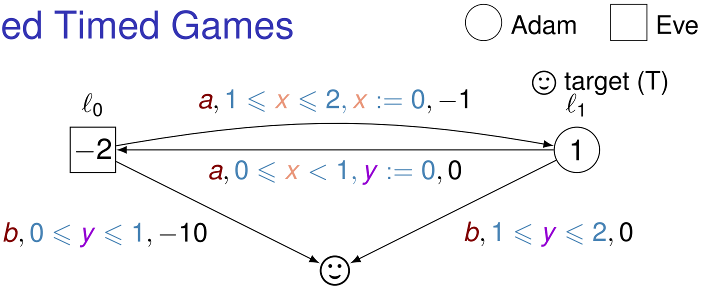
Weighted Timed Games



Play ρ

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow[1 \times 0.5 + 0]{0.5,\ a} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right) \xrightarrow[-2 \times 1.25 - 1]{1.25,\ a} \left(\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}\right) \xrightarrow[1 \times \frac{1}{3} + 0]{1/3,\ b} \left(☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}\right) \rightsquigarrow -\frac{8}{3}$$

Weighted Timed Games



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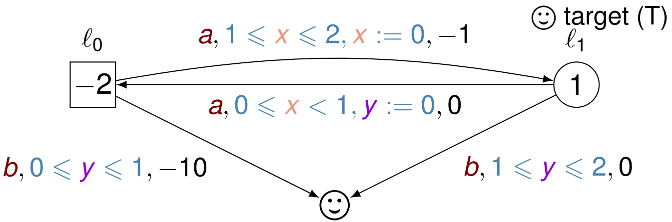
Deterministic strategy

Choose an edge and a delay

Weighted Timed Games

○ Adam □ Eve



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$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{pmatrix}1/3\\19/12\end{pmatrix})$$

Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Deterministic strategies: Min needs memory

σ Min

τ Max

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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Deterministic value

$$\mathrm{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(c, \sigma, \tau))$$

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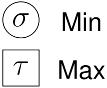

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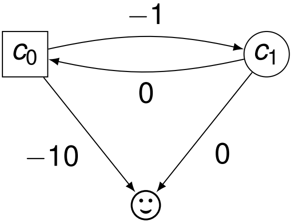

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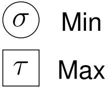

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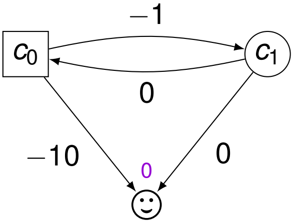

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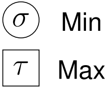

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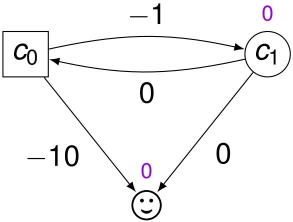

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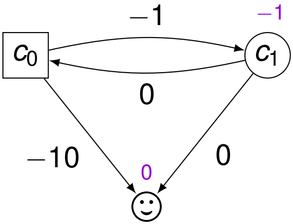

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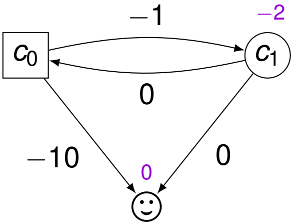

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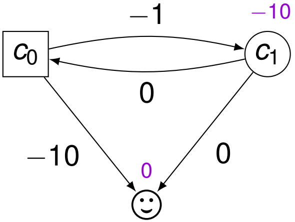

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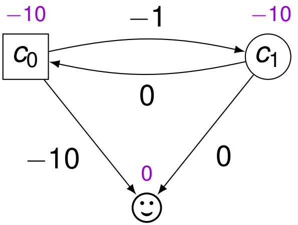

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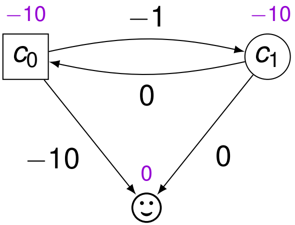
Optimal strategy

$$\mathsf{dVal}^{\sigma^*}(c) \leqslant \mathsf{dVal}(c)$$



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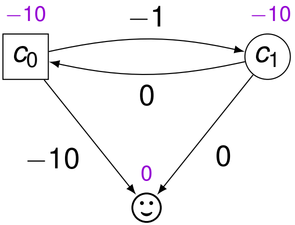

Optimal strategy for Min

An optimal strategy for Min may require finite memory.

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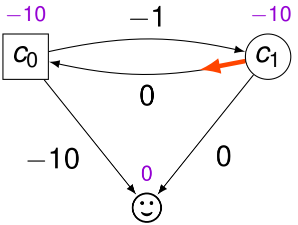

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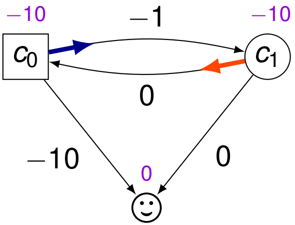

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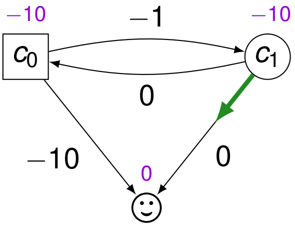

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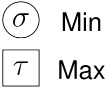

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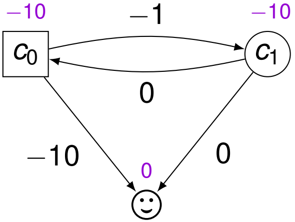

Optimal strategy for Min

Switching strategy:

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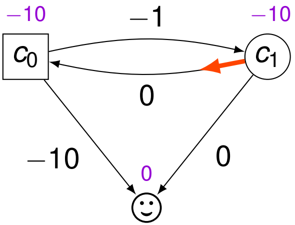

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$

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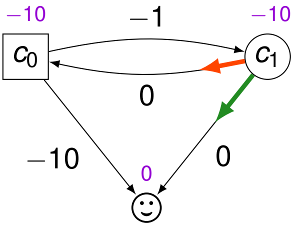

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺

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Optimal strategy

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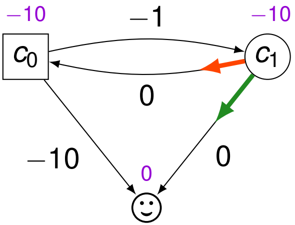

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

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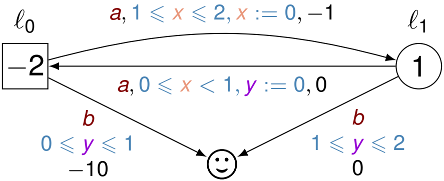
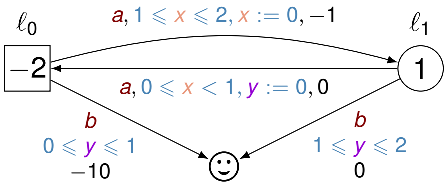
Stochastic strategies

Min Max



Stochastic strategies

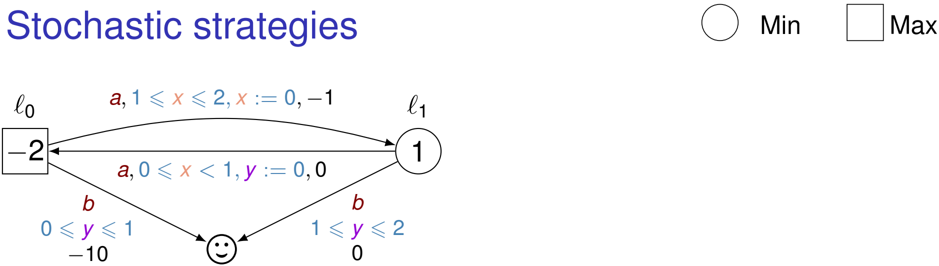


Stochastic strategy

Distribution over possible choices

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies



Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution

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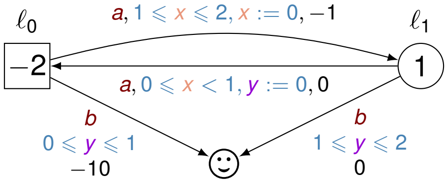
Stochastic strategies



Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

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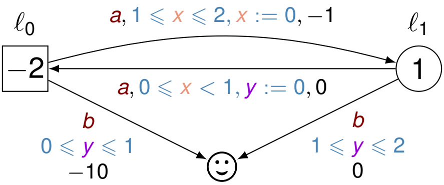
Stochastic strategies



In $(\ell_1, (0, 0))$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

Stochastic strategy

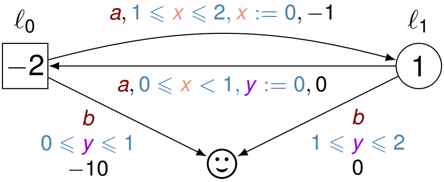
Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies

○ Min □ Max



In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- a: choose t with $\mathcal{U}([0, 1[)$

Stochastic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

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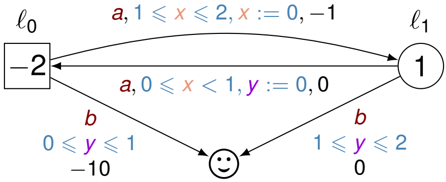
Stochastic strategies



In $(\ell_1, (0, 0))$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

- *a*: choose t with $\mathcal{U}([0, 1[)$
- *b*: choose t with $\delta_{1.5}$

Stochastic strategy

Distribution over possible choices

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2. Delay for *a*: infinite distribution

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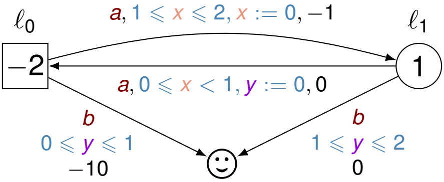
Stochastic strategies

η Min θ Max



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Stochastic strategy

Distribution over possible choices

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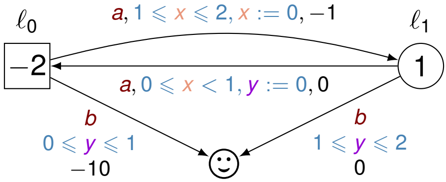
When we fix two strategies

Stochastic strategies

η Min θ Max



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Stochastic strategy

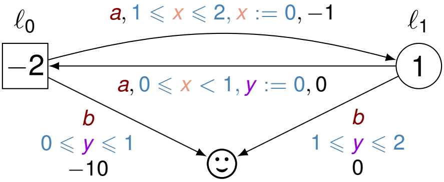
Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

When we fix two strategies

- Infinite Markov Chain

Stochastic strategies



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Stochastic strategy

Distribution over possible choices

1. Edge a: finite distribution
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When we fix two strategies

- Infinite Markov Chain
- Replace **SP**(Play(c, η, θ)) by $\mathbb{E}_c^{\eta,\theta}$(**SP**)

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min θ Max

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min θ Max

$\eta, \theta : C^*C \to \Delta(C)$

Distribution over possible choices

1. Edge a: finite distribution $\eta_E(c)$
2. Delay for a: infinite distribution: $\eta_{\mathbb{R}^+}(c, a)$

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# Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$ 	η Min 	θ Max

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Probability of a path

$$\mathbb{P}_c^{\eta,\theta}(a\,\pi) = \int_{t \in I(c,a)} \eta_E(c)(a)\, \mathbb{P}_{c_1}^{\eta,\theta}(\pi)\, \mathrm{d}\eta_{\mathbb{R}^+}(c, a)(t)$$

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

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Expectation of **SP** in a path

$$\mathbb{E}_c^{\eta,\theta}(a\,\pi) = \int_{t \in I(c,a)} \eta_E(c)(a) \left[(t\, \mathrm{wt}(c) + \mathrm{wt}(a))\, \mathbb{P}_{c_1}^{\eta,\theta}(\pi) + \mathbb{E}_{c_1}^{\eta,\theta}(\pi) \right] \mathrm{d}\eta_{\mathbb{R}^+}(c, a)(t)$$

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$$\mathbb{E}_c^{\eta,\theta}(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c^{\eta,\theta}(\pi)$$

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min θ Max

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Restrictions on strategies for Min

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Expectation of SP

$$\mathbb{E}_c^{\eta,\theta}(\mathbf{SP}) = \sum_{\pi} \mathbb{E}_c^{\eta,\theta}(\pi)$$

Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond ☺) = 1$

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

$\bigcirc\!\!\!\!\eta$ Min $\boxed{\theta}$ Max

$\eta, \theta : C^* C \to \Delta(C)$

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Expectation of **SP**

$$\mathbb{E}_c^{\eta,\theta}(\mathbf{SP}) = \sum_{\substack{\pi \\ \pi \models \diamond \smiley}} \mathbb{E}_c^{\eta,\theta}(\pi)$$

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Convergence ?

Restrictions on strategies for Min

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$|\mathbb{E}_c^{\eta,\theta}(\pi)|$

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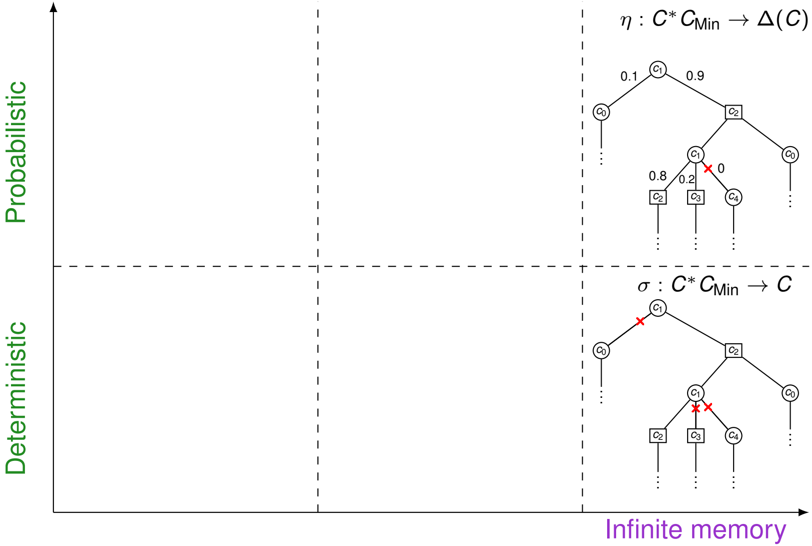
Convergence ?

$$|\mathbb{E}_c^{\eta,\theta}(\pi)| \leqslant \underbrace{k|\pi|}_{|\mathbf{SP}(\pi)|} \underbrace{\alpha^{-|\pi|}}_{\mathbb{P}_c^{\eta,\theta}(\pi)}$$

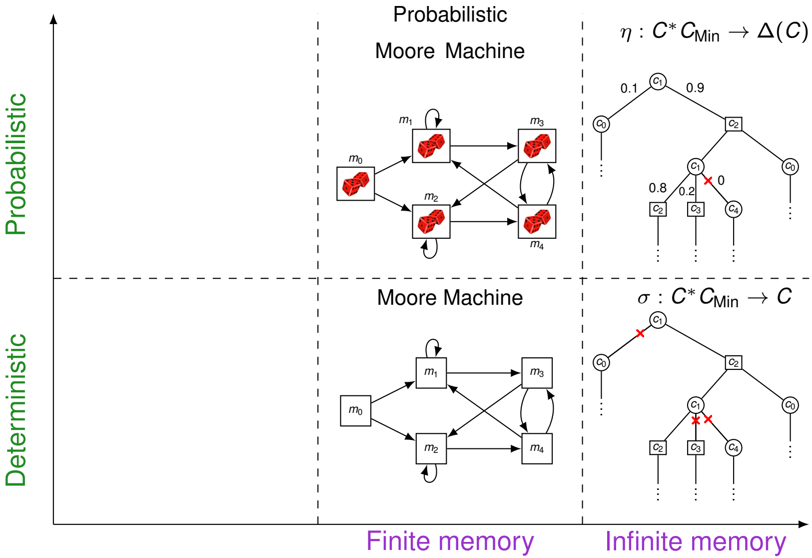
Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond ☺) = 1$
- ☺ must be reached quickly enough

Zoology of strategies



Zoology of strategies



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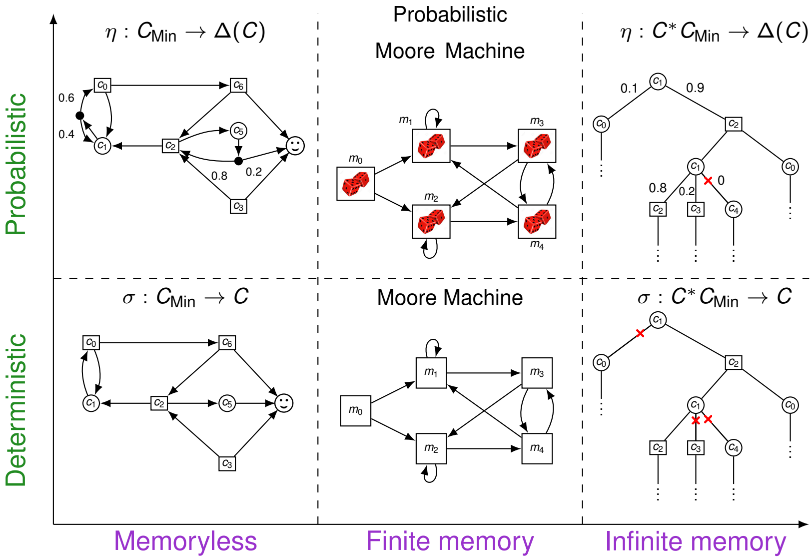
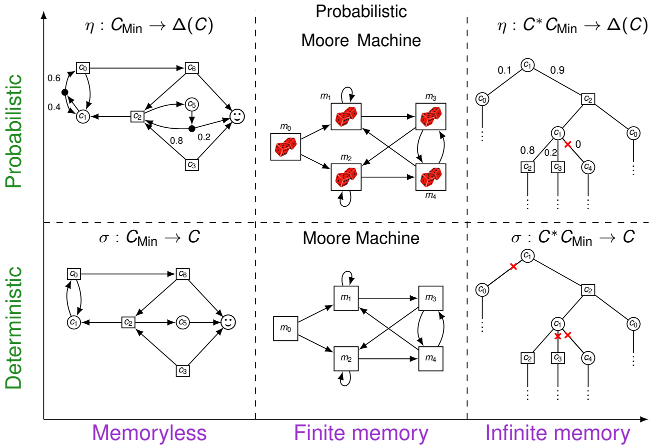


Stochastic values



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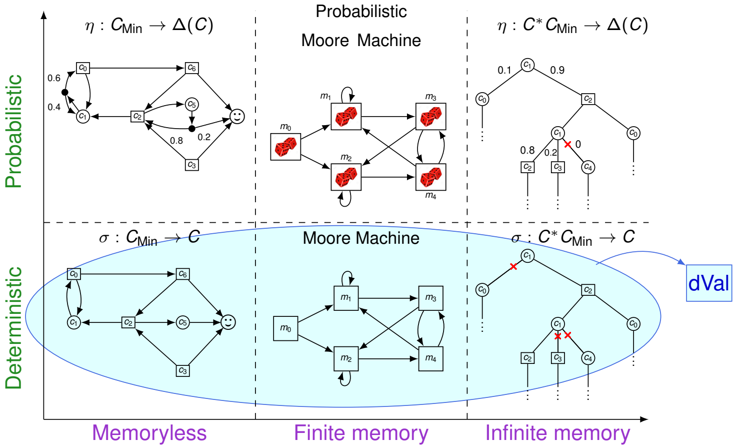
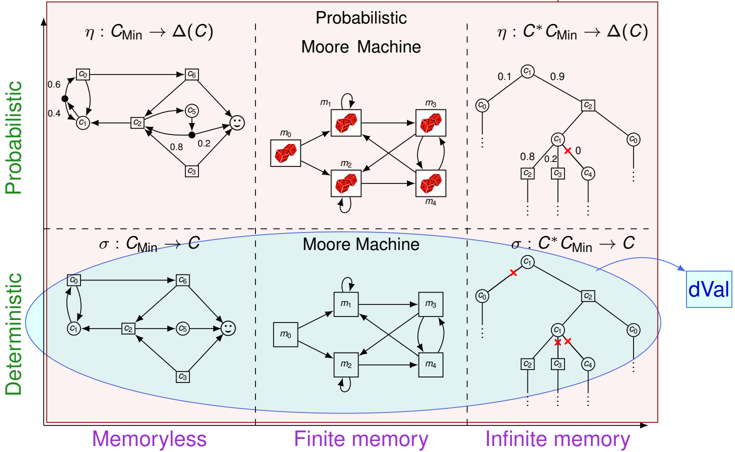


Stochastic values

$\text{Val} = \inf_\eta \sup_\theta \mathbb{E}_c^{\eta,\theta}(\textbf{SP})$



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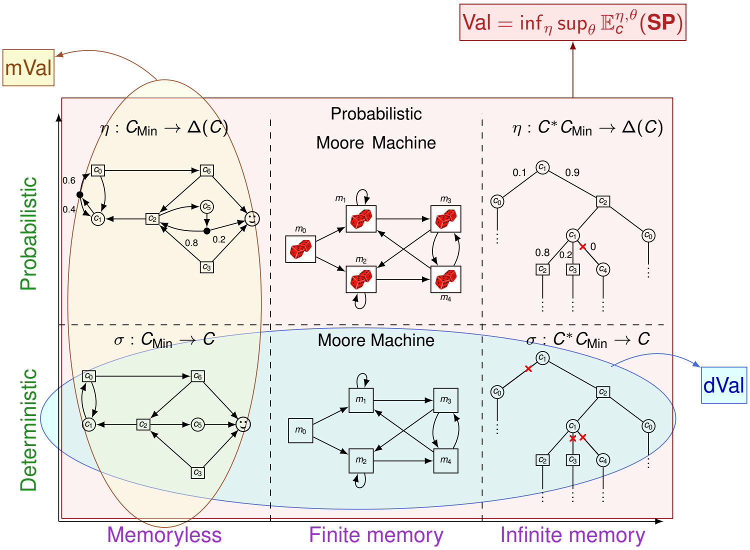


Contribution

$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

$$\text{dVal} = \text{Val} = \text{mVal}$$

Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives

$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Contribution

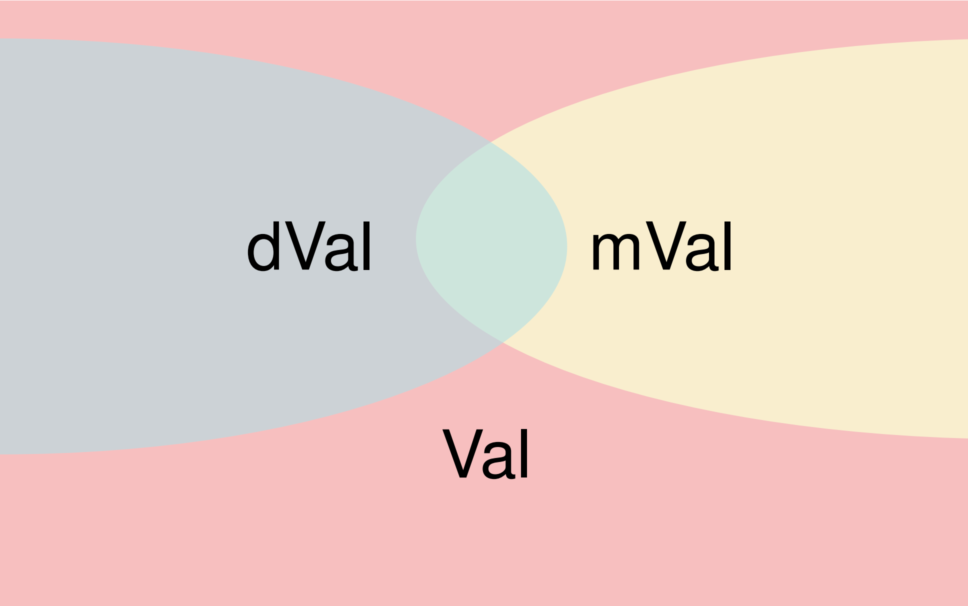
Trade-off between memory and randomness

- Stochastic games with qualitative objectives
- Reachability Timed Games

$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Contribution



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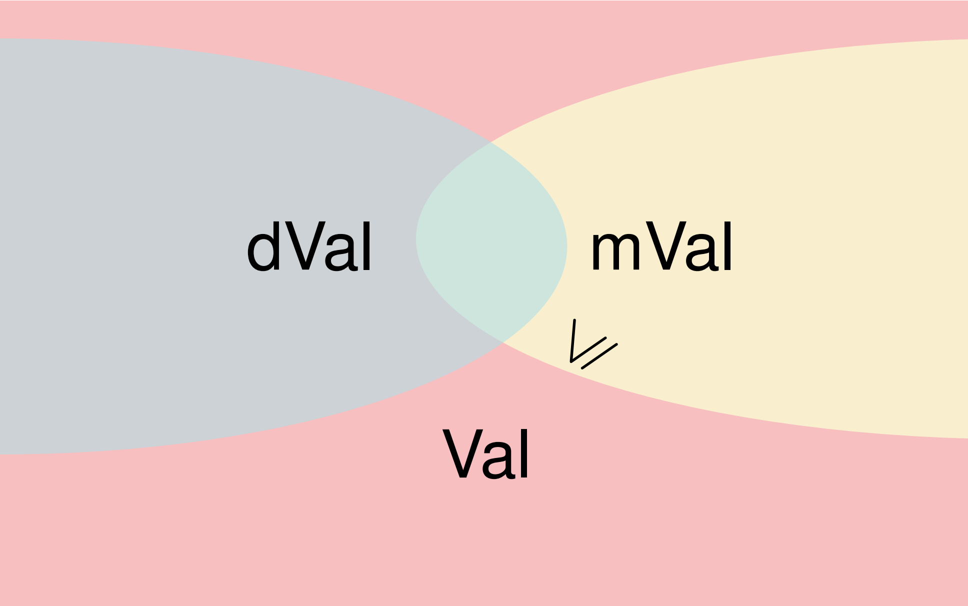

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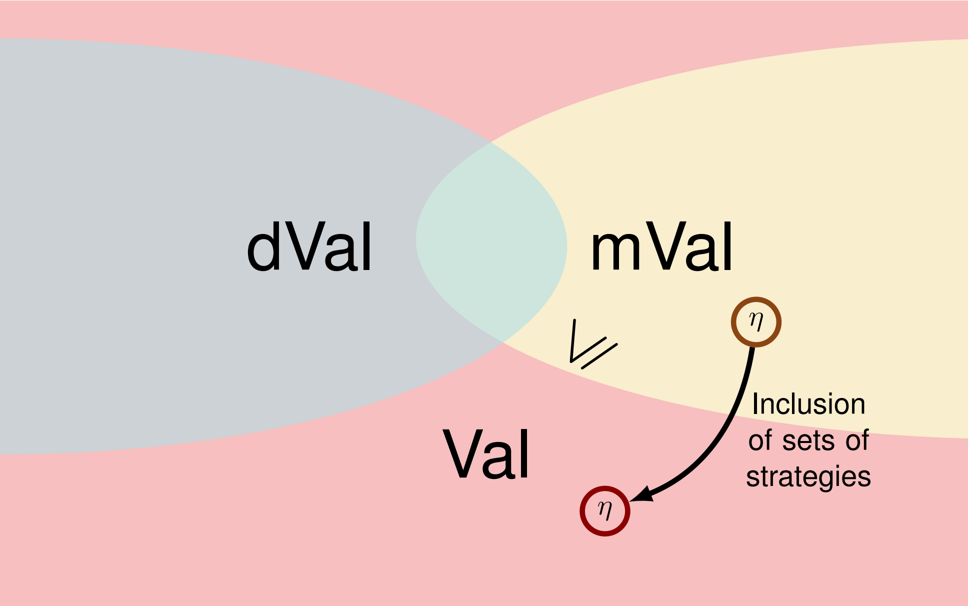

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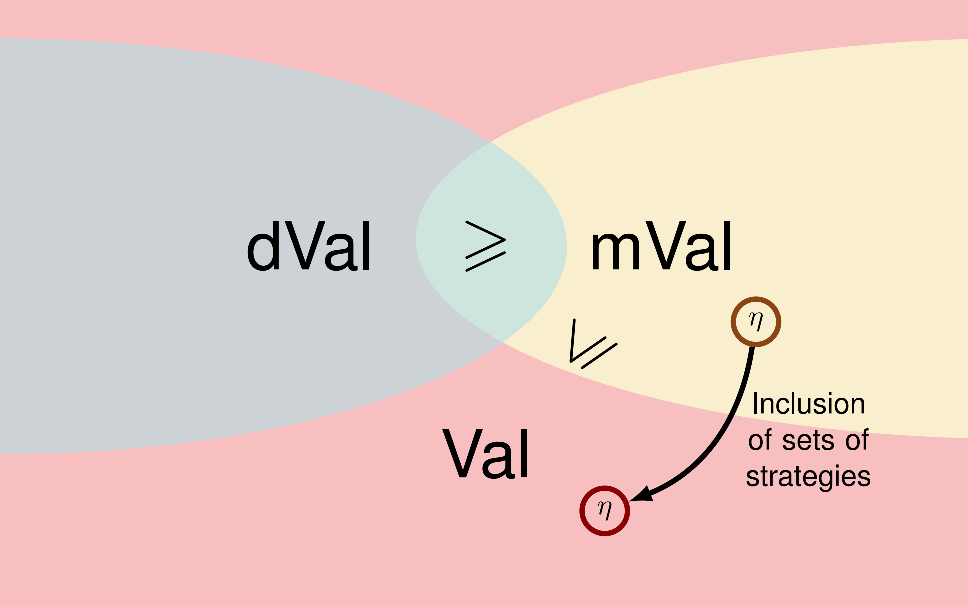

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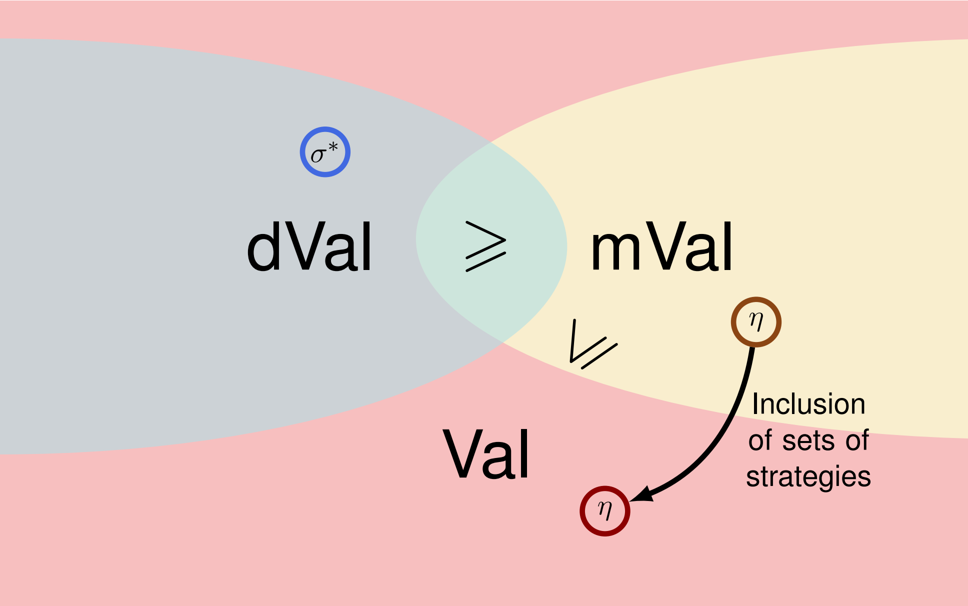
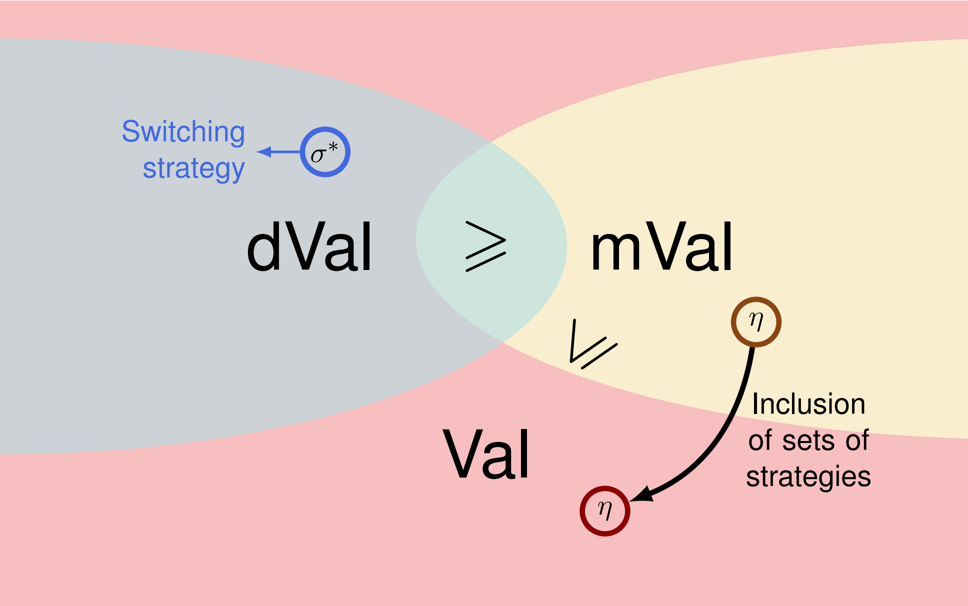


Contribution



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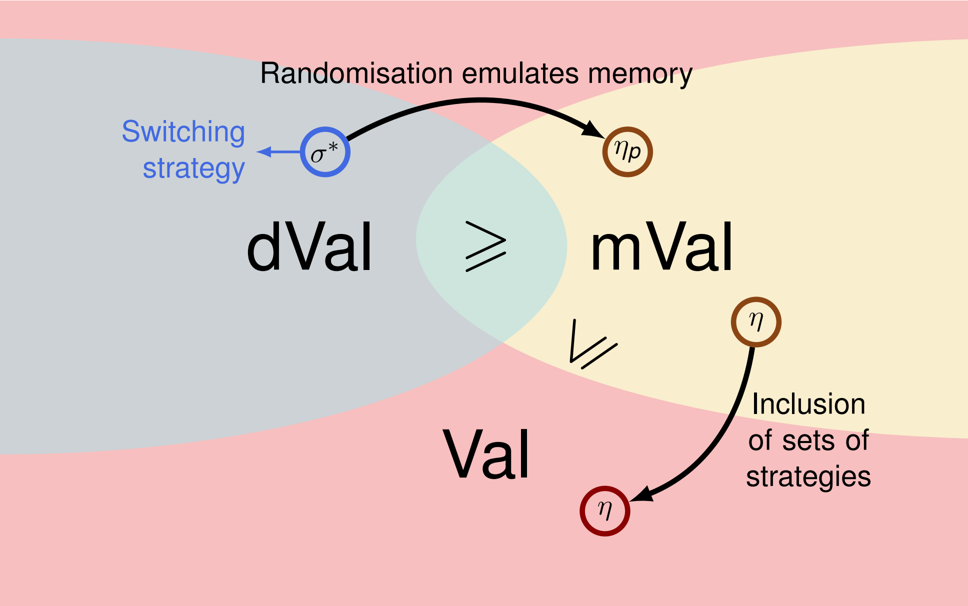

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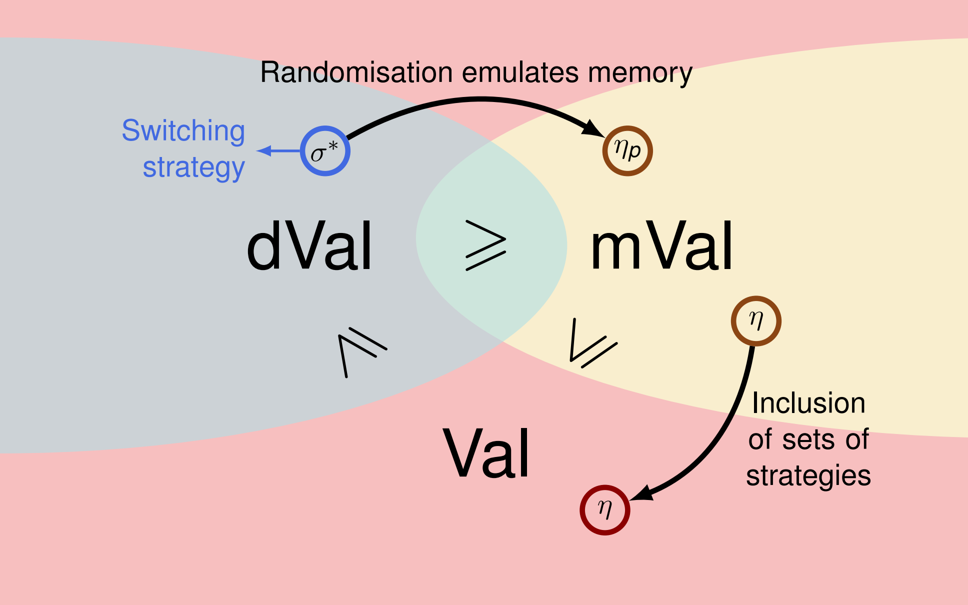

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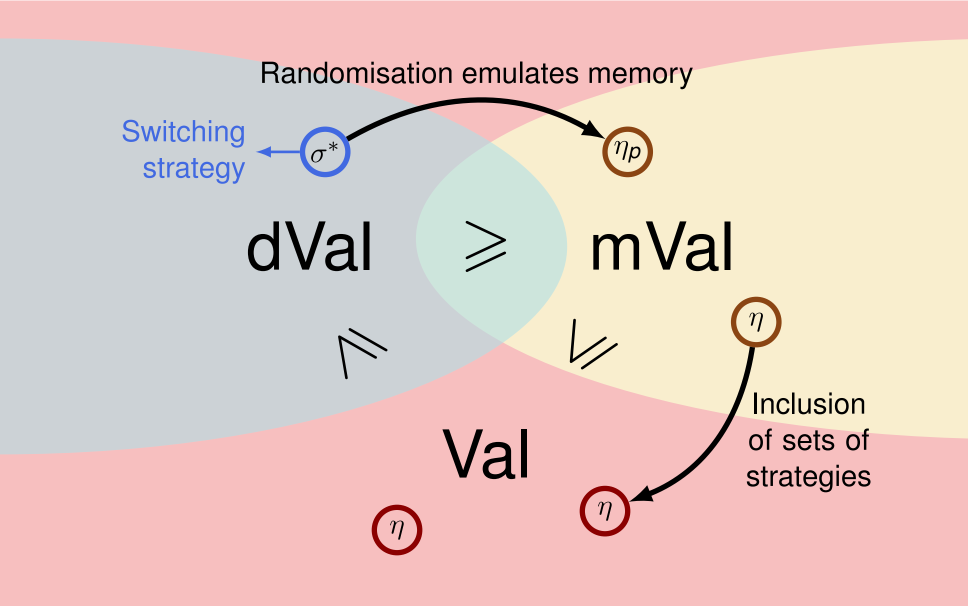

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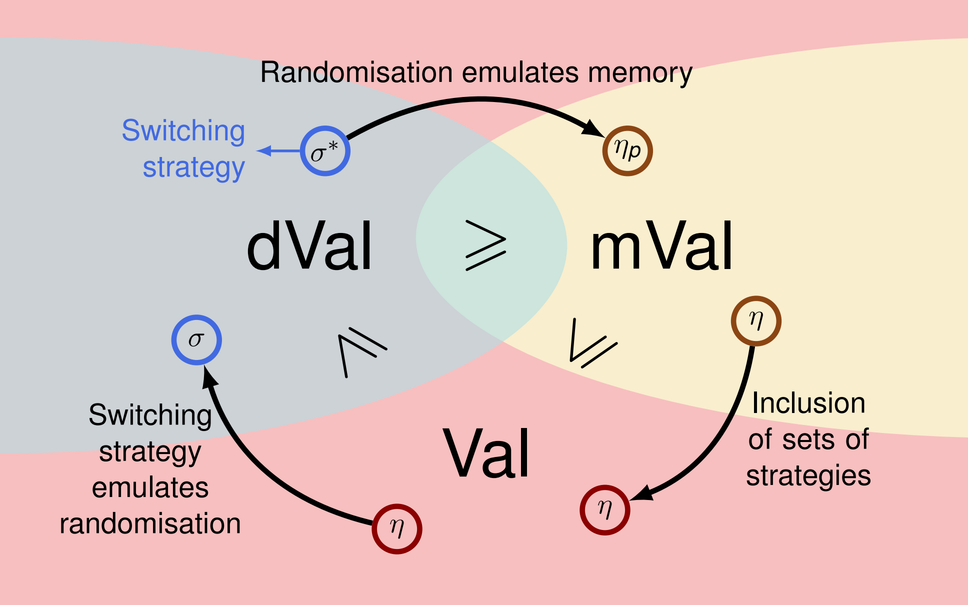

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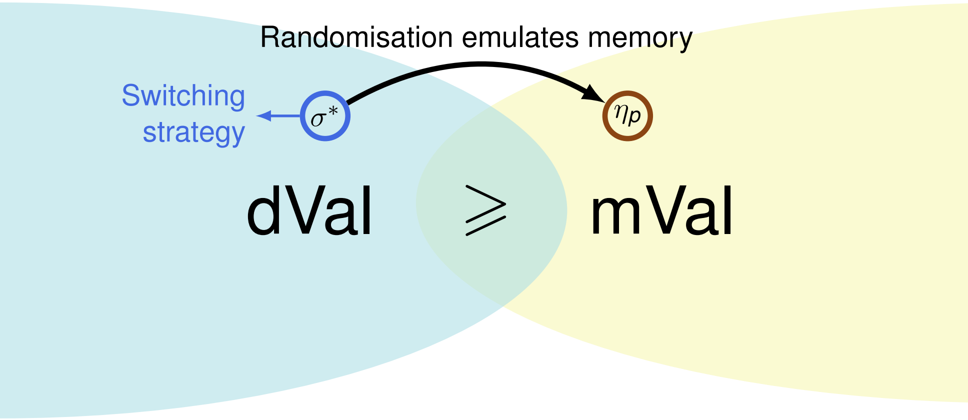

Randomisation emulates memory

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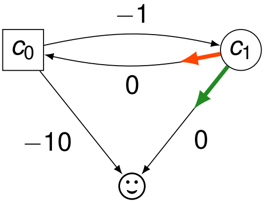

Strategy η_p

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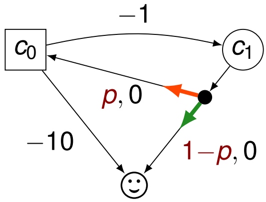

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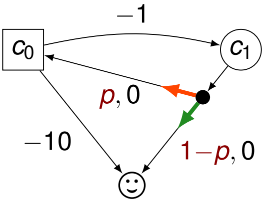

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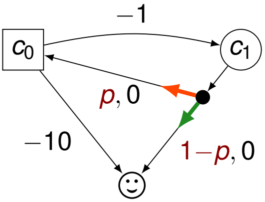

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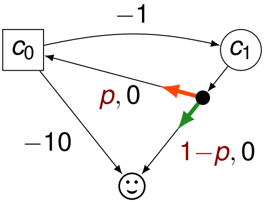

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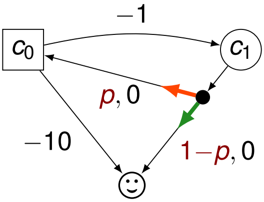
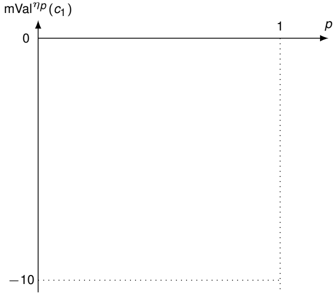


Computation of $\mathrm{mVal}^{\eta_p}(c_1)$



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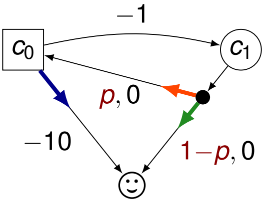

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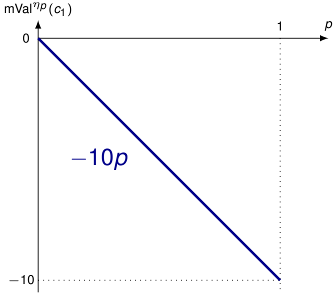

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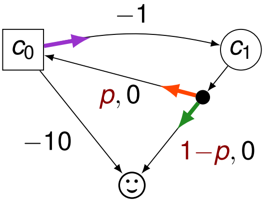

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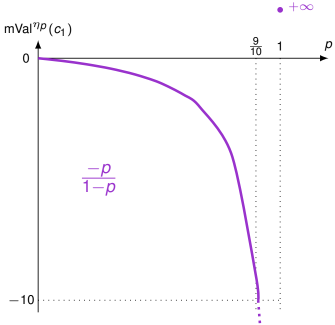

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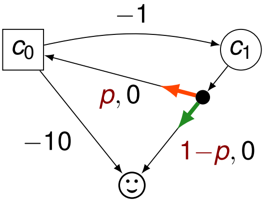

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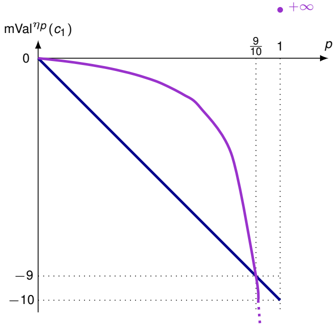

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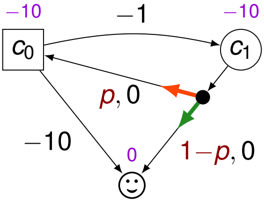

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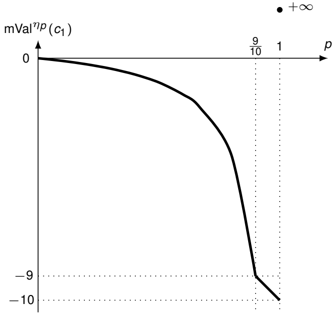

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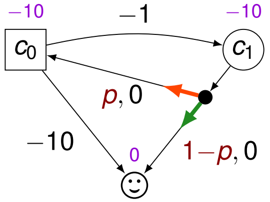

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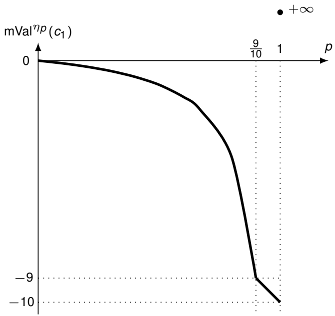

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Claim

For all c,

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E}_c^{\eta_p, \tau}(\mathbf{SP}) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$$

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Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

$$\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) = \sum_{\substack{\rho \\ \rho \models \diamond \smiley}} \mathbf{SP}(\rho)\,\mathbb{P}(\rho)$$

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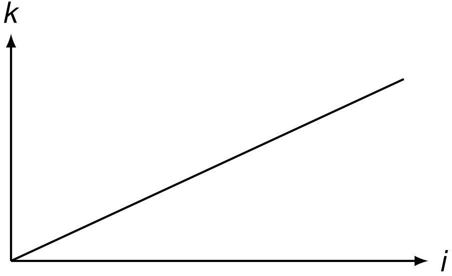

k size of play reaching the target

i number of choices given by σ_2

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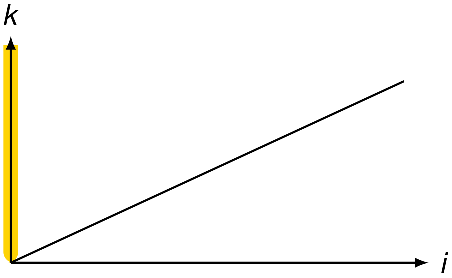

Yellow zone

All plays conforming to σ_1

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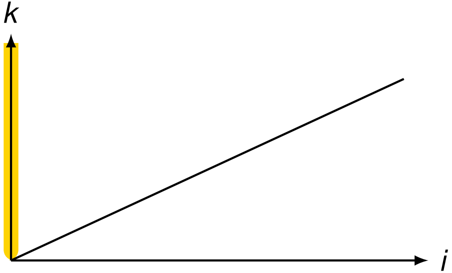

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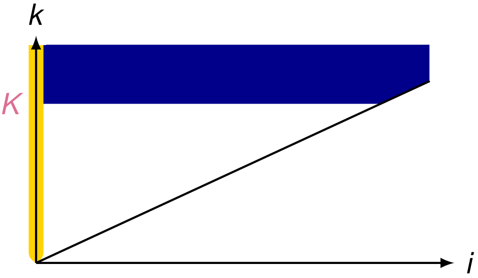

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Blue zone

Plays with many negative cycles

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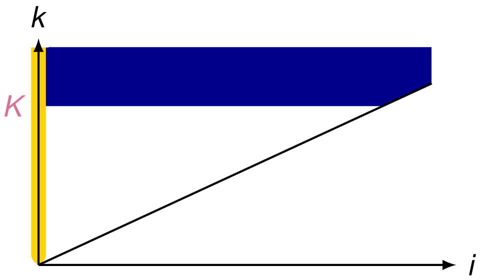

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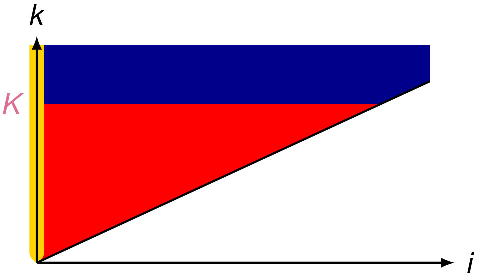

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Plays with many negative cycles

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Red zone

Rest of plays

Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

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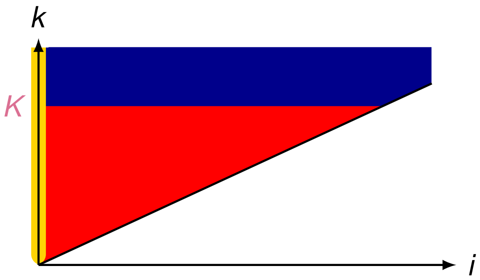

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Red zone

Rest of plays $\quad \mathbb{E} \xrightarrow[\substack{p \to 1 \\ p<1}]{} 0$

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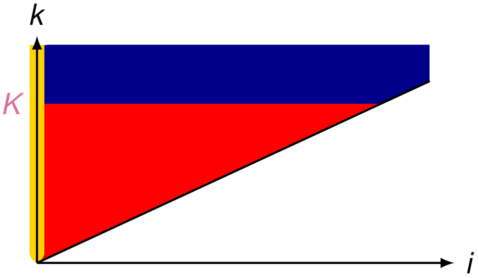

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$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} + \mathbb{E} \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2 K \rangle}(c)$$

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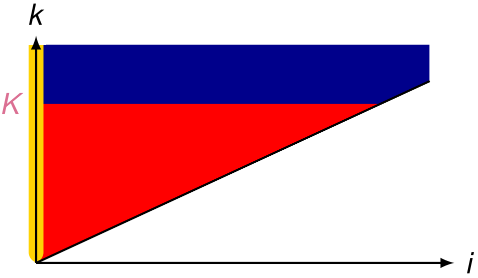

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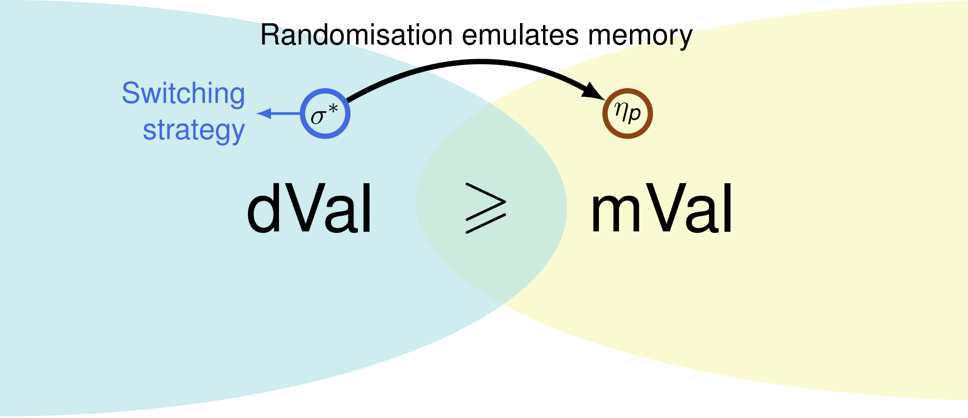
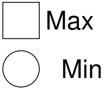
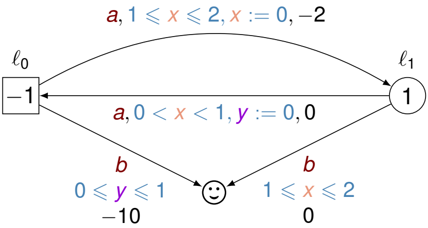
Contribution



Existence of an ε-optimal switching strategy





Switching strategy

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

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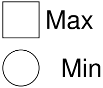

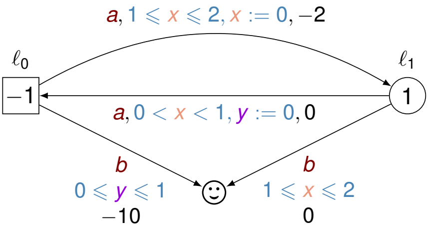

Divergent weighted timed game

All SCCs contain only cycles with a weight $\leqslant -1$ or $\geqslant 1$

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Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

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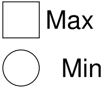

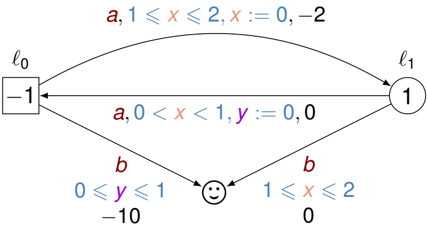

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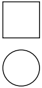
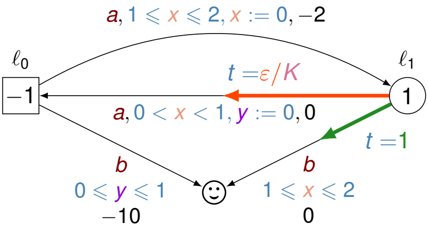
Theorem

Min has an ε-optimal switching strategy

Existence of an ε-optimal switching strategy





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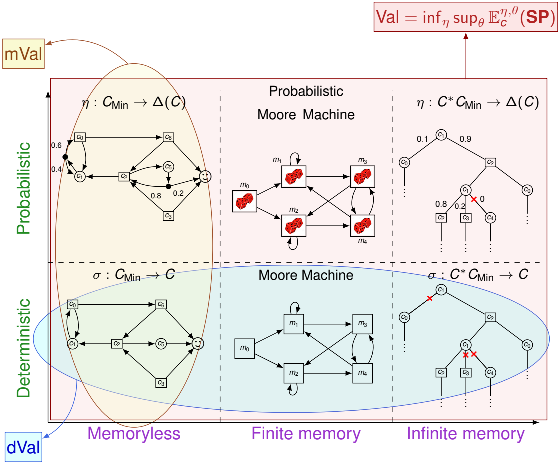
Theorem

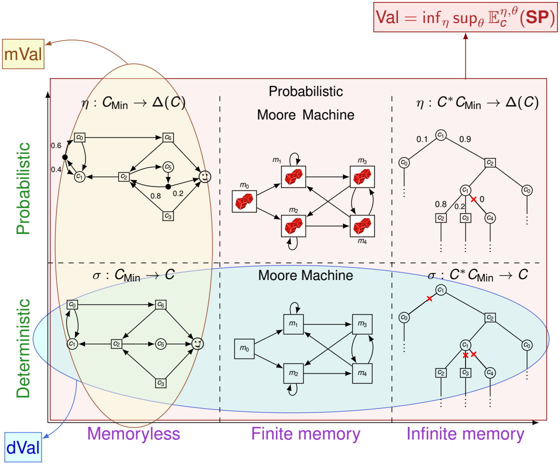
Min has an ε-optimal switching strategy : $\langle \sigma_1, \sigma_2, K \rangle$

Summary



- Definition of $\mathbb{P}_c^{\eta,\theta}(\pi)$
- Definition of $\mathbb{E}_c^{\eta,\theta}(\pi)$
- Definition of $\mathbb{E}_c^{\eta,\theta}(\textbf{SP})$
- Safety conditions on strategies

Summary



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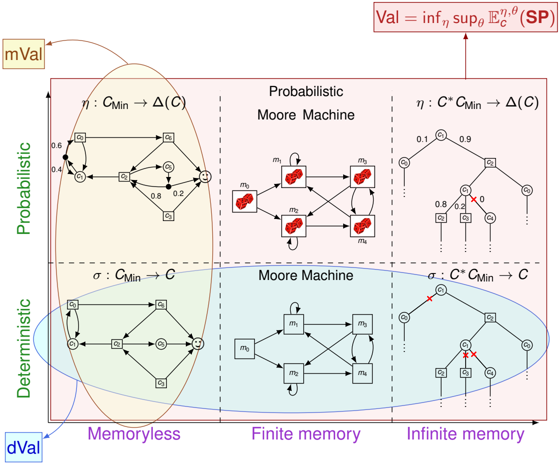
Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Summary



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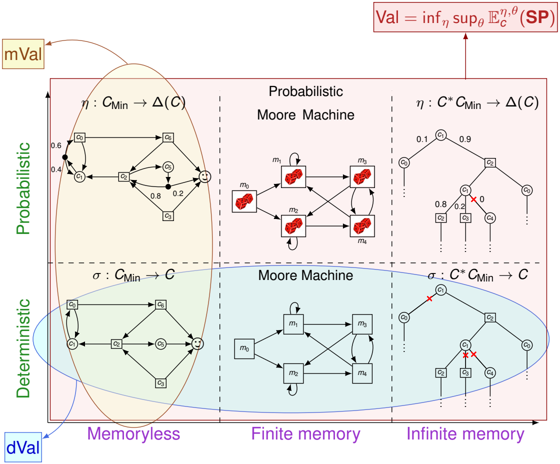
For which classes of games?

- Finite shortest path games

Theorem: $\mathsf{Val} = \mathsf{dVal} = \mathsf{mVal}$

Summary



- Definition of $\mathbb{P}_c^{\eta,\theta}(\pi)$
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- Safety conditions on strategies

For which classes of games?

- Finite shortest path games
- Divergent weighted timed games

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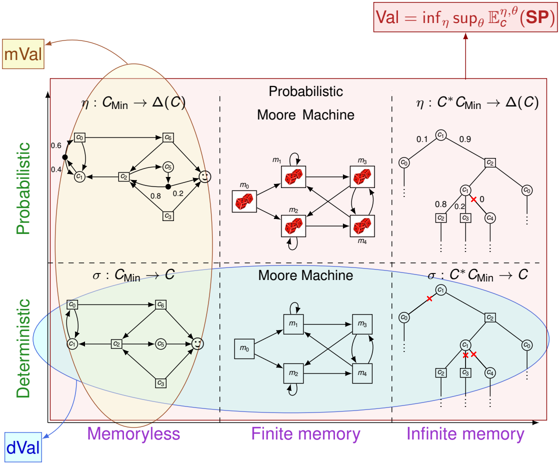
Summary: perspectives



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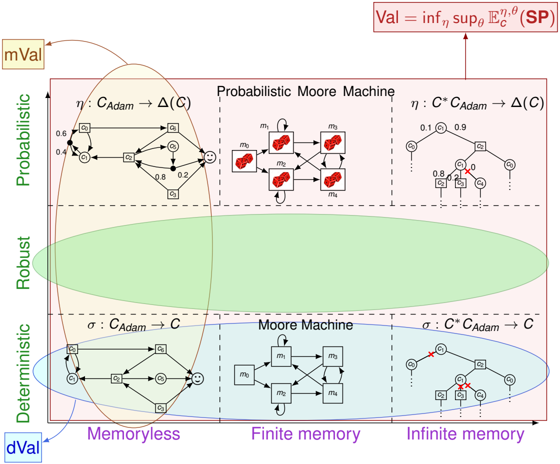

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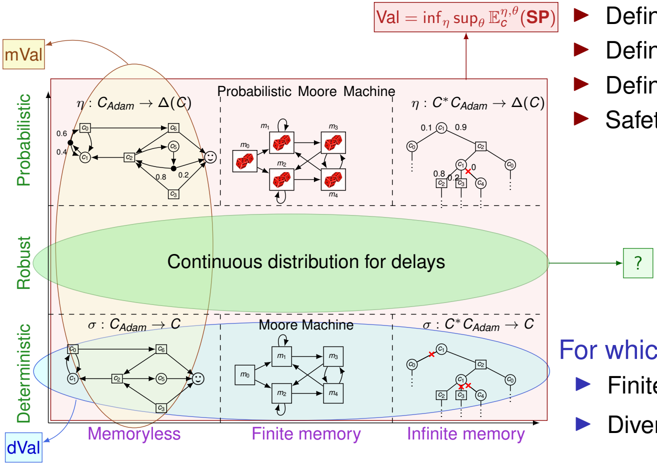
Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Summary: perspectives



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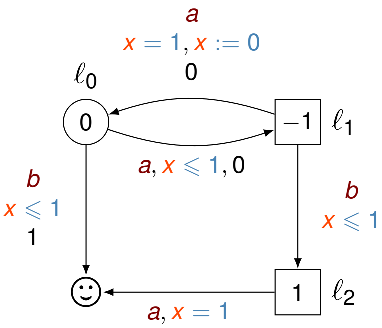
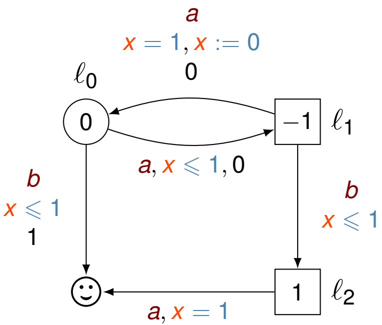
Value problem in 1-clock Weighted Timed Games



Value problem in 1-clock Weighted Timed Games

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



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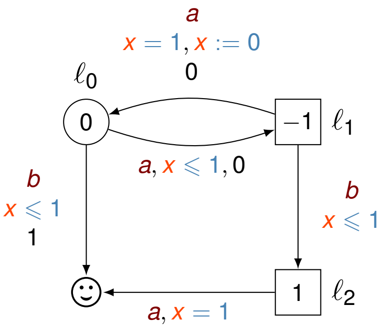

State of the art

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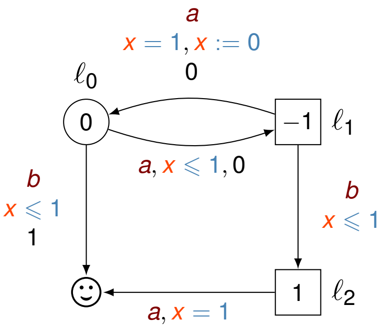

State of the art

☹ Undecidable for at least two clocks

On Optimal Timed Strategies, T. Brihaye, V. Bruyère and J.-F. Raskin, 2005, FORMATS

On the Value Problem in Weighted Timed Games, P. Bouyer, S. Jaziri, and N. Markey, 2015, CONCUR.

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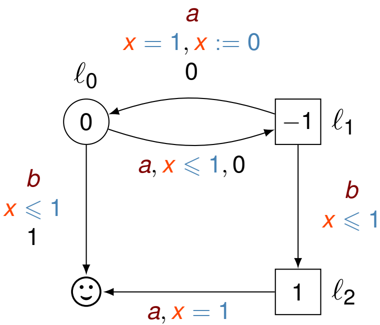

State of the art

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Almost Optimal Strategies in One Clock Priced Timed Games, P. Bouyer, K. Larsen, N. Markey, and J. Rasmussen, 2006, FSTTCS

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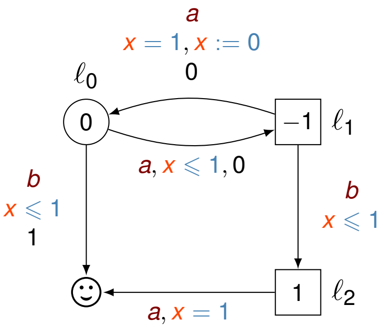

State of the art

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Simple Priced Timed Games Are Not That Simple, T. Brihaye, G. Geeraerts, A. Haddad, E. Lefaucheux, and B. Monmege, 2015, FSTTCS

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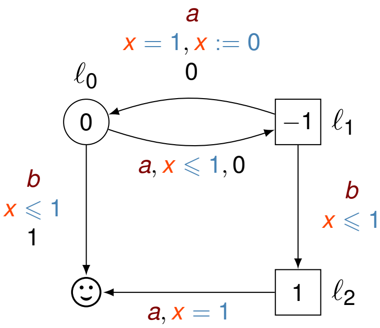

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One-clock priced timed games are PSPACE-hard., J. Fearnley, R. Ibsen-Jensen, and R. Savani, 2020, LICS

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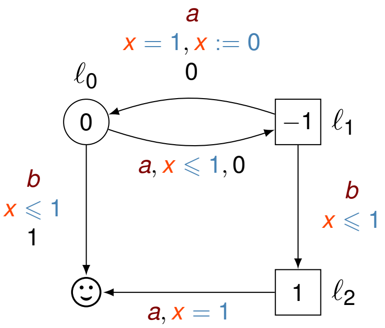

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Decidable in 1-clock Weighted Timed Games

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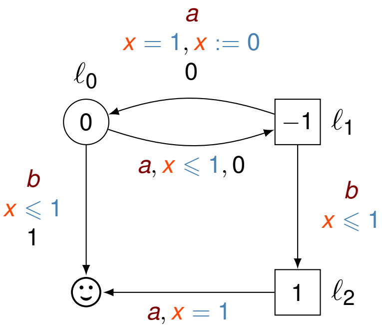

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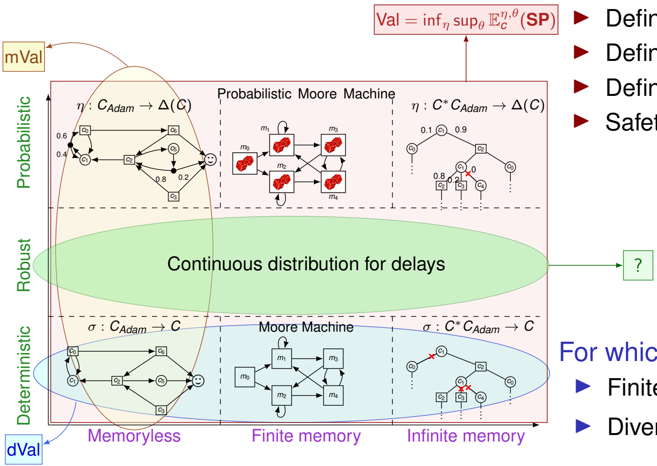
Decidable in 1-clock Weighted Timed Games

$c \mapsto \mathrm{dVal}(c)$ is computable in exponential time

Summary : perspectives



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