

Playing Stochastically in Weighted Timed Games to Emulate Memory

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Motivation: game theory for synthesis



Classical approach

Check the correctness
of a system



Game theory

Interaction between two
antagonistic agents:
environment and controller

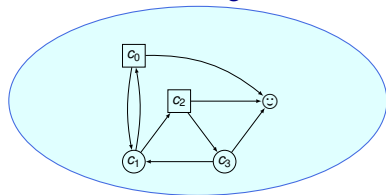


Code synthesis

Correct by
construction:
synthesis of
controller

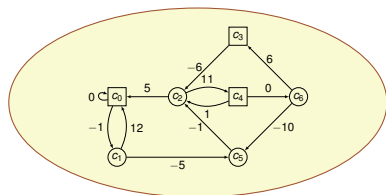
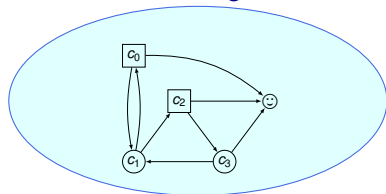
Different classes of games

Qualitative games



Different classes of games

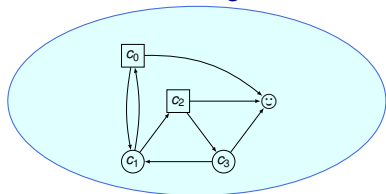
Qualitative games



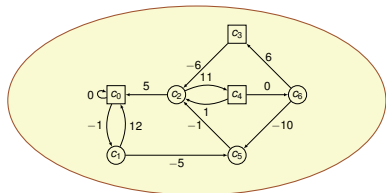
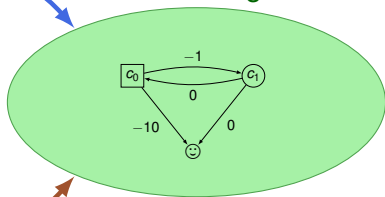
Quantitative games

Different classes of games

Qualitative games



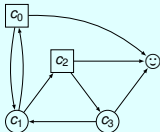
Shortest-Path games



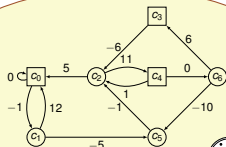
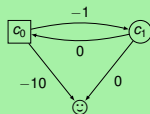
Quantitative games

Different classes of games

Qualitative games



Shortest-Path games

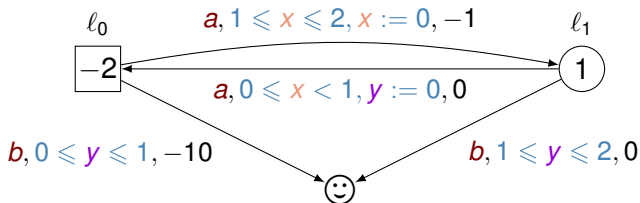


Quantitative games



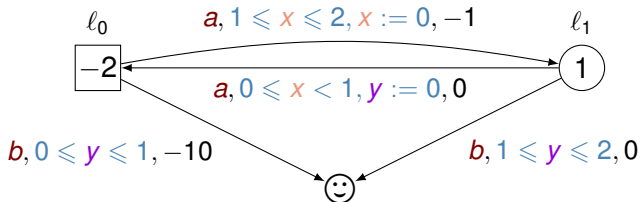
Weighted Timed Games

○ Min □ Max 😊 target



Weighted Timed Games

○ Min □ Max 😊 target

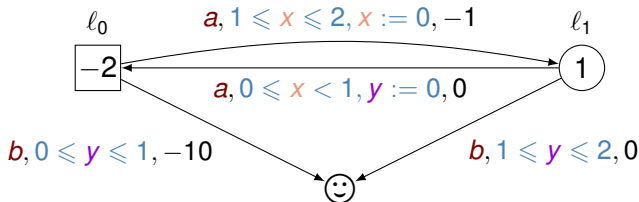


Play ρ

$(l_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix})$

Weighted Timed Games

○ Min □ Max 😊 target

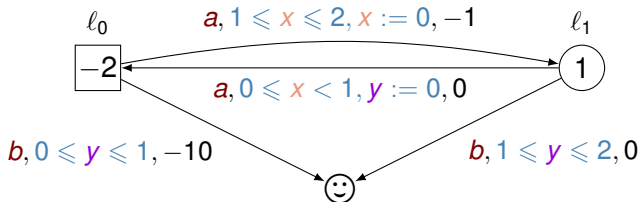


Play ρ

$$(l_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5, a}$$

Weighted Timed Games

○ Min □ Max 😊 target

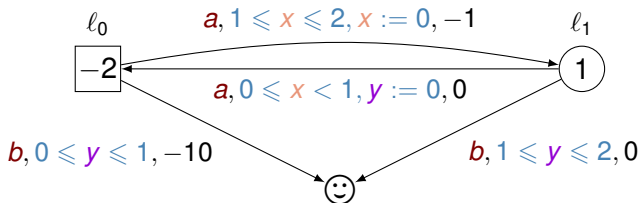


Play ρ

$$(l_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5, a} (l_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix})$$

Weighted Timed Games

○ Min □ Max 😊 target

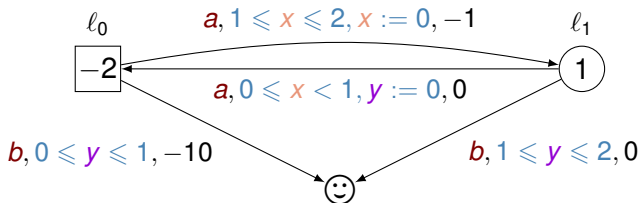


Play ρ

$$(l_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5, a} (l_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow{1.25, a} (l_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow{1/3, b} (\text{😊}, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix})$$

Weighted Timed Games

○ Min □ Max 😊 target



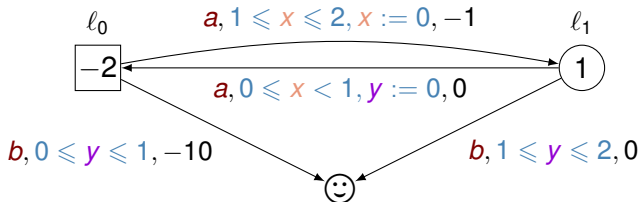
Play ρ

$$(l_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) \xrightarrow{0.5, a} (l_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow{1.25, a} (l_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow{1/3, b} (\text{😊}, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix})$$

$1 \times 0.5 + 0$

Weighted Timed Games

○ Min □ Max 😊 target

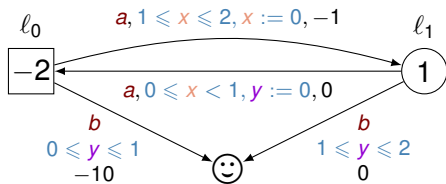


Play ρ

$$\begin{aligned}
 (l_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}) &\xrightarrow[1 \times 0.5 + 0]{0.5, a} (l_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}) \xrightarrow[-2 \times 1.25 - 1]{1.25, a} (l_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}) \xrightarrow[1 \times \frac{1}{3} + 0]{1/3, b} (\text{😊}, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}) \rightsquigarrow -\frac{8}{3}
 \end{aligned}$$

Strategies

○ Min □ Max

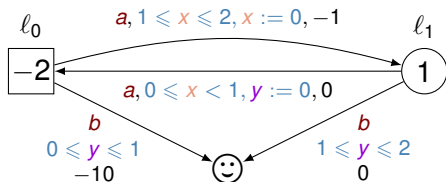


Deterministic strategy

Choose an edge and a delay

Strategies

○ Min □ Max



Deterministic strategy

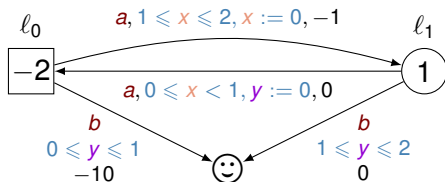
Choose an edge and a delay

In $(l_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

Deterministic strategy

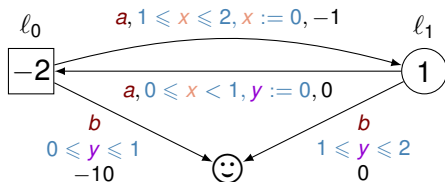
Choose an edge and a delay

In $(l_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a : finite distribution

Deterministic strategy

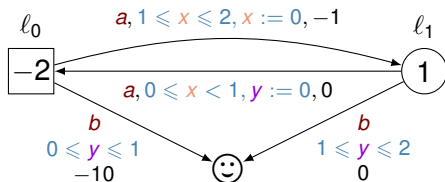
Choose an edge and a delay

In $(l_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a : finite distribution
2. Delay for a : infinite distribution

Deterministic strategy

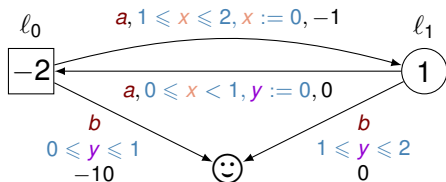
Choose an edge and a delay

In $(l_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a : finite distribution
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Deterministic strategy

Choose an edge and a delay

In $(l_1, (0, 0))$

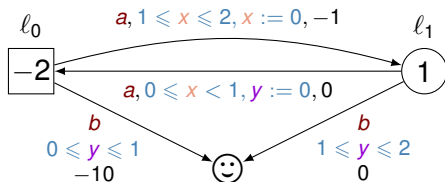
Choose a with $t = \frac{1}{3}$

In $(l_1, (0, 0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a : finite distribution
2. Delay for a : infinite distribution

Deterministic strategy

Choose an edge and a delay

In $(l_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

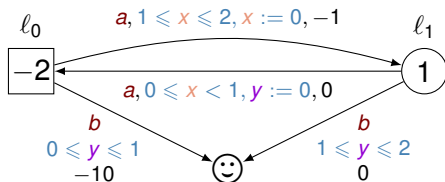
In $(l_1, (0, 0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- a : choose t with $\mathcal{U}([0, 1])$

Strategies

○ Min □ Max



Probabilistic strategy

Distribution over possible choices

1. Edge a : finite distribution
2. Delay for a : infinite distribution

Deterministic strategy

Choose an edge and a delay

In $(l_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

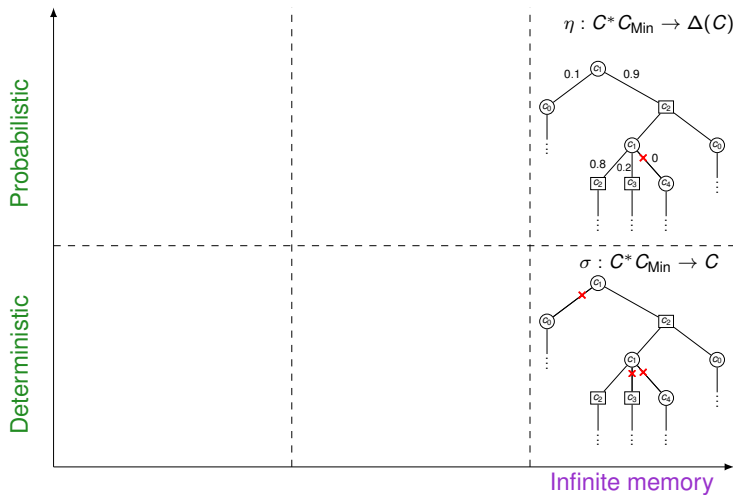
In $(l_1, (0, 0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- ▶ a : choose t with $\mathcal{U}([0, 1])$
- ▶ b : choose t with $\delta_{1.5}$

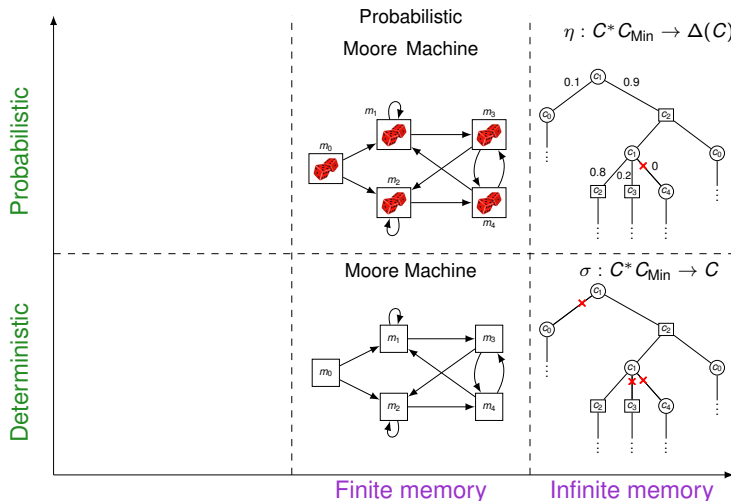
Zoology of strategies

$$C = L \times \mathbb{R}^{|C|}$$



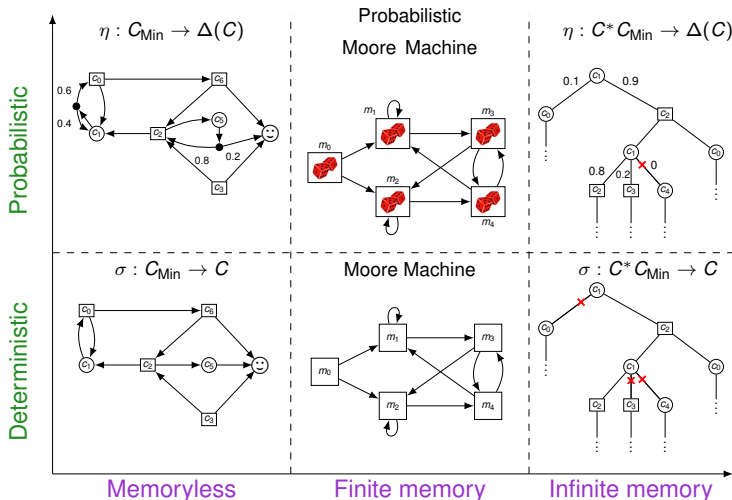
Zoology of strategies

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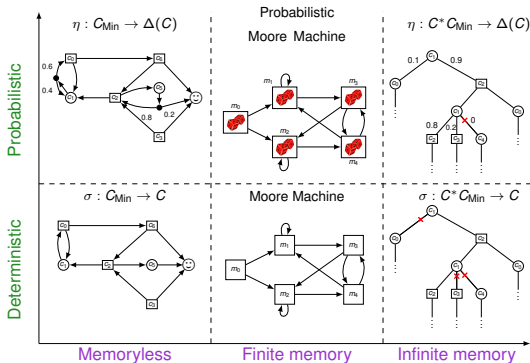
Zoology of strategies

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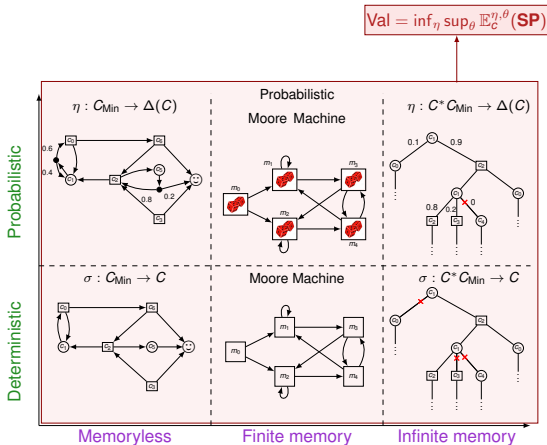
Contribution

$$C = L \times \mathbb{R}^{|C|}$$



Contribution

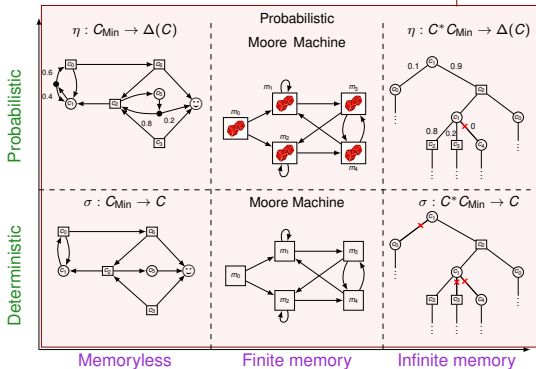
$$C = L \times \mathbb{R}^{|C|}$$



Contribution

$$\text{Val} = \inf_{\eta} \sup_{\theta} \mathbb{E}_C^{\eta, \theta}(\mathbf{SP})$$

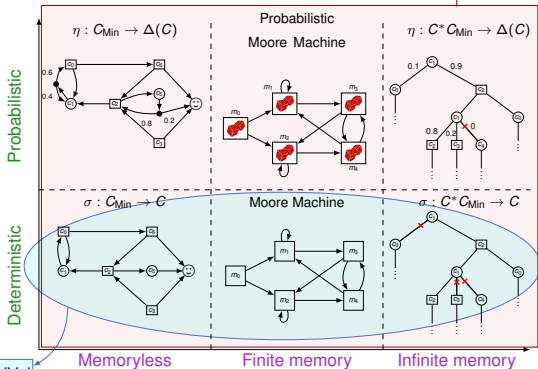
- Definition of $\mathbb{P}_C^{\eta, \theta}(\pi)$
- Definition of $\mathbb{E}_C^{\eta, \theta}(\pi)$
- Definition of $\mathbb{E}_C^{\eta, \theta}(\mathbf{SP})$
- Safety conditions on strategies



Contribution

$$\text{Val} = \inf_{\eta} \sup_{\theta} \mathbb{E}_C^{\eta, \theta}(\text{SP})$$

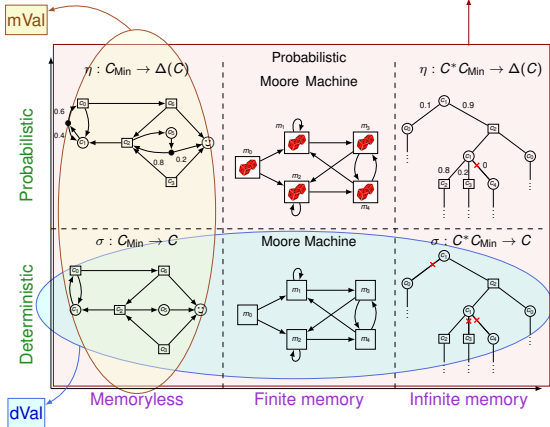
- ▶ Definition of $\mathbb{P}_C^{\eta, \theta}(\pi)$
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Contribution

$$\text{Val} = \inf_{\eta} \sup_{\theta} \mathbb{E}_c^{\eta, \theta}(\text{SP})$$

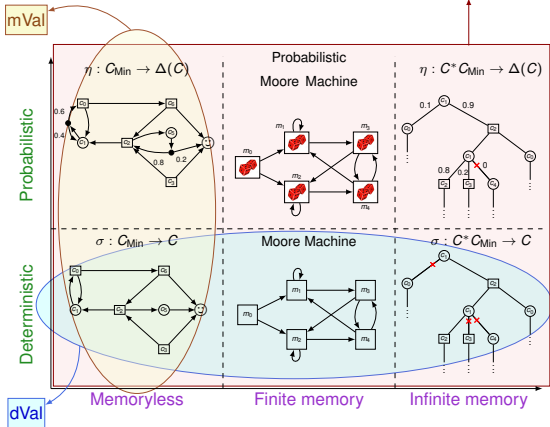
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Contribution

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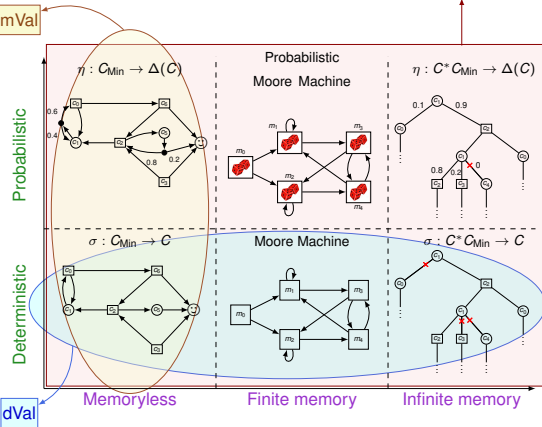


Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Contribution

$$\text{Val} = \inf_{\eta} \sup_{\theta} \mathbb{E}_c^{\eta, \theta}(\text{SP})$$

- ▶ Definition of $\mathbb{P}_c^{\eta, \theta}(\pi)$
- ▶ Definition of $\mathbb{E}_c^{\eta, \theta}(\pi)$
- ▶ Definition of $\mathbb{E}_c^{\eta, \theta}(\text{SP})$
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For which classes of games?

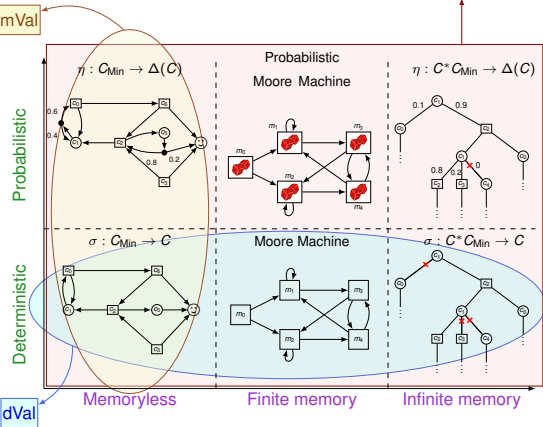
- ▶ Divergent weighted timed games

Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Contribution

$$\text{Val} = \inf_{\eta} \sup_{\theta} \mathbb{E}_c^{\eta, \theta}(\text{SP})$$

- ▶ Definition of $\mathbb{P}_c^{\eta, \theta}(\pi)$
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For which classes of games?

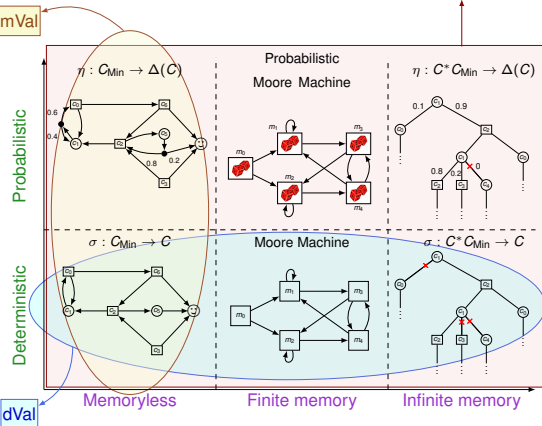
- ▶ Divergent weighted timed games
- ▶ Finite shortest path games

Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Contribution

$$\text{Val} = \inf_{\eta} \sup_{\theta} \mathbb{E}_c^{\eta, \theta}(\text{SP})$$

- ▶ Definition of $\mathbb{P}_c^{\eta, \theta}(\pi)$
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For which classes of games?

- ▶ Divergent weighted timed games
- ▶ Finite shortest path games

Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Thank you! Questions?