Playing Stochastically in Weighted Timed Games to Emulate Memory

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TickTac meeting
March 4, 2022

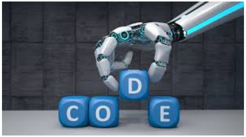
Motivation : game theory for synthesis

Classical approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents : environment and controller

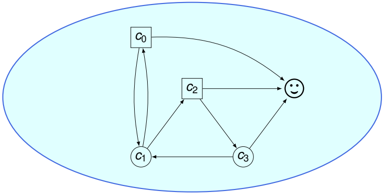


Code synthesis

Correct by construction : synthesis of controller

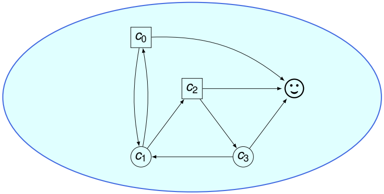
Different classes of games

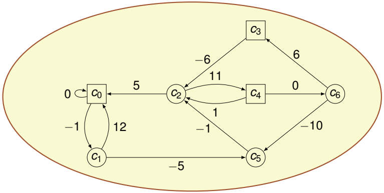
Qualitative games



Different classes of games

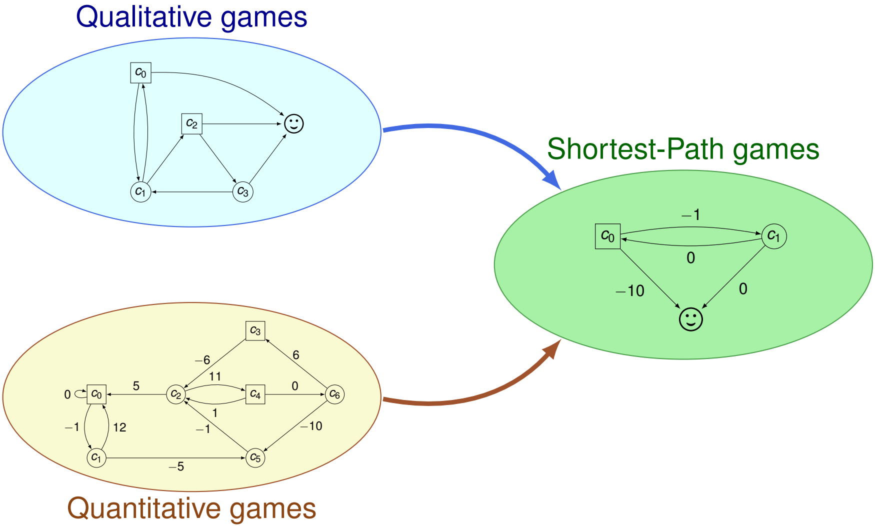
Qualitative games



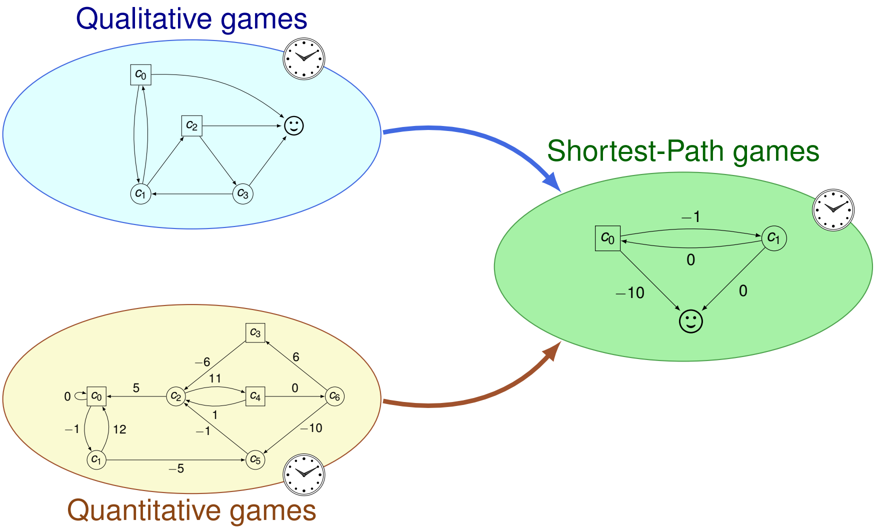


Quantitative games

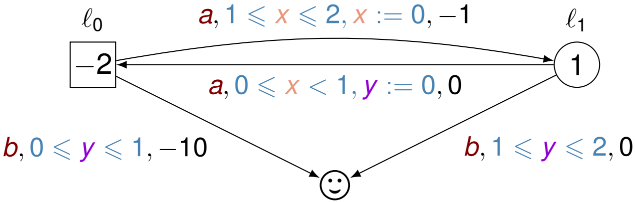
Different classes of games



Different classes of games



Weighted Timed Games



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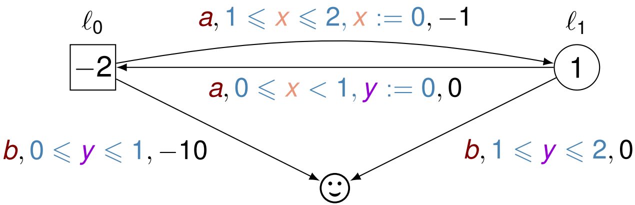
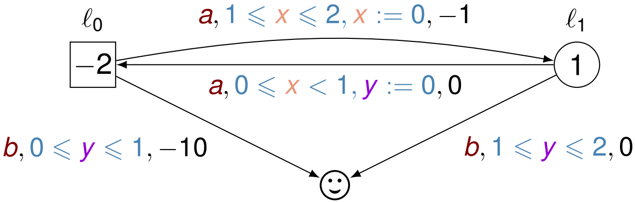


Play ρ

$(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix})$

Weighted Timed Games

Min Max target

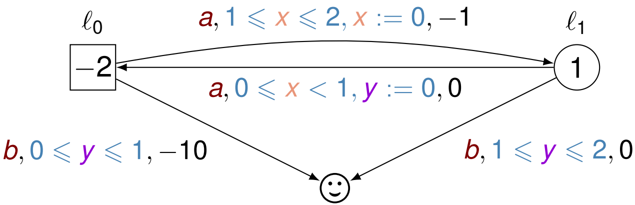


Play ρ

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{0.5,\, a} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix})$$

Weighted Timed Games

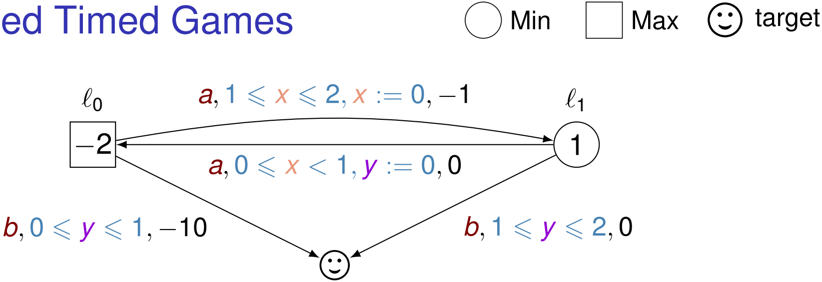
◯ Min ☐ Max ☺ target



Play ρ

$$(\ell_1, \begin{pmatrix}0\\0\end{pmatrix}) \xrightarrow{0.5,\, a} (\ell_0, \begin{pmatrix}0.5\\0\end{pmatrix}) \xrightarrow{1.25,\, a} (\ell_1, \begin{pmatrix}0\\1.25\end{pmatrix}) \xrightarrow{1/3,\, b} (☺, \begin{pmatrix}1/3\\19/12\end{pmatrix})$$

Weighted Timed Games



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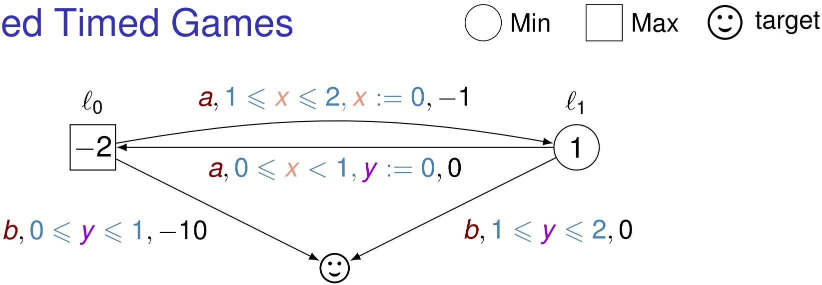
$$1 \times 0.5 + 0$$

Weighted Timed Games



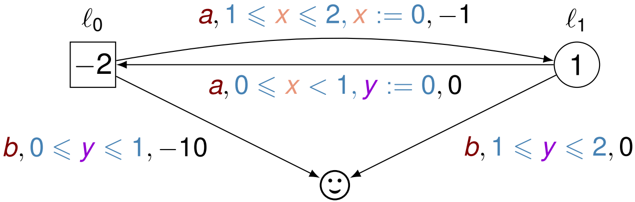
Play ρ

$$\left(\ell_1, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \xrightarrow{0.5,\, a} \left(\ell_0, \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}\right) \xrightarrow{1.25,\, a} \left(\ell_1, \begin{pmatrix} 0 \\ 1.25 \end{pmatrix}\right) \xrightarrow{1/3,\, b} \left(☺, \begin{pmatrix} 1/3 \\ 19/12 \end{pmatrix}\right) \rightsquigarrow -\frac{8}{3}$$

$$1 \times 0.5 + 0 \qquad -2 \times 1.25 - 1 \qquad 1 \times \tfrac{1}{3} + 0$$

Weighted Timed Games

○ Min □ Max ☺ target



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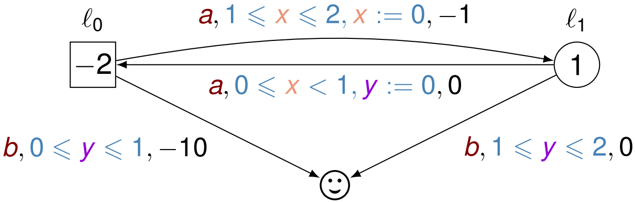
Deterministic strategy

Choose an edge and a delay

Weighted Timed Games



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Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

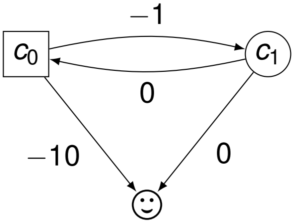
Deterministic strategies: Min needs memory

σ Min

τ Max

Deterministic value

$$\mathrm{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathbf{SP}(\mathrm{Play}(c, \sigma, \tau))$$



Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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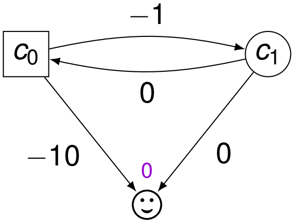

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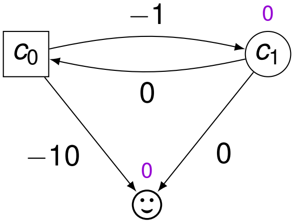

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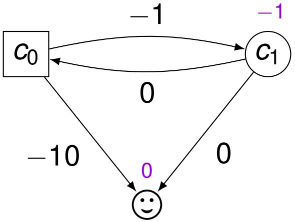

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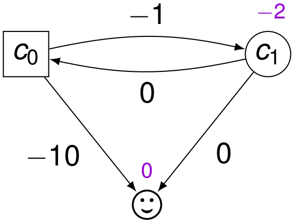

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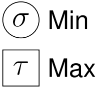

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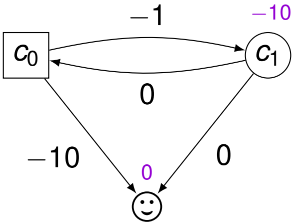

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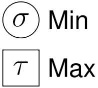

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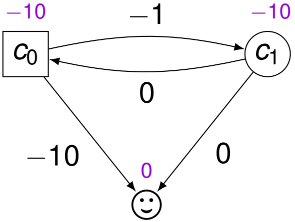

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$\bigcirc\ \sigma$ Min

$\square\ \tau$ Max

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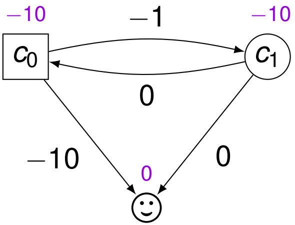
Optimal strategy

$$\mathsf{dVal}^{\sigma^*}(c) \leqslant \mathsf{dVal}(c)$$



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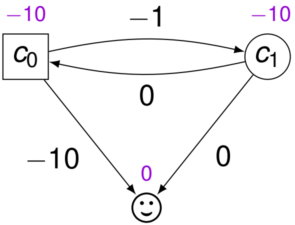

Optimal strategy for Min

An optimal strategy for Min may require finite memory.

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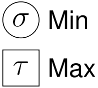

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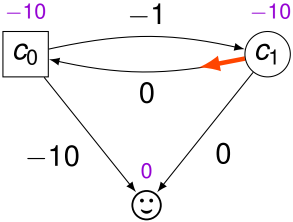

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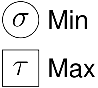

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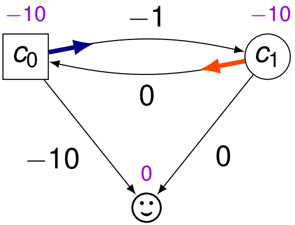

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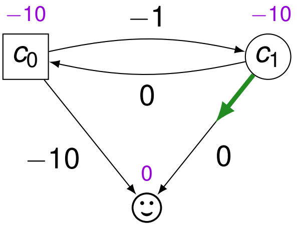

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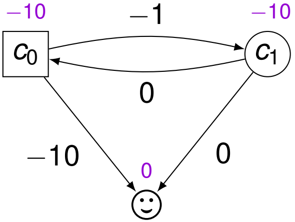

Optimal strategy for Min

Switching strategy:

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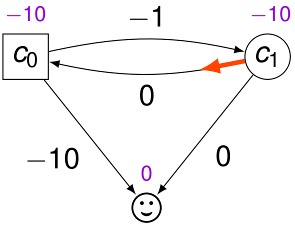

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$

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Deterministic strategies: Min needs memory

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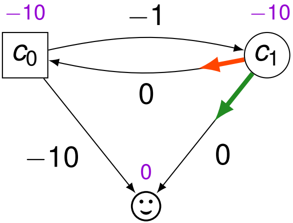

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺

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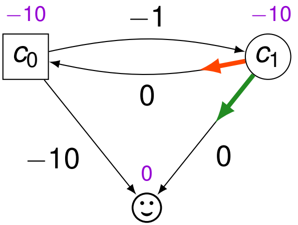

Optimal strategy for Min

Switching strategy:

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

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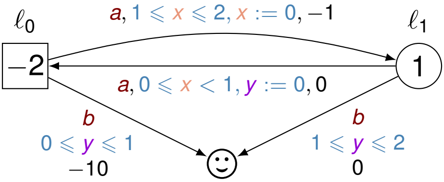
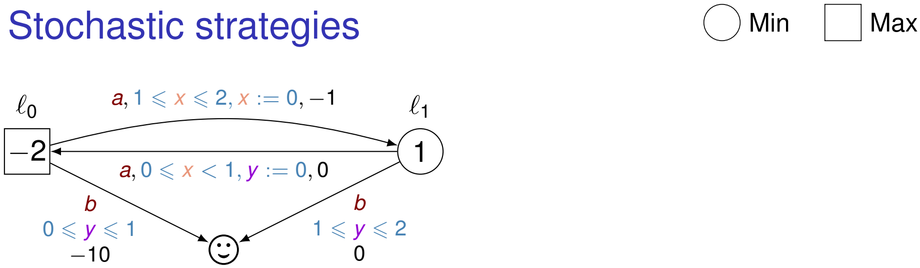
Stochastic strategies

Min Max



Stochastic strategies

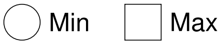


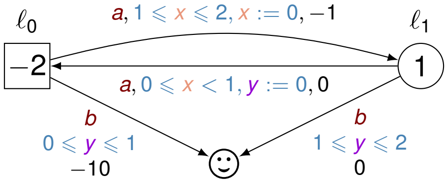
Stochastic strategy

Distribution over possible choices

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies





Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution

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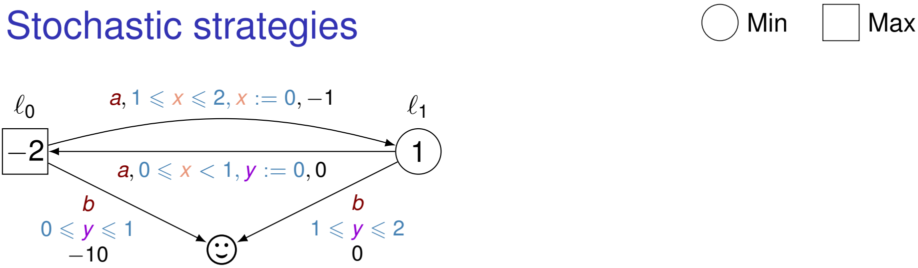
Stochastic strategies



Stochastic strategy

Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

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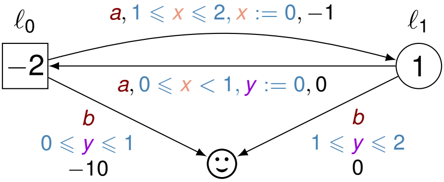
Stochastic strategies

◯ Min ☐ Max



In $(\ell_1, (0, 0))$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

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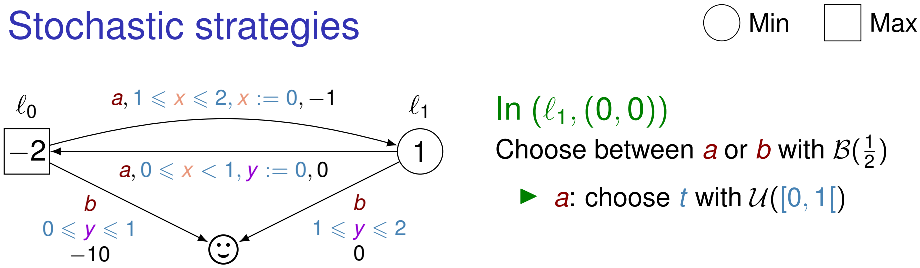


In $(\ell_1, (0, 0))$

Choose between a or b with $\mathcal{B}(\frac{1}{2})$

- a: choose t with $\mathcal{U}([0, 1[)$

Stochastic strategy

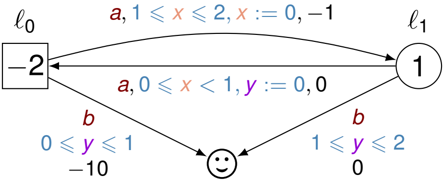
Distribution over possible choices

1. Edge a: finite distribution
2. Delay for a: infinite distribution

Stochastic Timed Automata, N. Bertrand, P. Bouyer, T. Brihaye, Q. Menet, C. Baier, M. Grosser, and M. Jurdzinzki, 2014, Logical Methods in Computer Science

Stochastic strategies

○ Min □ Max



In $(\ell_1, (0, 0))$

Choose between *a* or *b* with $\mathcal{B}(\frac{1}{2})$

- *a*: choose t with $\mathcal{U}([0, 1[)$
- *b*: choose t with $\delta_{1.5}$

Stochastic strategy

Distribution over possible choices

1. Edge *a*: finite distribution
2. Delay for *a*: infinite distribution

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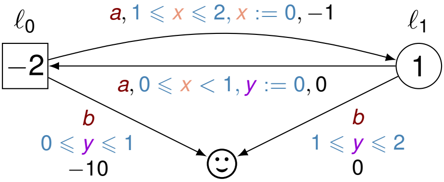
Stochastic strategies

η Min θ Max



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When we fix two strategies

Stochastic strategies

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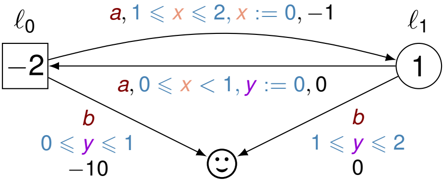

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When we fix two strategies

- Infinite Markov Chain

Stochastic strategies

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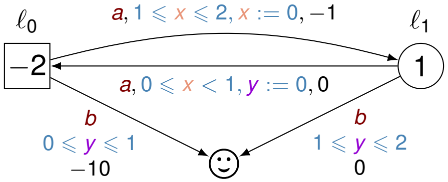

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When we fix two strategies

- Infinite Markov Chain
- Replace $\mathbf{SP}(\mathrm{Play}(c, \eta, \theta))$ by $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$

η Min θ Max

Existence of the expectation: $\mathbb{E}_c^{\eta,\theta}(\mathbf{SP})$ $\quad$ η Min $\quad$ θ Max

$\eta, \theta : C^* C \to \Delta(C)$

Distribution over possible choices

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Expectation of **SP** in a path

$$\mathbb{E}_c^{\eta,\theta}(a\,\pi) = \int_{t \in I(c,a)} \eta_E(c)(a) \left[(t\,\mathrm{wt}(c) + \mathrm{wt}(a))\, \mathbb{P}_{c_1}^{\eta,\theta}(\pi) + \mathbb{E}_{c_1}^{\eta,\theta}(\pi)\right] \mathrm{d}\eta_{\mathbb{R}^+}(c, a)(t)$$

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Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond ☺) = 1$

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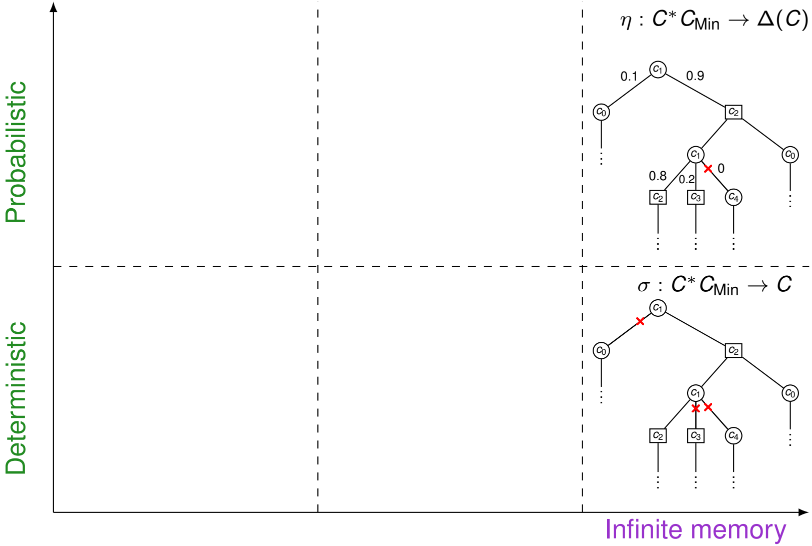
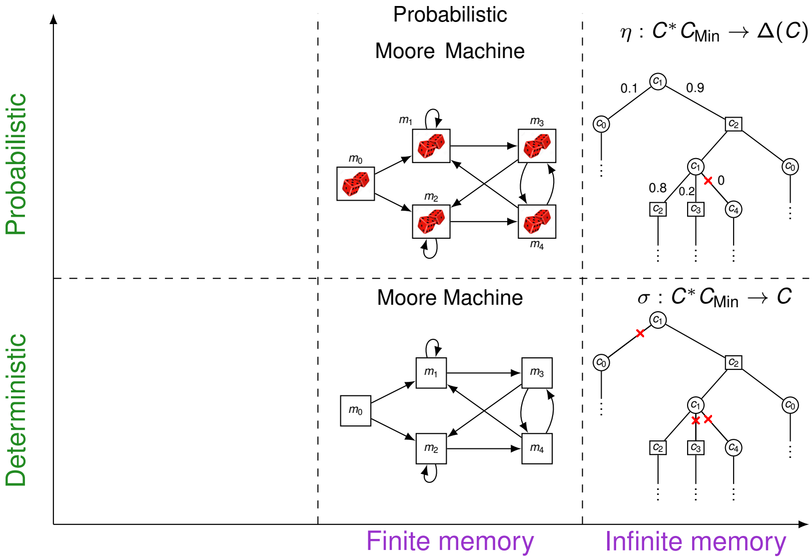
Restrictions on strategies for Min

- For all θ, $\mathbb{P}_c^{\eta,\theta}(\diamond ☺) = 1$
- ☺ must be reached quickly enough

Zoology of strategies



Zoology of strategies



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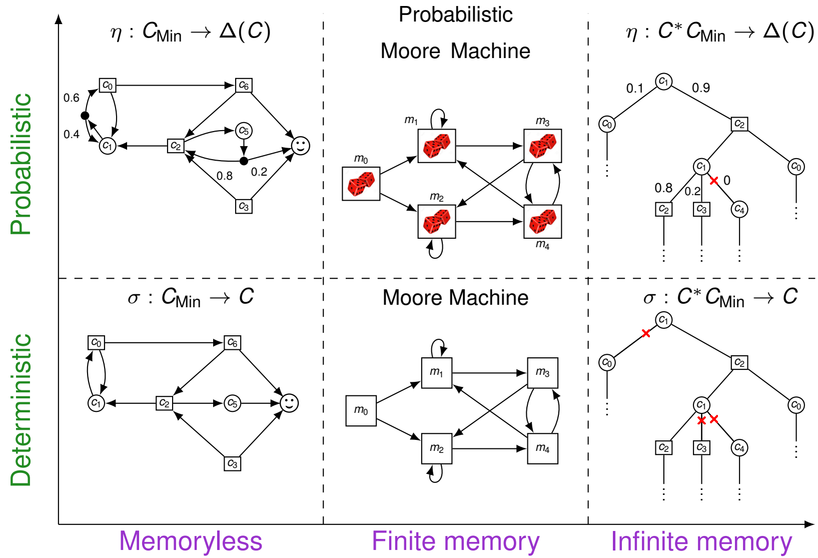
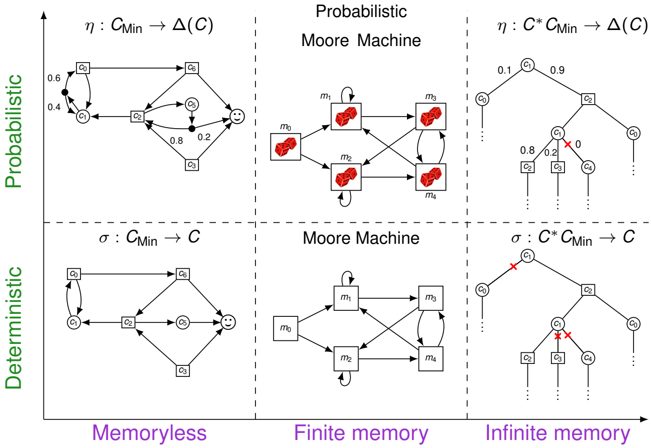


Stochastic values



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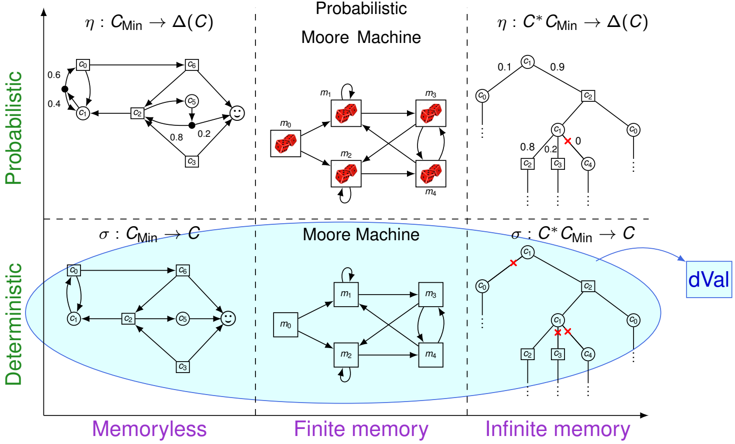

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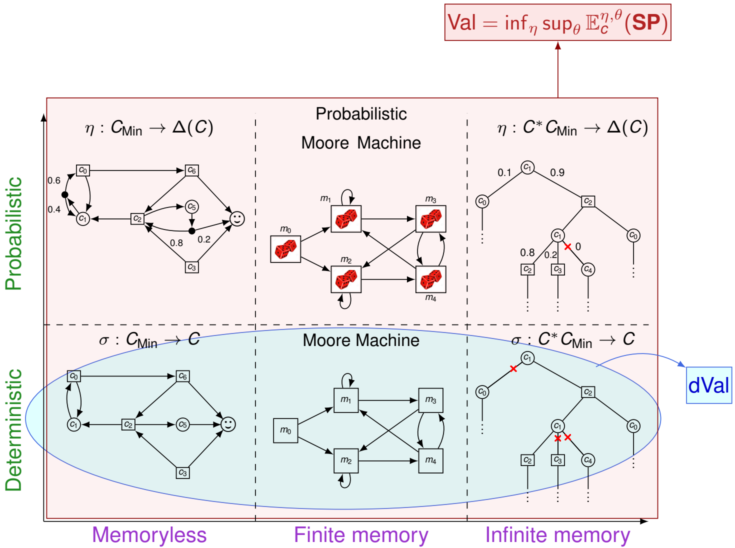

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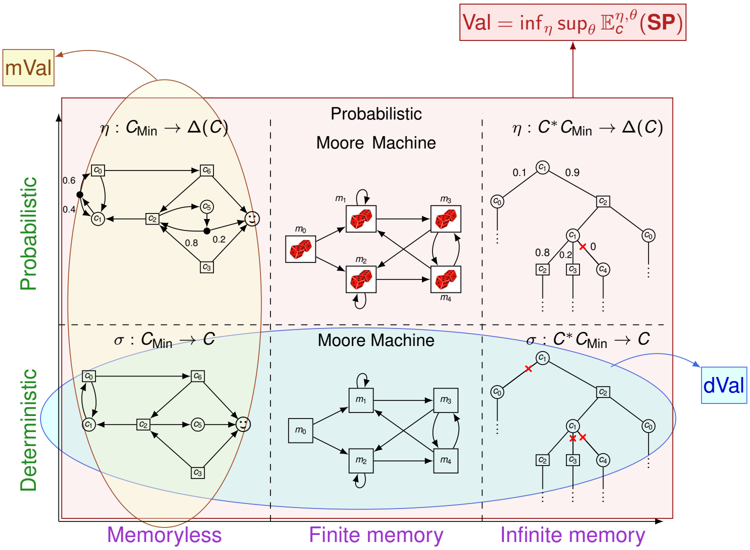

Contribution

$$\text{dVal} \quad = \quad \text{Val} \quad = \quad \text{mVal}$$

Contribution

Trade-off between memory and randomness

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Contribution

Trade-off between memory and randomness

- Stochastic games with qualitative objectives

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Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

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Trade-off between memory and randomness

- Stochastic games with qualitative objectives
- Reachability Timed Games

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Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

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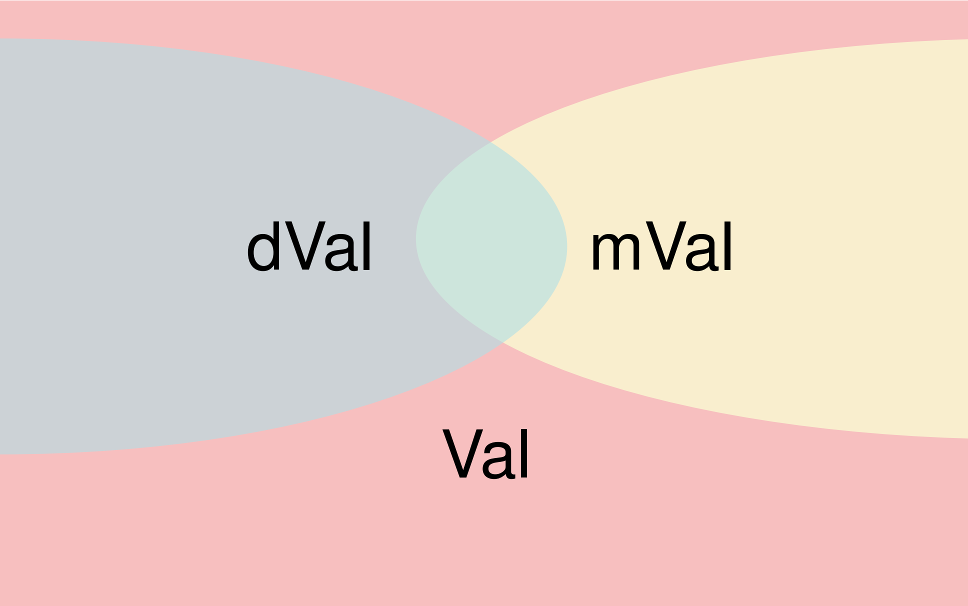

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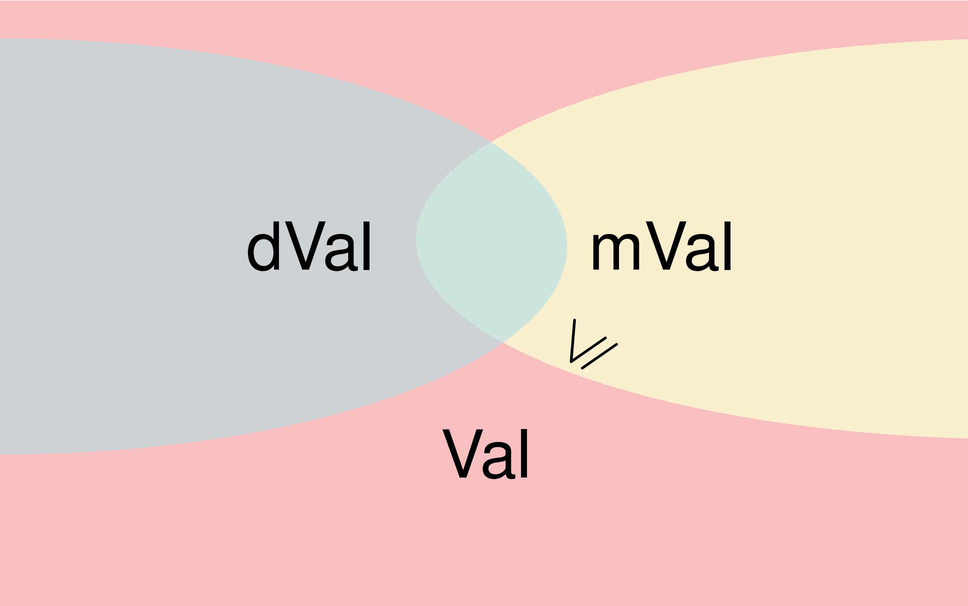

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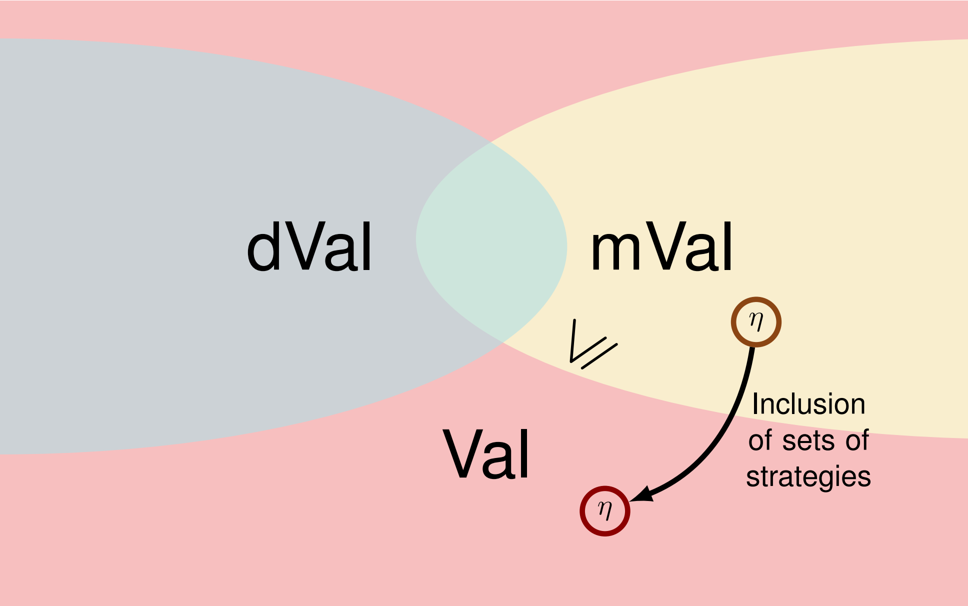

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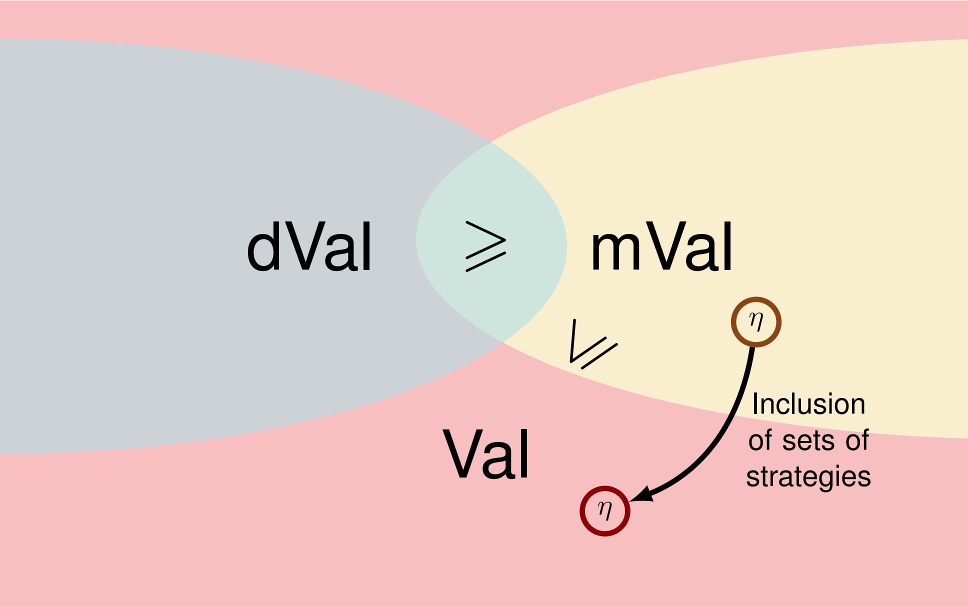

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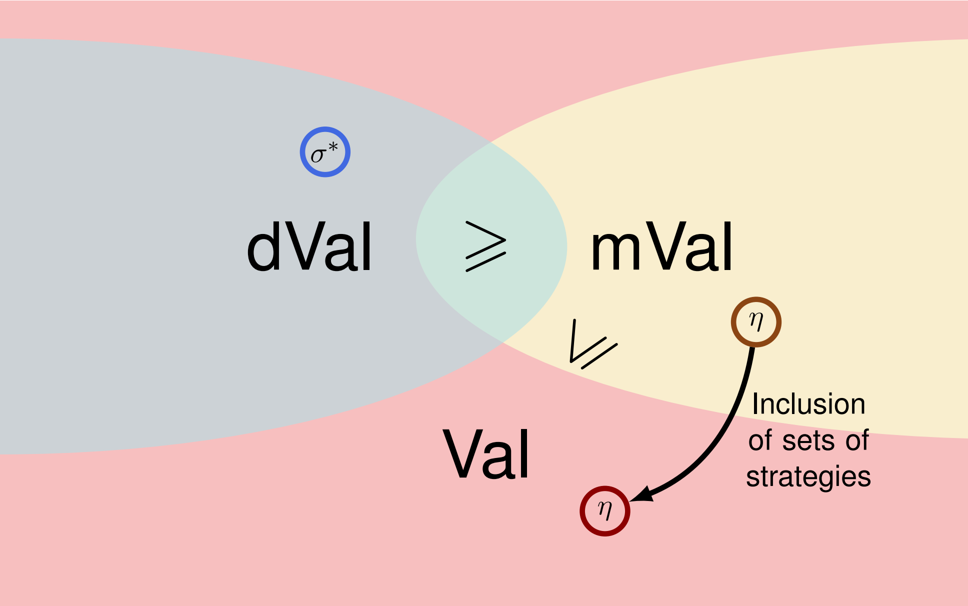

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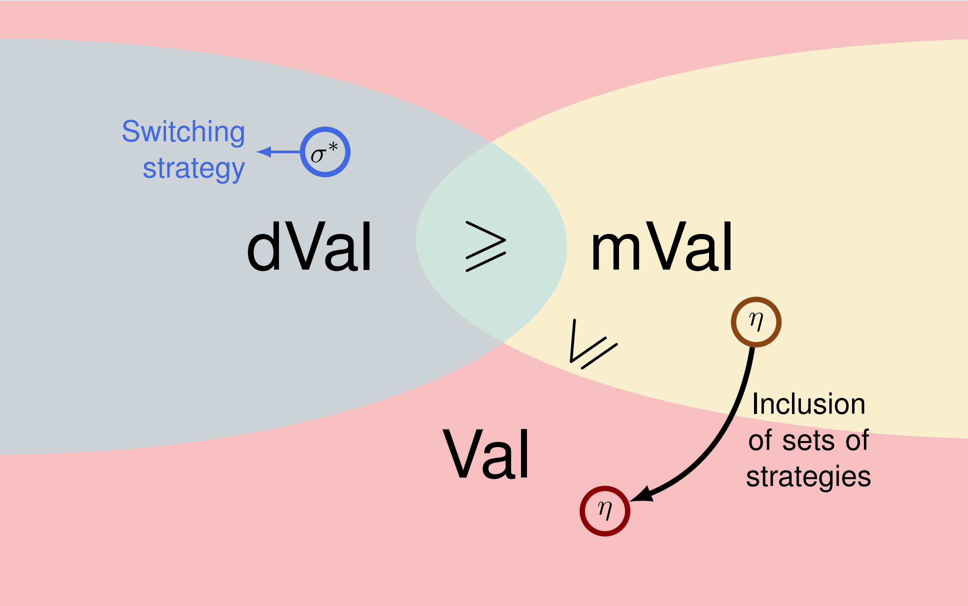

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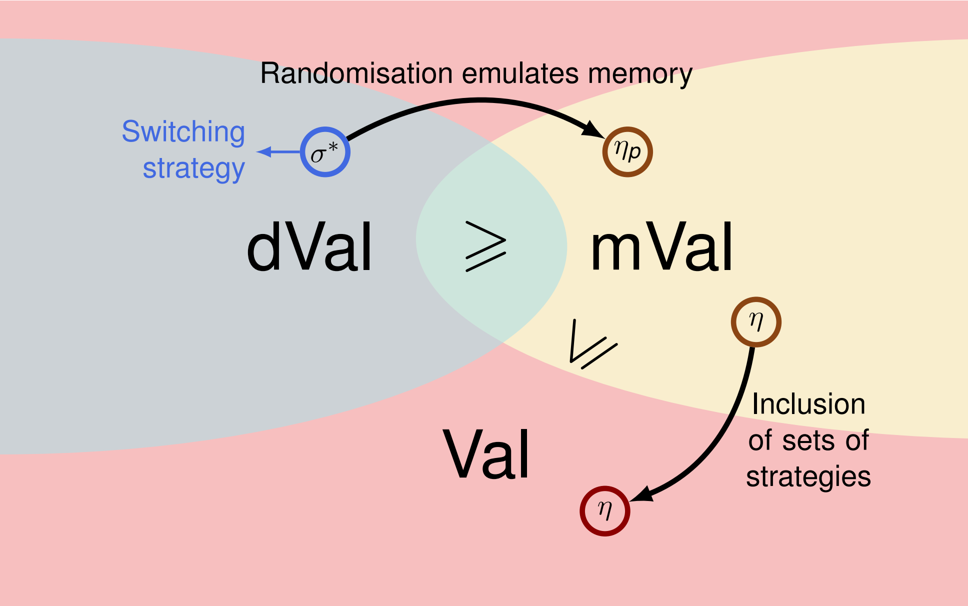

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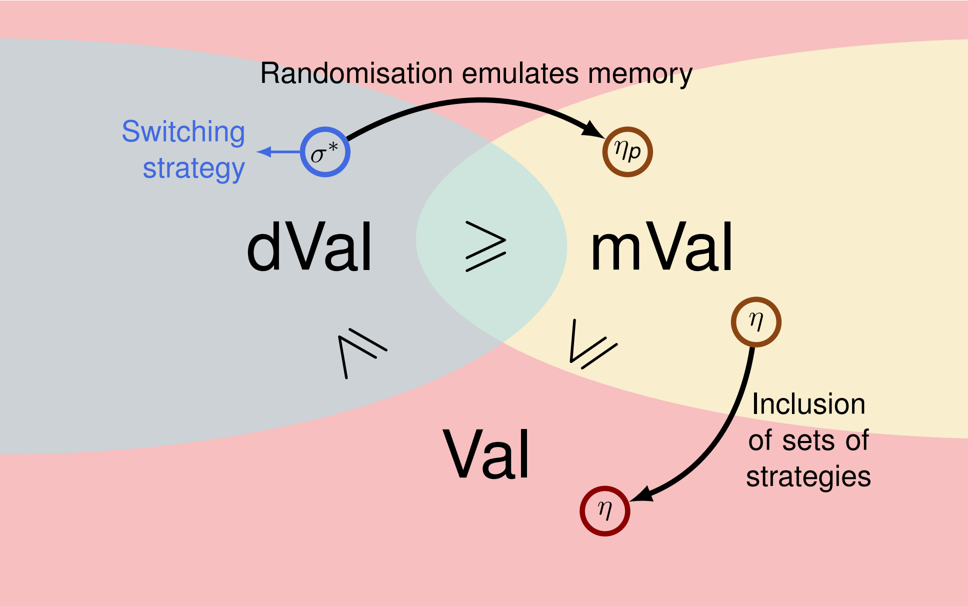

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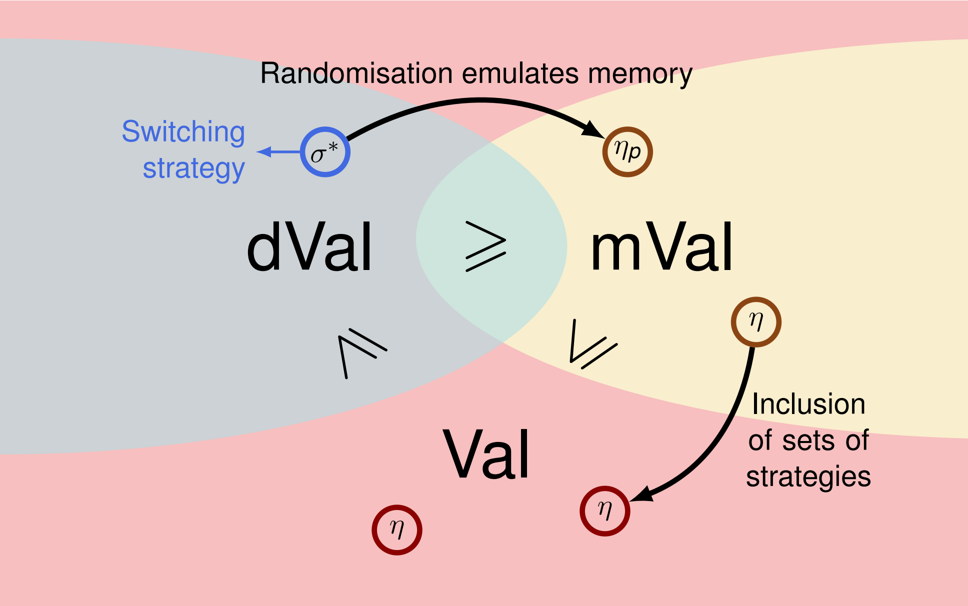

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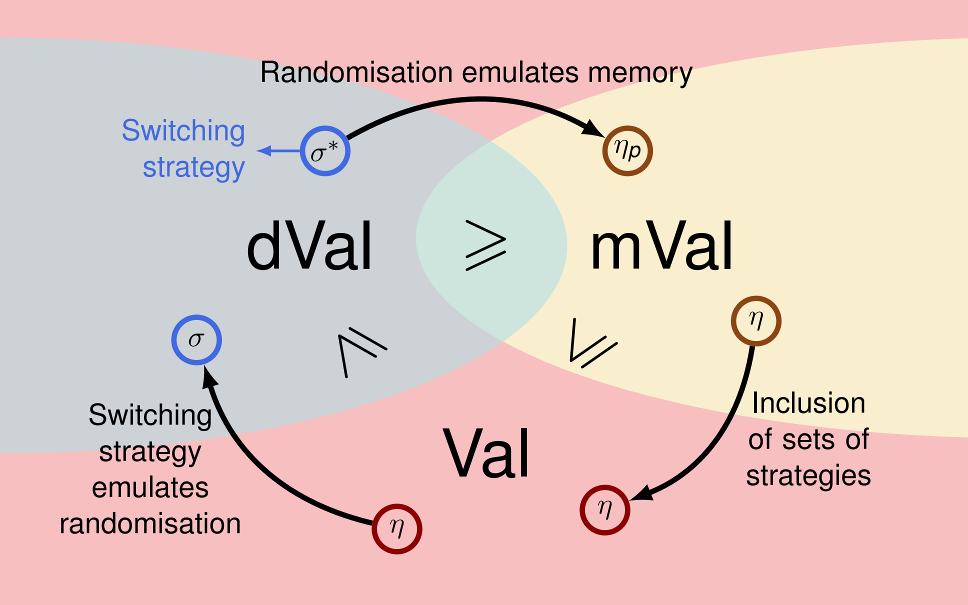

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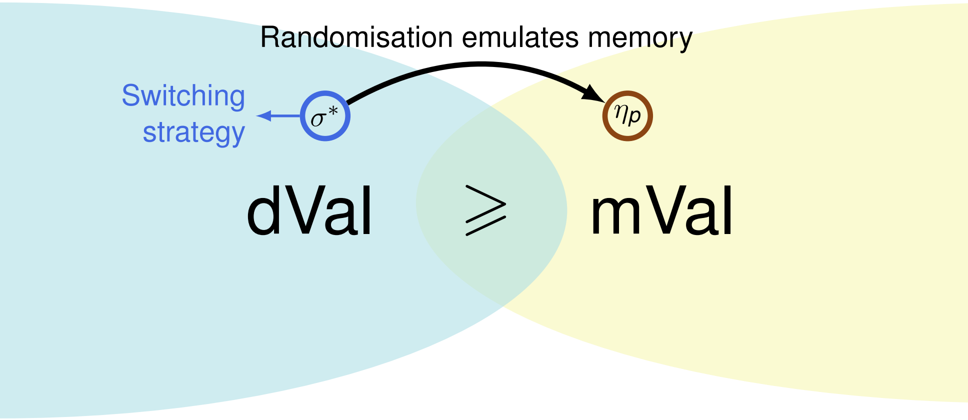
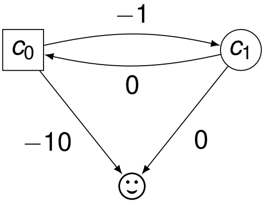


Randomisation emulates memory



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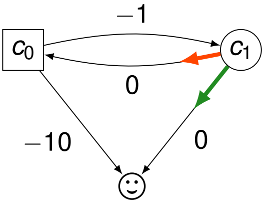

Strategy η_p

Let $\langle \sigma_1, \sigma_2, K \rangle$ be an optimal switching strategy,

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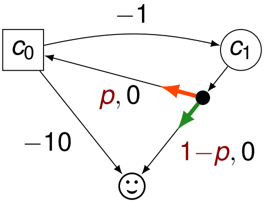

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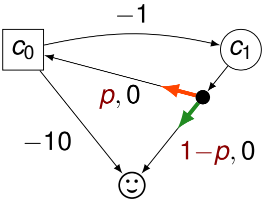

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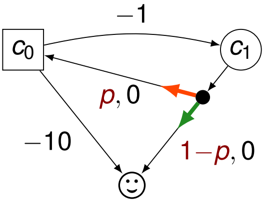
Properties of η_p

- For all θ, $\mathbb{P}_c^{\eta_p,\theta}(\diamond ☺) = 1$

Randomisation emulates memory

○ Min □ Max



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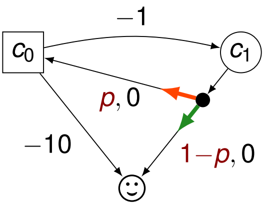

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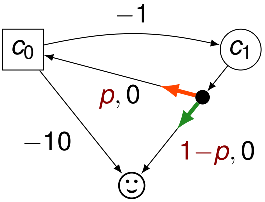
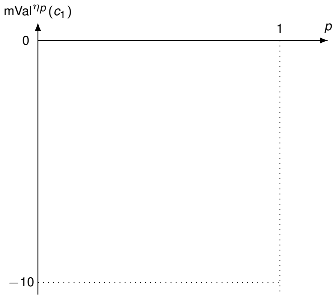


Computation of $\mathrm{mVal}^{\eta_p}(c_1)$



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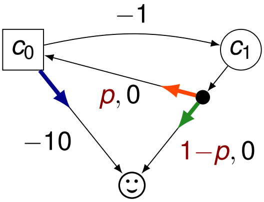

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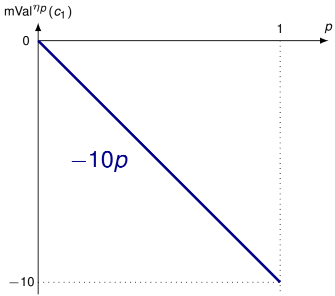

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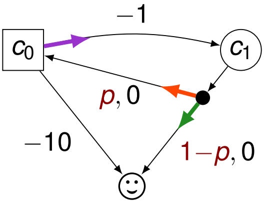

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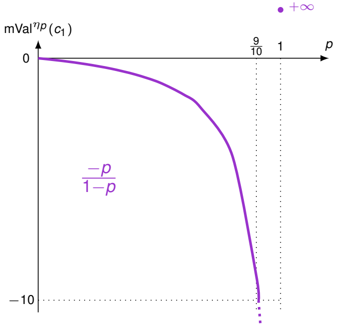

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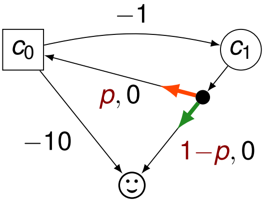

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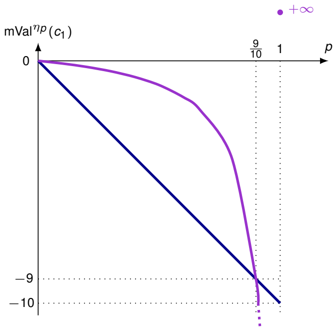

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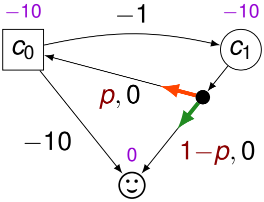

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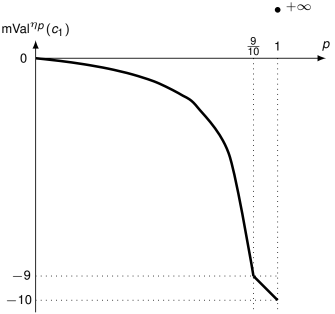

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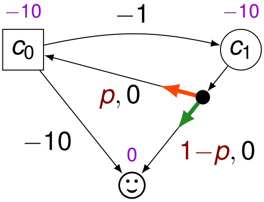

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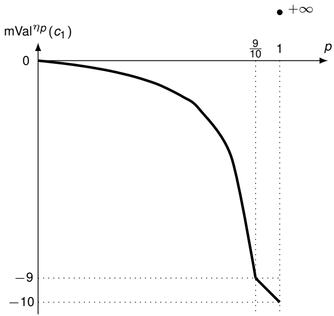

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Claim

For all c,

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E}_c^{\eta_p, \tau}(\mathbf{SP}) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$$

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Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

$$\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) = \sum_{\rho} \mathbf{SP}(\rho)\,\mathbb{P}(\rho)$$

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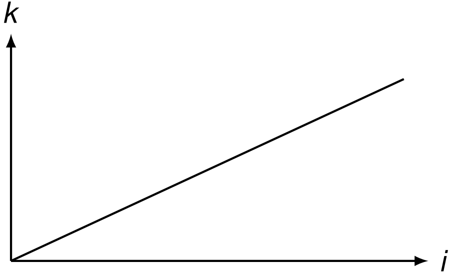

k size of play reaching the target

i number of choices given by σ_2

Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

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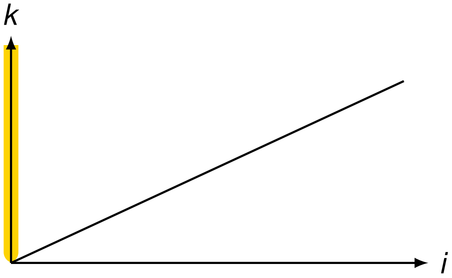

Yellow zone

All plays conforming to σ_1

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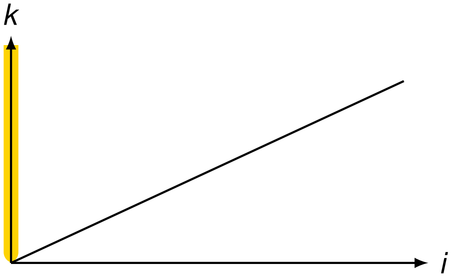

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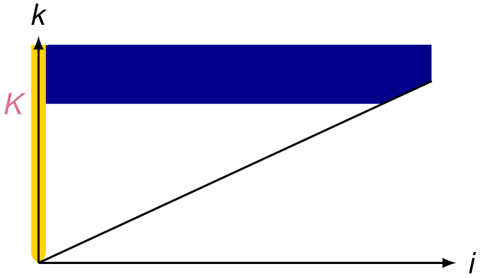

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Blue zone

Plays with many negative cycles

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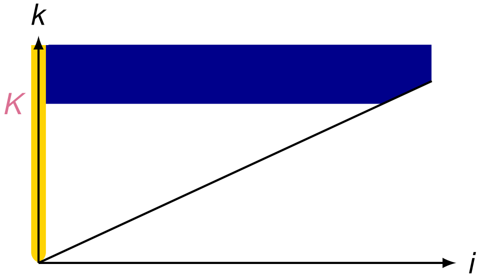

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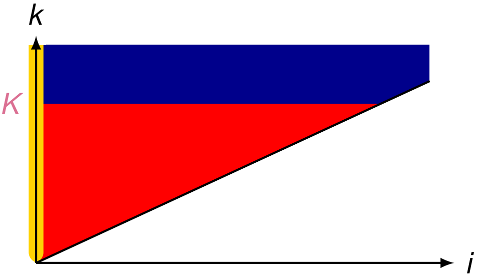

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Red zone

Rest of plays

Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

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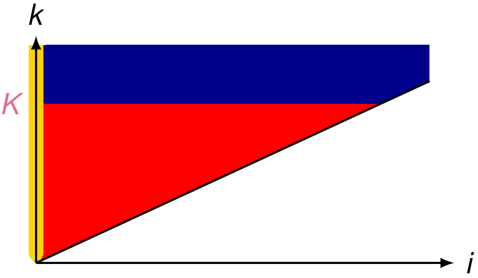

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Red zone

Rest of plays $\quad \mathbb{E} \underset{\substack{p \to 1 \\ p<1}}{\longrightarrow} 0$

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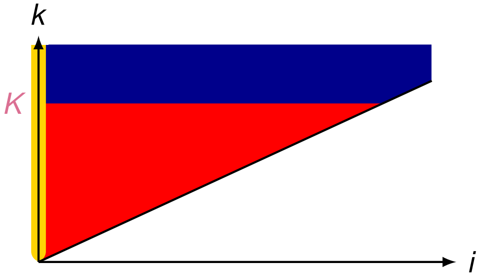

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All plays conforming to σ_1

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$

Blue zone

Plays with many negative cycles

$\mathbf{SP}(\rho) \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2, K \rangle}(c)$

$$\lim_{\substack{p \to 1 \\ p < 1}} \mathbb{E} + \mathbb{E} \leqslant \mathrm{dVal}^{\langle \sigma_1, \sigma_2 K \rangle}(c)$$

Red zone

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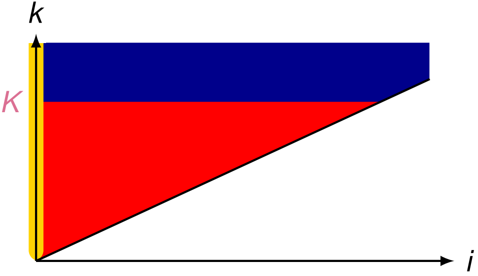
Computation of the expectation $\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP})$

$$\mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) = \sum_\rho \mathbf{SP}(\rho)\,\mathbb{P}(\rho) = \mathbb{E} + \mathbb{E} + \mathbb{E} \Rightarrow \lim_{\substack{p\to 1\\ p<1}} \mathbb{E}_c^{\eta_p,\tau}(\mathbf{SP}) \leqslant \mathrm{dVal}^{\langle \sigma_1,\sigma_2,K\rangle}(c)$$



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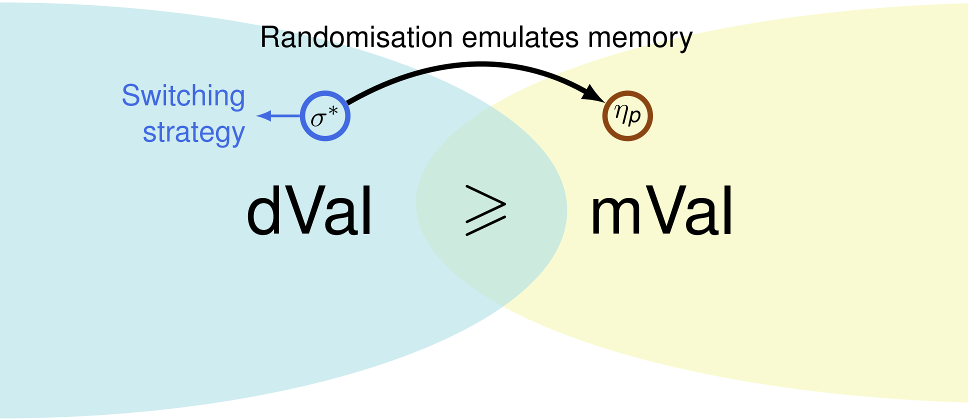
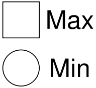
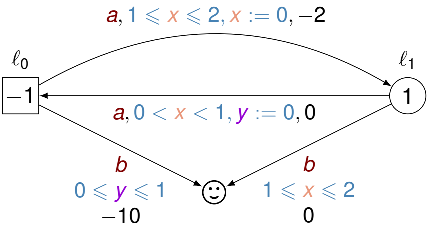
Contribution



Existence of an ε-optimal switching strategy





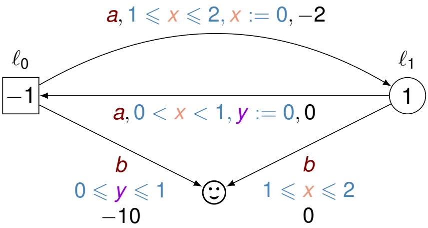
Switching strategy

- σ_1: reach cycle with a weight $\leqslant -1$
- σ_2: reach ☺
- K: number of turns before switch

Existence of an ε-optimal switching strategy

□ Max

○ Min



Divergent weighted timed game

All SCCs contain only cycles with a weight $\leqslant -1$ or $\geqslant 1$

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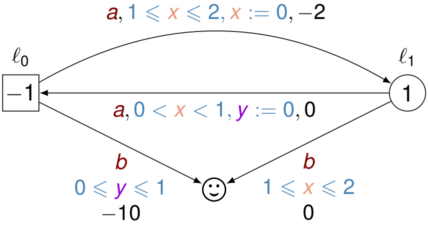
Optimal Reachability in Divergent Weighted Timed Games, D. Busatto-Gaston, B. Monmege and P.-A. Reynier, 2017, ETAPS

Existence of an ε-optimal switching strategy

Max

Min



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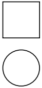
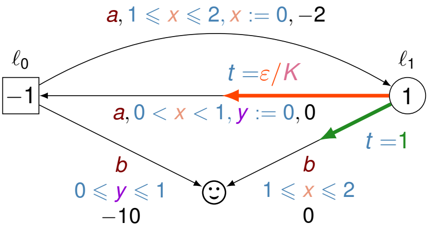
Theorem

Min has an ε-optimal switching strategy

Existence of an ε-optimal switching strategy





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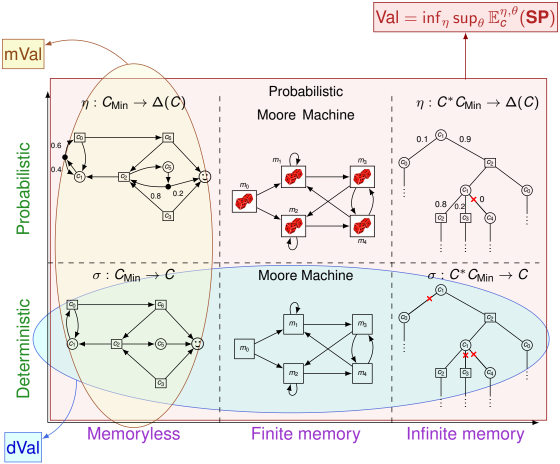
Min has an ε-optimal switching strategy : $\langle \sigma_1, \sigma_2, K \rangle$

Summary



- Definition of $\mathbb{P}_c^{\eta,\theta}(\pi)$
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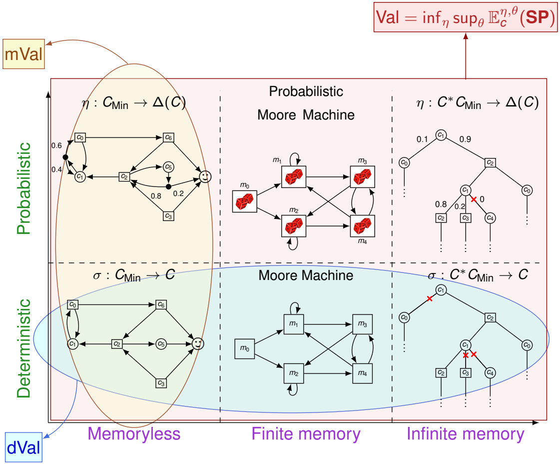
Summary



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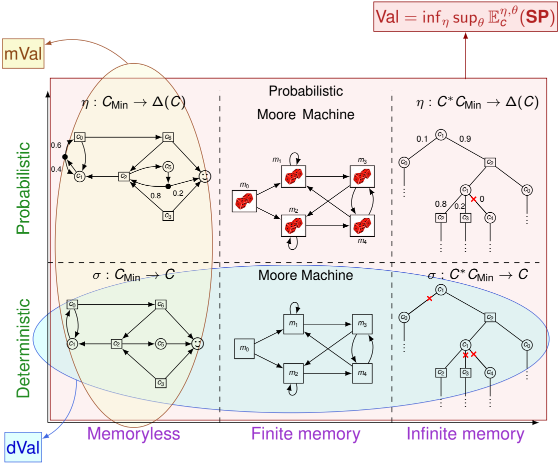
Summary



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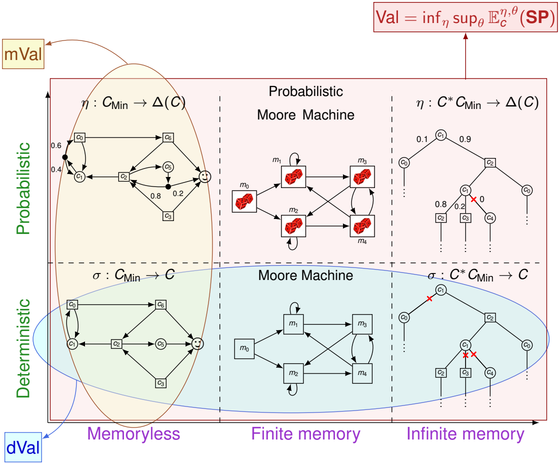
For which classes of games?

- Finite shortest path games

Theorem: $\mathsf{Val} = \mathsf{dVal} = \mathsf{mVal}$

Summary



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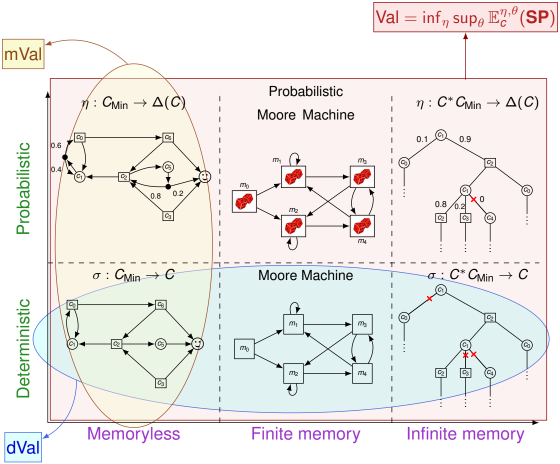
Summary: perspectives



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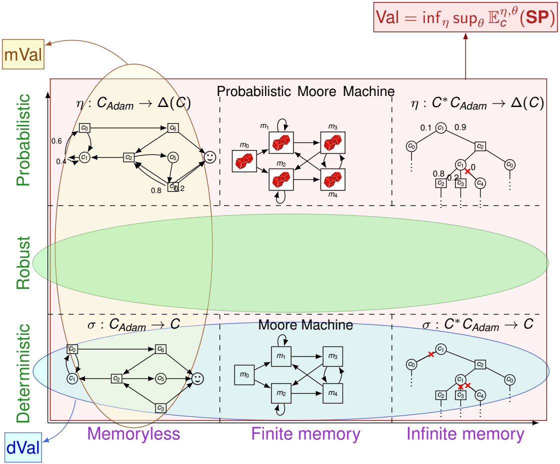

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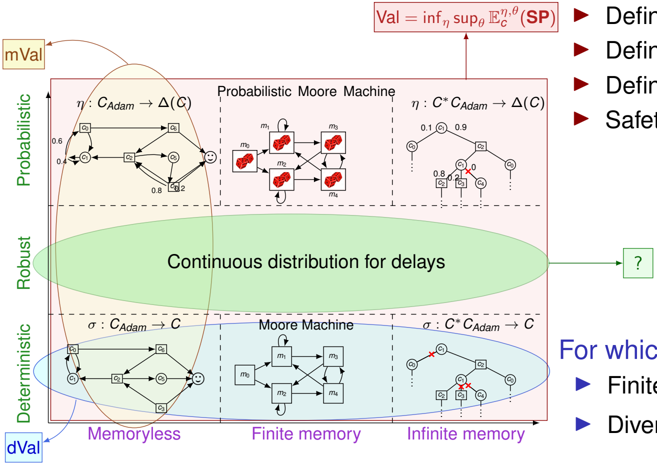
Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Summary: perspectives



- Definition of $\mathbb{P}_c^{\eta,\theta}(\pi)$
- Definition of $\mathbb{E}_c^{\eta,\theta}(\pi)$
- Definition of $\mathbb{E}_c^{\eta,\theta}(\textbf{SP})$
- Safety conditions on strategies

For which classes of games?

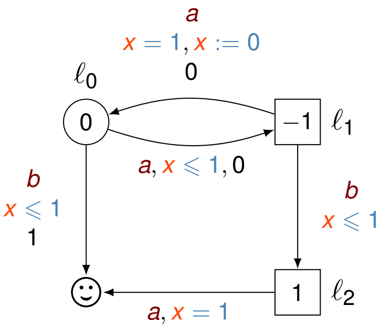
- Finite shortest path games
- Divergent weighted timed games

Theorem: $\text{Val} = \text{dVal} = \text{mVal}$

Trading Infinite Memory for Uniform Randomness in Timed Games, K. Chatterjee, T. Henzinger and S. Vinayak, 2008, HSCC

Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

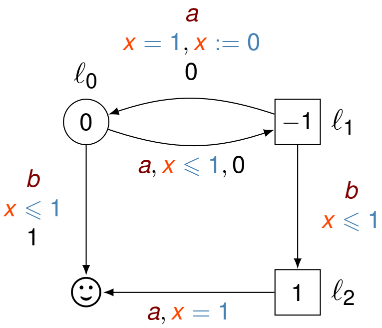


Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

Value Problem

Decide if $\mathsf{dVal}(c) \leqslant \lambda$?



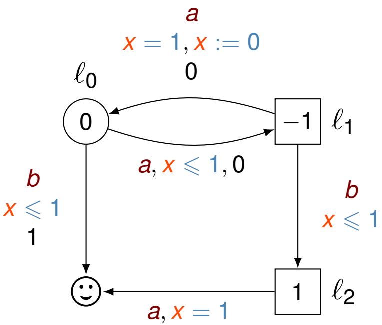
Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

State of the art

Value Problem

Decide if $\mathsf{dVal}(c) \leqslant \lambda$?



Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

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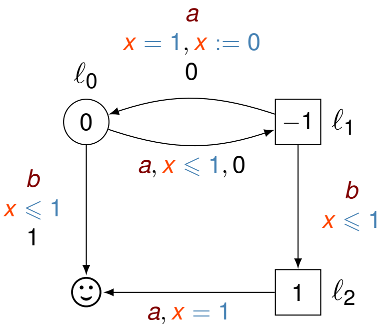


State of the art

☹ Undecidable for at least two clocks

On Optimal Timed Strategies, T. Brihaye, V. Bruyère and J.-F. Raskin, 2005, FORMATS

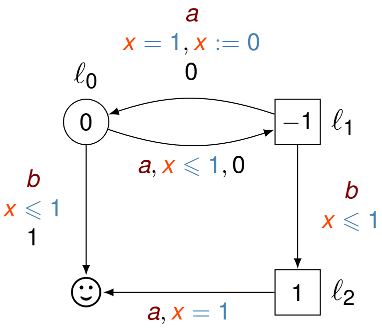
On the Value Problem in Weighted Timed Games, P. Bouyer, S. Jaziri, and N. Markey, 2015, CONCUR.

Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



State of the art

- ☹ Undecidable for at least two clocks
- ☺ Decidable for 1-clock with non-negative weights

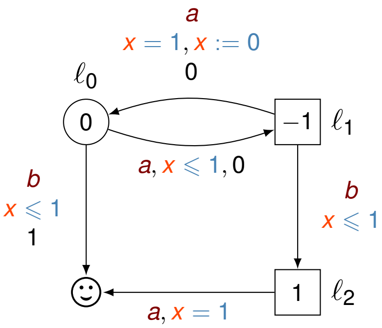
Almost Optimal Strategies in One Clock Priced Timed Games, P. Bouyer, K. Larsen, N. Markey, and J. Rasmussen, 2006, FSTTCS

Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



State of the art

- ☹ Undecidable for at least two clocks
- ☺ Decidable for 1-clock with non-negative weights
- ☺ Decidable for 1-clock but without reset cycles

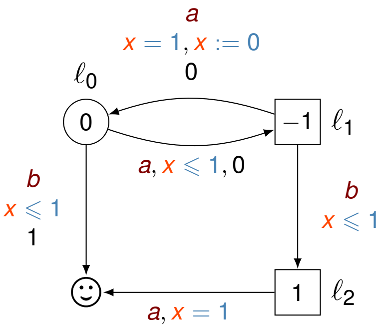
Simple Priced Timed Games Are Not That Simple, T. Brihaye, G. Geeraerts, A. Haddad, E. Lefaucheux, and B. Monmege, 2015, FSTTCS

Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



State of the art

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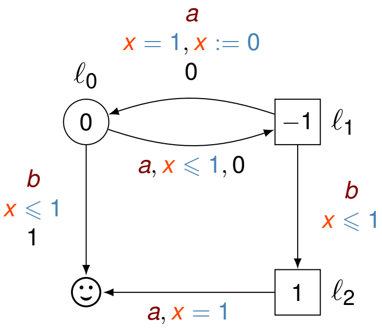
Decidable in 1-clock Weighted Timed Games

Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



State of the art

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Decidable in 1-clock Weighted Timed Games

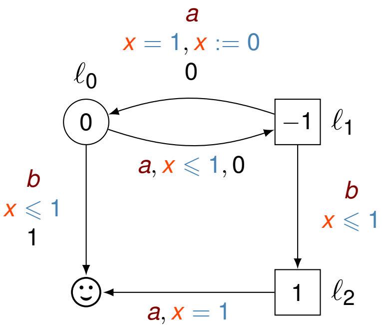
$c \mapsto \mathrm{dVal}(c)$ is computable in exponential time

Value problem in 1-clock Weighted Timed Games

Ongoing work: submitted

Value Problem

Decide if $\mathrm{dVal}(c) \leqslant \lambda$?



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Decidable in 1-clock Weighted Timed Games

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Thank you! Questions?