Robustness in Weighted Timed Games

Work in progress

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ANR Ticktac meeting
June 9, 2023

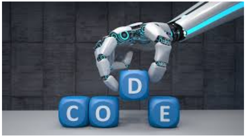
Motivation: game theory for synthesis

Classical approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents: environment and controller

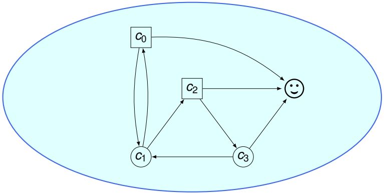


Code synthesis

Correct by construction: synthesis of controller

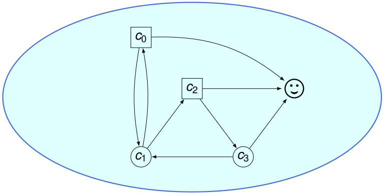
Different classes of games

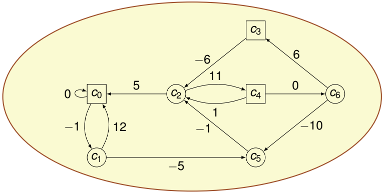
Qualitative games



Different classes of games

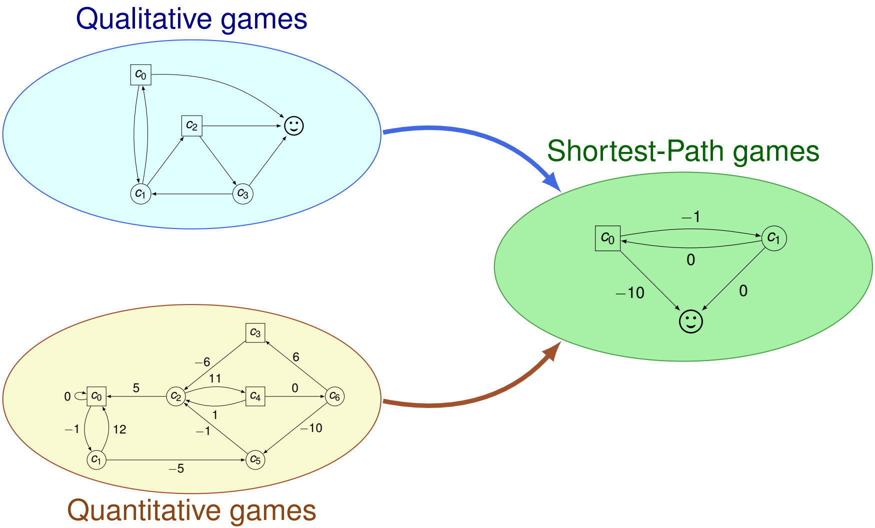
Qualitative games



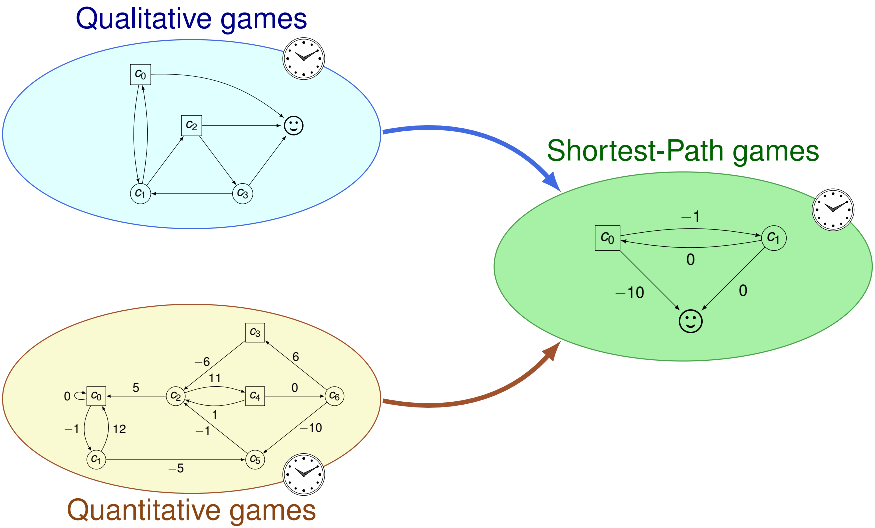


Quantitative games

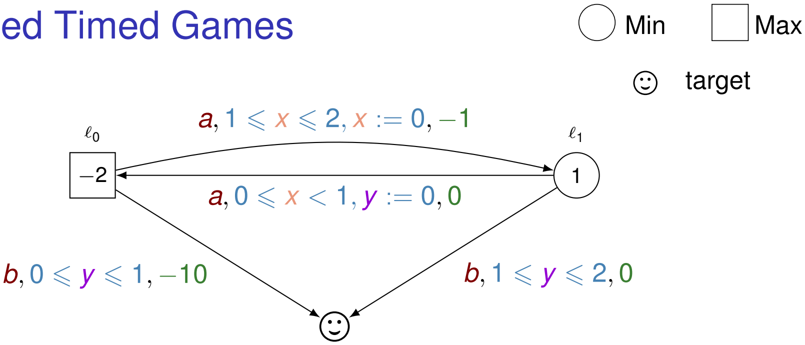
Different classes of games



Different classes of games



Weighted Timed Games



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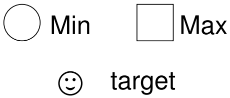
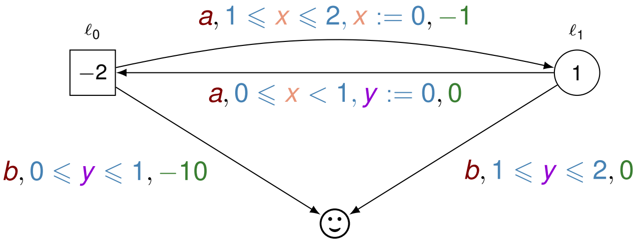
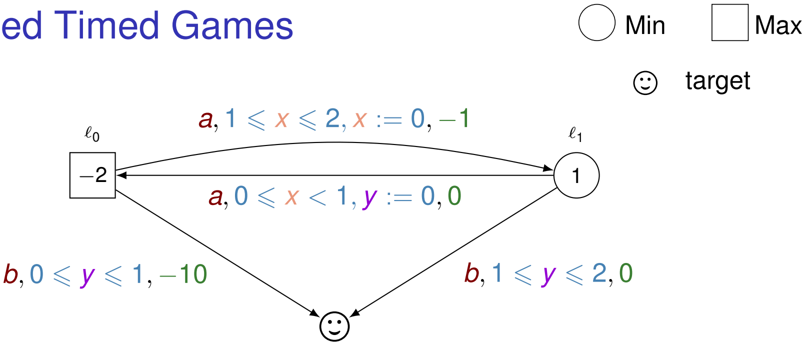




Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix})$$

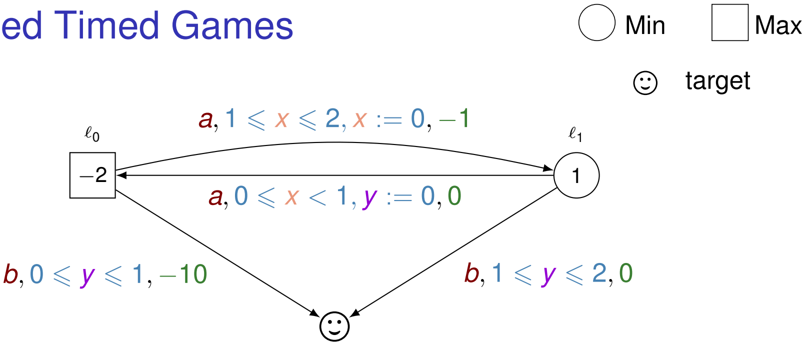
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a}$$

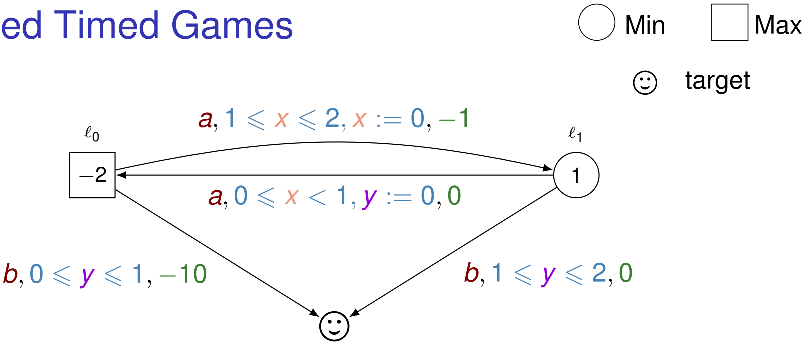
Weighted Timed Games



Play ρ

$$(\ell_1, \begin{bmatrix} 0 \\ 0 \end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix} 0.5 \\ 0 \end{bmatrix})$$

Weighted Timed Games



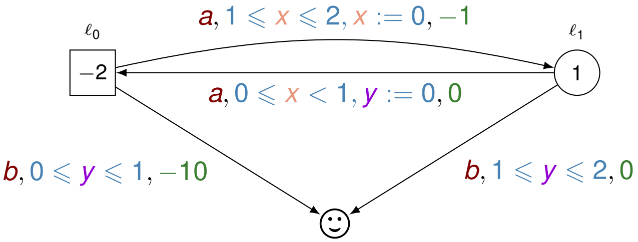
Play ρ

$$(\ell_1, \begin{bmatrix}0\\0\end{bmatrix}) \xrightarrow{0.5,\ a} (\ell_0, \begin{bmatrix}0.5\\0\end{bmatrix}) \xrightarrow{1.25,\ a} (\ell_1, \begin{bmatrix}0\\1.25\end{bmatrix}) \xrightarrow{1/3,\ b} (☺, \begin{bmatrix}1/3\\19/12\end{bmatrix})$$

Weighted Timed Games

○ Min ☐ Max

☺ target



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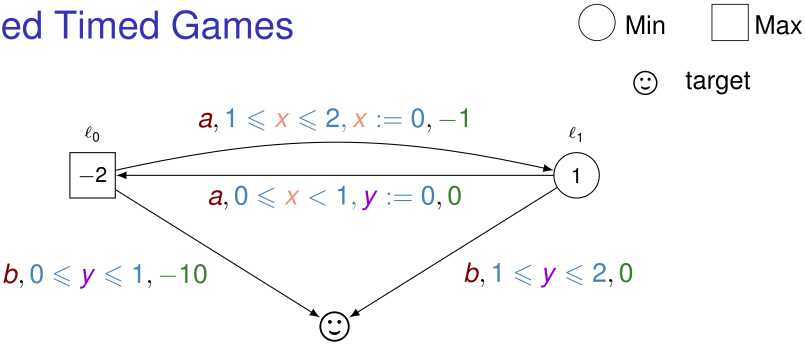
$+ \qquad\qquad +$

Weighted Timed Games



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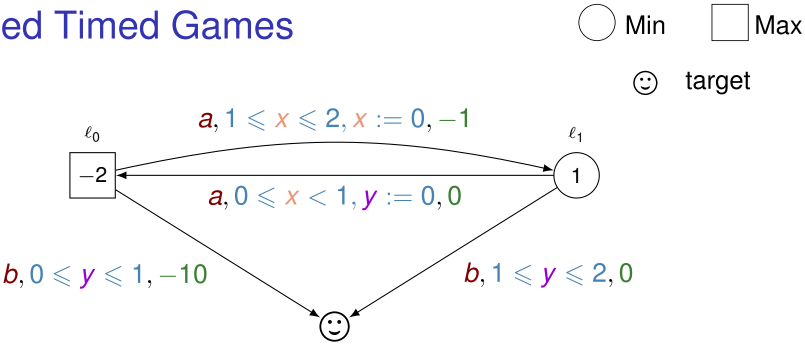
$$1 \times 0.5 + 0 \quad + \qquad\qquad + $$

Weighted Timed Games



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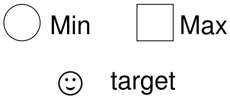
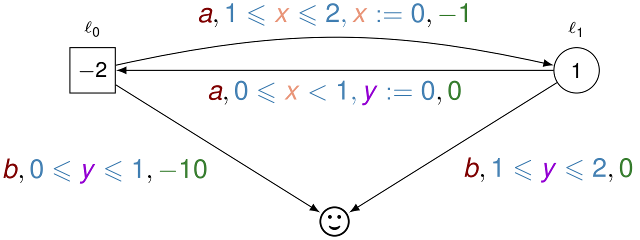
$$1 \times 0.5 + 0 \quad + \quad -2 \times 1.25 - 1 \quad + \quad 1 \times \tfrac{1}{3} + 0$$

Weighted Timed Games





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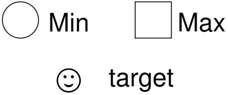
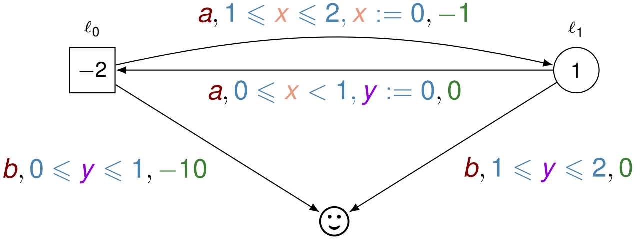
Deterministic strategy

Choose an edge and a delay

Weighted Timed Games





Play ρ

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Deterministic strategy

Choose an edge and a delay

In $(\ell_1, (0, 0))$

Choose a with $t = \frac{1}{3}$

Deterministic value problem

σ Min τ Max

Deciding if $\mathsf{dVal}(c) \leqslant \lambda$?

Deterministic value problem

σ Min τ Max

Deciding if dVal(c) $\leqslant \lambda$?

Deterministic value

$$\mathrm{dVal}(c) = \inf_{\sigma} \sup_{\tau} \mathrm{Payoff}(\mathrm{Play}(c, \sigma, \tau))$$

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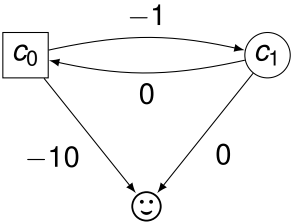

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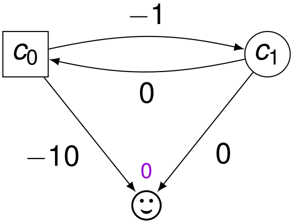

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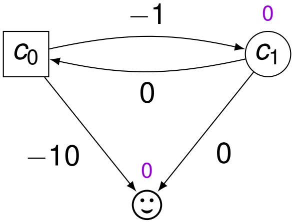

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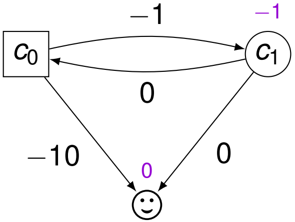

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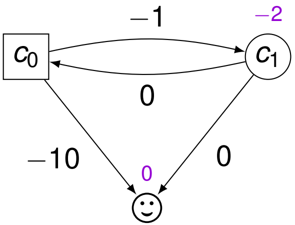

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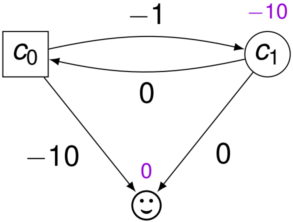

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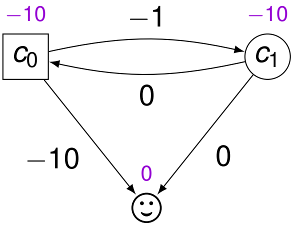

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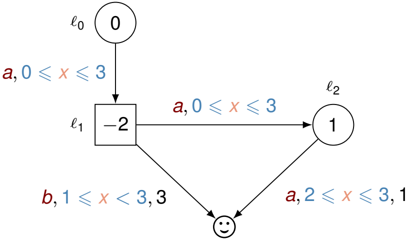

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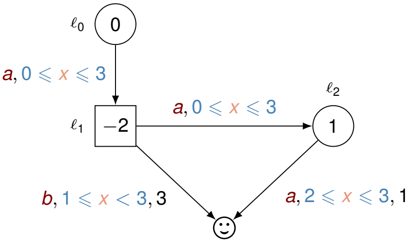

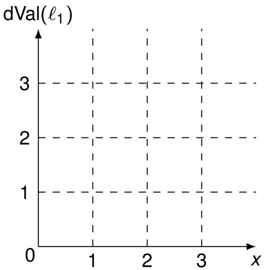

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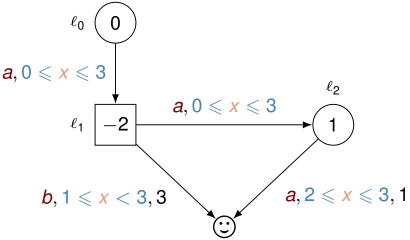

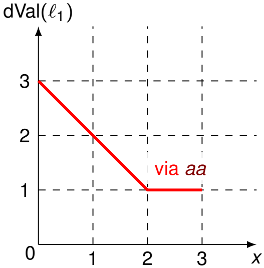

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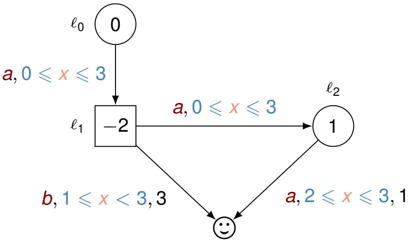

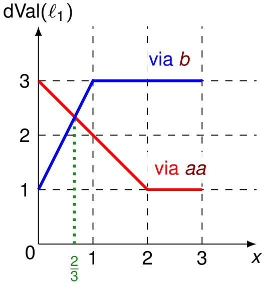

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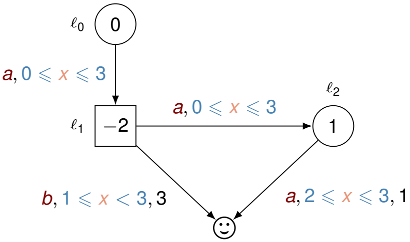

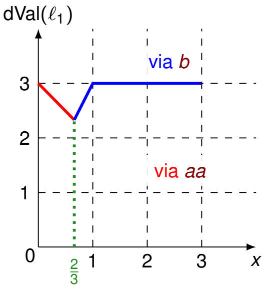

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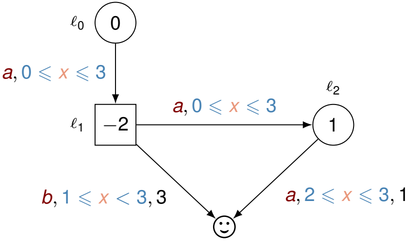
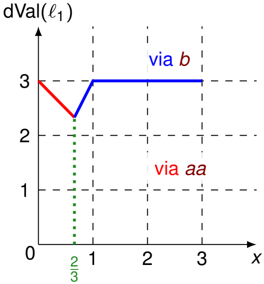
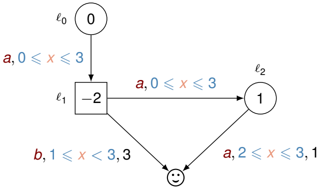


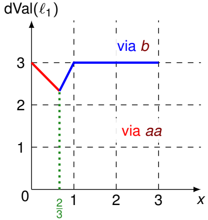


Optimal play

$$(\ell_0, 0) \xrightarrow{2/3,\ a} (\ell_1, \tfrac{2}{3}) \xrightarrow{1/3,\ b} (☺, 1)$$

Robustness in weighted timed games





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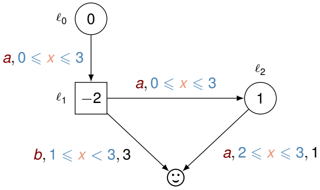
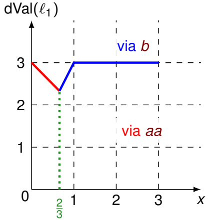
Robustness in weighted timed games

Min Max





Model too precise

Optimal play

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Robustness in weighted timed games

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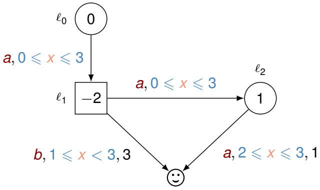

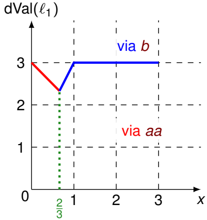

Model too precise

Give to Max the power to perturb the delay chosen by Min

Optimal play

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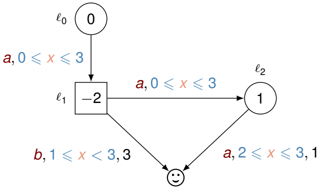
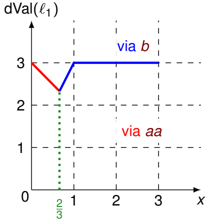
Robustness in weighted timed games

Min Max





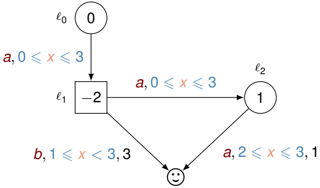
Model too precise

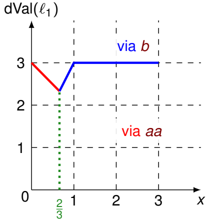
Give to Max the power to perturb the delay chosen by Min

Excessive semantics

Check the guard before
the perturbation

Robustness in weighted timed games





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Excessive play

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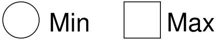

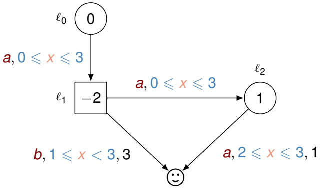

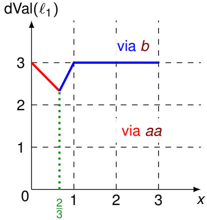

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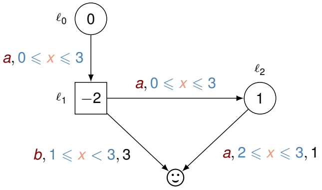
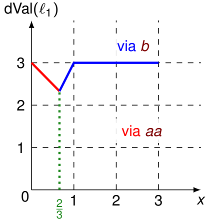
Min Max





Model too precise

Give to Max the power to perturb the delay chosen by Min

Excessive semantics

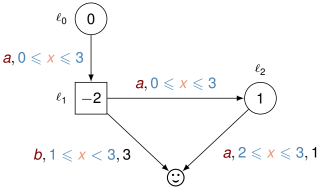
Check the guard before the perturbation

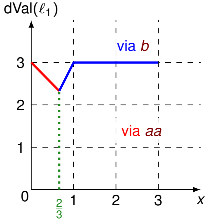
Excessive play

$$(\ell_0, 0) \xrightarrow{2/3,\ a} \overset{1/3}{\rightsquigarrow} (\ell_1, 1) \xrightarrow{0,\ b} (\smile, 1)$$

Robustness in weighted timed games

Min Max





Model too precise

Give to Max the power to perturb the delay chosen by Min

Excessive semantics

Check the guard before the perturbation

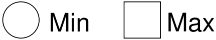
Conservative semantics

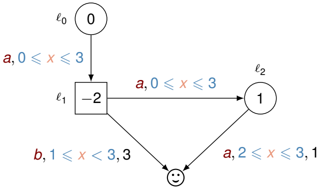
Check the guard after the perturbation

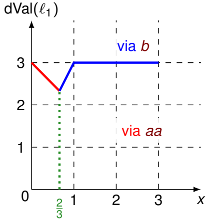
Excessive play

$$(\ell_0, 0) \xrightarrow{2/3,\ a} \overset{1/3}{\rightsquigarrow} (\ell_1, 1) \xrightarrow{0,\ b} (\text{☺}, 1)$$

Robustness in weighted timed games







Model too precise

Give to Max the power to perturb the delay chosen by Min

Excessive semantics

Check the guard before the perturbation

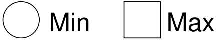
Conservative semantics

Check the guard after the perturbation

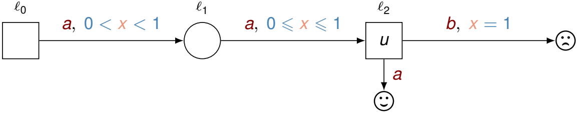
Excessive play and conservative

$$(\ell_0, 0) \xrightarrow{2/3,\ a} \overset{1/3}{\rightsquigarrow} (\ell_1, 1) \xrightarrow{0,\ b} (☺, 1)$$

Robust reachability



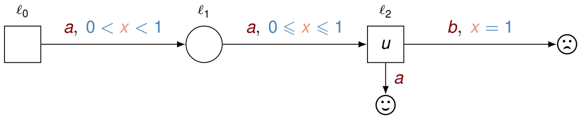
Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

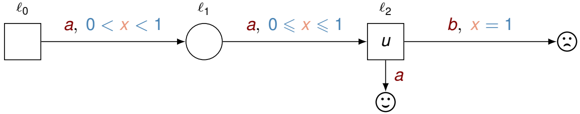


Exact play

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

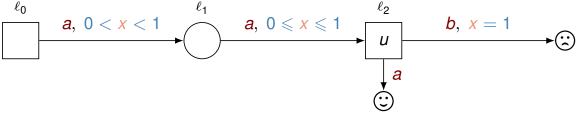


Exact play $(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

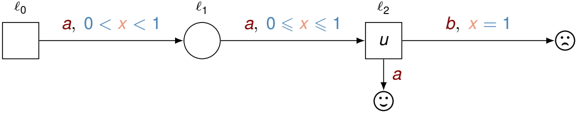


Exact play $(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Robust reachability

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



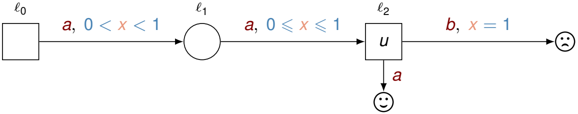
Exact play $(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play

Robust reachability

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



Exact play $(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

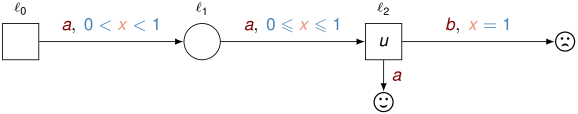
winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1 - \delta)$

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



Exact play

$(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play

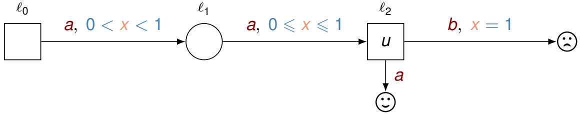
$(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

winning strategy for Max: reach ℓ_1 in at least $1 - \delta$

Robust reachability

◯ Min ▢ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



Exact play $(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

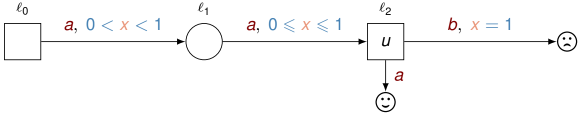
winning strategy for Max: reach ℓ_1 in at least $1 - \delta$

Excessive play

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



Exact play $(\ell_0, 0) \xrightarrow{0.75,\ a} (\ell_1, 0.75) \xrightarrow{0,\ b} (\ell_2, 0.75) \xrightarrow{0,\ b} (☺, 0.75)$

winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

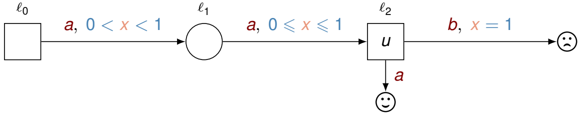
winning strategy for Max: reach ℓ_1 in at least $1-\delta$

Excessive play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



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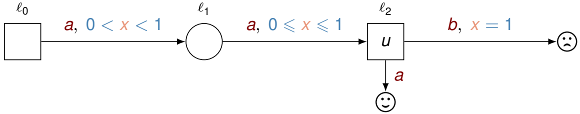
Excessive play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta) \xrightarrow{0,\ a}$

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



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winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

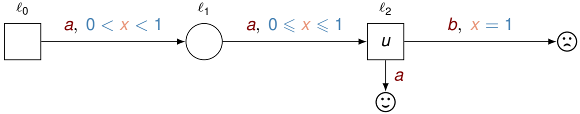
winning strategy for Max: reach ℓ_1 in at least $1-\delta$

Excessive play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta) \xrightarrow{0,\ a} \overset{\delta}{\rightsquigarrow} (\ell_2, 1)$

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



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winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

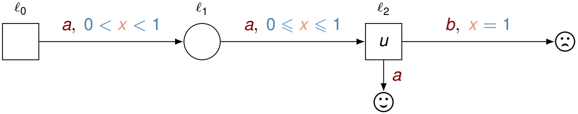
winning strategy for Max: reach ℓ_1 in at least $1-\delta$

Excessive play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta) \xrightarrow{0,\ a} \overset{\delta}{\rightsquigarrow} (\ell_2, 1) \xrightarrow{0,\ b} (☹, 1)$

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?



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winning strategy for Min: $\sigma(\ell_1, \nu) = (0,\ a)$

Conservative play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta)$

winning strategy for Max: reach ℓ_1 in at least $1-\delta$

Excessive play $(\ell_0, 0) \xrightarrow{1-\delta,\ a} (\ell_1, 1-\delta) \xrightarrow{0,\ a} \overset{\delta}{\rightsquigarrow} (\ell_2, 1) \xrightarrow{0,\ b} (☹, 1)$

winning strategy for Max: reach ℓ_1 in at least $1-\delta$, then ℓ_2 in 1

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c		
TG	EXPTIME-c		

A Theory of Timed Automata, R. Alur and D. Dill, 1994, Theoretical Computer Science

Reachability-Time Games on Timed Automata, M. Jurdziński and A. Trivedi, 2007, ICALP

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	
TG	EXPTIME-c		

Robust Controller Synthesis in Timed Automata, O. Sankur, P. Bouyer, N.s Markey, and PA. Reynier, 2013, CONCUR

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
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Robust Controller Synthesis in Timed Automata, O. Sankur, P. Bouyer, N.s Markey, and PA. Reynier, 2013, CONCUR

Robust Reachability in Timed Automata: Game-Based Approach, P. Bouyer, N. Markey, and O. Sankur, 2015, TCS

Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c		EXPTIME-c

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Robust reachability

○ Min □ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c	EXPTIME-c

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c hardness	EXPTIME-c

Probabilistic Robust Timed Game, Y. Oualhadj, PA. Reynier, and O. Sankur, 2014, CONCUR

Robust reachability

Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c hardness easiness	EXPTIME-c

Robust reachability

○ Min ☐ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c hardness easiness	EXPTIME-c

Encoding conservative semantics into excess one

Robust reachability

◯ Min ☐ Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c hardness easiness	EXPTIME-c

Encoding conservative semantics into excess one

Max controls a posteriori the delay chosen by Min

Robust reachability

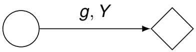
Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c hardness easiness	EXPTIME-c

Encoding conservative semantics into excess one

Max controls a posteriori the delay chosen by Min



Robust reachability

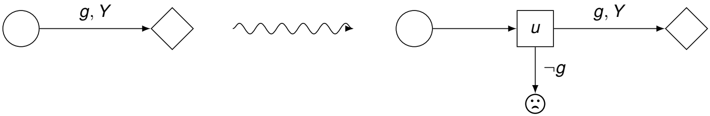
Min Max

Deciding if exists $\delta > 0$ such that Min reaches ☺ when Max perturbs with $[0, 2\delta]$?

	exact	conservative	excessive
TA	PSPACE-c	PSPACE-c	EXPTIME-c
TG	EXPTIME-c	EXPTIME-c hardness easiness	EXPTIME-c

Encoding conservative semantics into excess one

Max controls a posteriori the delay chosen by Min



Robust value problem

σ Min τ Max

Deciding if $\mathsf{rVal}(c) \leqslant \lambda$?

Robust value

$$\mathsf{rVal}(c) = \inf_{\sigma} \sup_{\tau} \mathsf{Payoff}(\mathsf{Play}(c, \sigma, \tau))$$

Robust value problem

χ Min ζ Max

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

Robust value

$$\mathrm{rVal}(c) = \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

Robust value problem

χ Min ζ Max

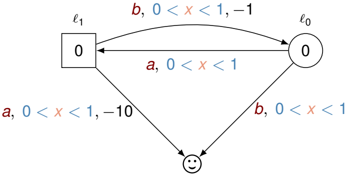
Deciding if rVal(c) $\leqslant \lambda$?

Robust value

$$\text{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \text{Payoff}(\text{Play}(c, \chi, \zeta))$$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

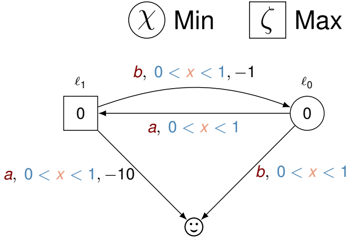


Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



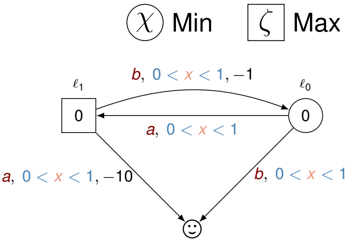
Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \; \inf_{\substack{\chi \\ \delta\text{-robust}}} \; \sup_{\substack{\zeta \\ \delta\text{-robust}}} \; \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

Deterministic value

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

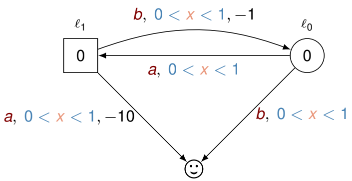
Deterministic value

Min can always choose $(\frac{1-\nu}{2},\ a)$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

χ Min ζ Max



Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

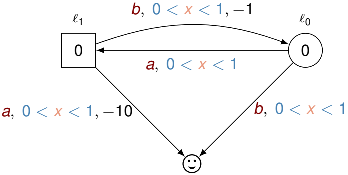
Deterministic value

Min can always choose $(\frac{1-\nu}{2},\ a)$

$\mathrm{dVal}(\ell_0, 0) = -10$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

Deterministic value

Min can always choose $(\frac{1-\nu}{2},\ a)$

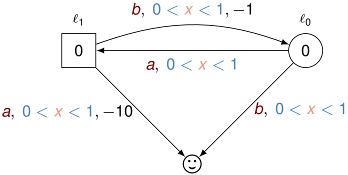
$\mathrm{dVal}(\ell_0, 0) = -10$

Conservative robust value

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

χ Min ζ Max



Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

Deterministic value

Min can always choose $(\frac{1-\nu}{2},\ a)$

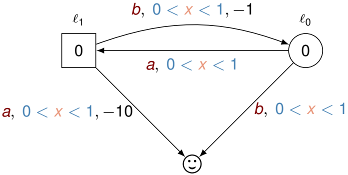
$\mathrm{dVal}(\ell_0, 0) = -10$

Conservative robust value

Min has no interest to reach ℓ_1

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

Deterministic value

Min can always choose $(\frac{1-\nu}{2}, a)$

$\mathrm{dVal}(\ell_0, 0) = -10$

Conservative robust value

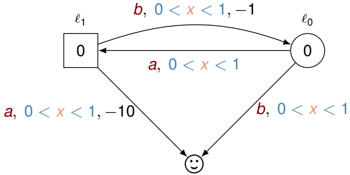
Min has no interest to reach ℓ_1

$$(\ell_0, 0) \xrightarrow{0.5,\ a}$$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

χ Min ζ Max



Robust value

$$\mathrm{rVal}(c) = \lim_{\substack{\delta \to 0 \\ \delta > 0}} \inf_{\substack{\chi \\ \delta\text{-robust}}} \sup_{\substack{\zeta \\ \delta\text{-robust}}} \mathrm{Payoff}(\mathrm{Play}(c, \chi, \zeta))$$

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$\mathrm{dVal}(\ell_0, 0) = -10$

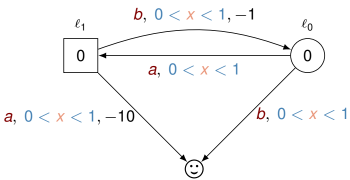
Conservative robust value

Min has no interest to reach ℓ_1

$$(\ell_0, 0) \xrightarrow{0.5,\ a} \overset{\delta}{\rightsquigarrow} (\ell_1, 0.5 + \delta)$$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



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Conservative robust value

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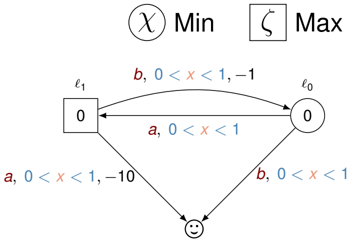
Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



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Conservative robust value

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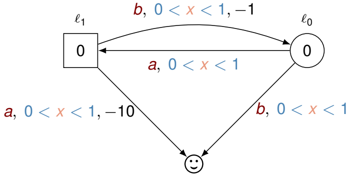
$$(\ell_0, 0) \xrightarrow{0.5,\ b} \overset{\delta}{\rightsquigarrow} (☺, .5 + \delta)$$

Robust value problem

χ Min ζ Max

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



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Deterministic value

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Conservative robust value

Min has no interest to reach ℓ_1

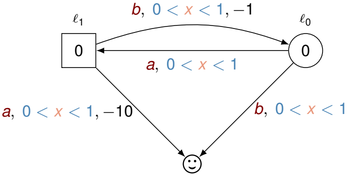
$\mathrm{rVal}(\ell_0, 0) = 0$

$$(\ell_0, 0) \xrightarrow{0.5,\ b} \overset{\delta}{\rightsquigarrow} (☺, .5 + \delta)$$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

χ Min ζ Max



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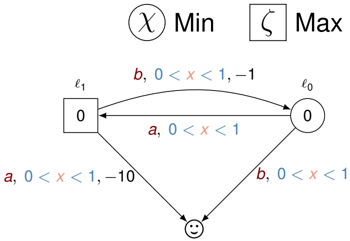
$\mathrm{rVal}(\ell_0, 0) = 0$

Excessive robust value

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



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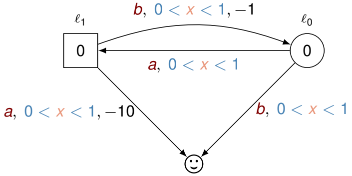
Excessive robust value

Min has no interest to reach two times ℓ_1

Robust value problem

χ Min ζ Max

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



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Conservative robust value

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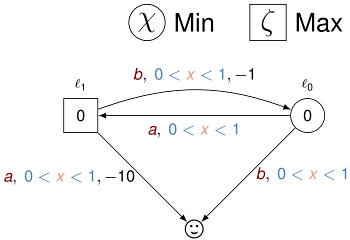
Excessive robust value

Min has no interest to reach two times ℓ_1

$$(\ell_0, 0) \xrightarrow{0.5,\ b} \overset{\delta}{\rightsquigarrow} (\ell_1, 0.5 + \delta) \xrightarrow{0.5-2\delta,\ a} (\ell_0, 1 - \delta)$$

Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?



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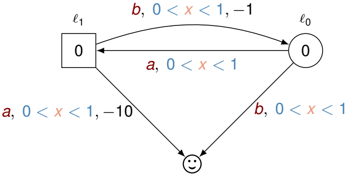
Robust value problem

Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

χ Min ζ Max



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Min has no interest to reach two times ℓ_1

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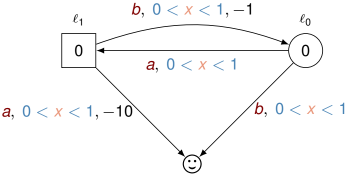
Deciding if $\mathrm{rVal}(c) \leqslant \lambda$?

χ Min ζ Max



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$\mathrm{rVal}(\ell_0, 0) = 0$

Excessive robust value

Min has no interest to reach two times ℓ_1

$\mathrm{rVal}(\ell_1, 0) = -1$

$$(\ell_0, 0) \xrightarrow{0.5,\ b} \overset{\delta}{\rightsquigarrow} (\ell_1, 0.5+\delta) \xrightarrow{0.5-2\delta,\ a} (\ell_0, 1-\delta) \xrightarrow{0,\ b} \overset{2\delta}{\rightsquigarrow} (☺, 1+\delta)$$

Robust value problem

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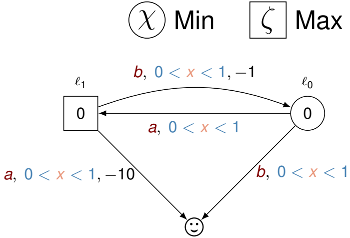

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	exact	conservative	excessive
WTG	Undecidable		
WTA	PSPACE-c		
Acyclic WTG	EXPTIME		
Strictly non-zeno WTG	EXPTIME		

On Optimal Timed Strategies, T. Brihaye, V. Bruyère and J.-F. Raskin, 2005, FORMATS

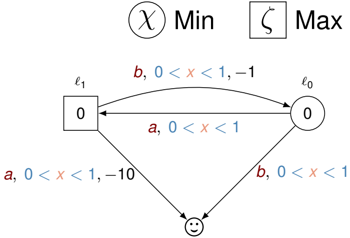
On the optimal reachability problem of weighted timed automata, P. Bouyer, T. Brihaye, V. Bruyère, and JF. Raskin, 2007, Formal Methods in System Design

A Theory of Timed Automata, R. Alur and D. Dill, 1994, Theoretical Computer Science

Optimal Strategies in Priced Timed Game Automata, P. Bouyer, F. Cassez, E. Fleury, and K. Larsen, 2004, TCS

Robust value problem

Deciding if $\mathsf{rVal}(c) \leqslant \lambda$?



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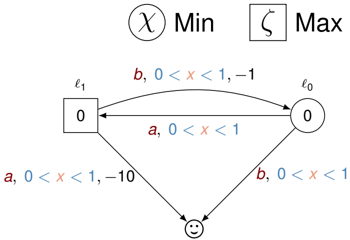
Robust Weighted Timed Automata and Games, P. Bouyer, N. Markey, and O. Sankur, 2013, FORMATS

Robust value problem

Deciding if rVal(c) $\leqslant \lambda$?



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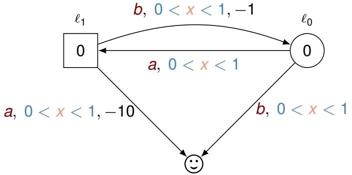
Robust value problem

Deciding if rVal(c) $\leqslant \lambda$?

χ Min $\quad$ ζ Max



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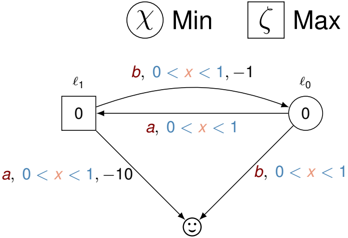

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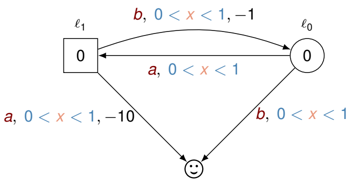

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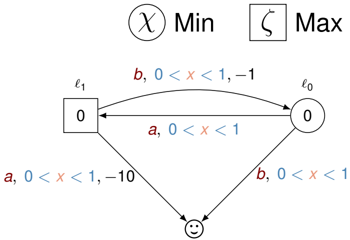

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Computing robust values in acyclic WTG

Symbolic computation

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A combination of two existing methods

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Cells

Optimal reachability for weighted timed games, R. Alur, M. Bernadsky and P. Madhusuda, 2004, ICALP

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Affine equations:
$y = \sum_i a_i x_i + b$

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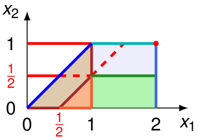

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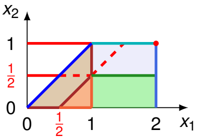

Shrunk DBM

Shrinking timed automata, O. Sankur, P. Bouyer, and N. Markey, 2011, FSTTCS

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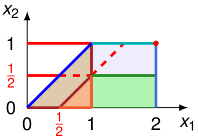

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Matrix: $M - \delta P$

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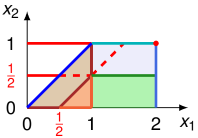

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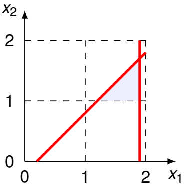

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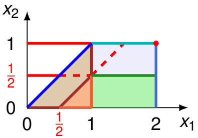

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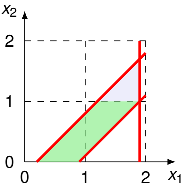

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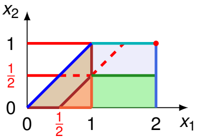

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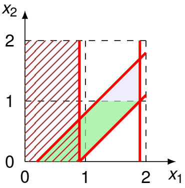

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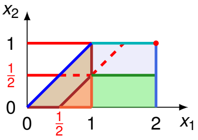

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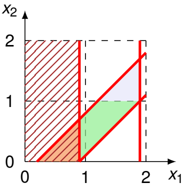

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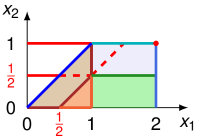

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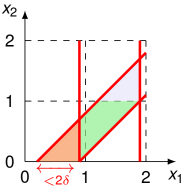
Matrix: $M - \delta P$



Shrinking timed automata, O. Sankur, P. Bouyer, and N. Markey, 2011, FSTTCS

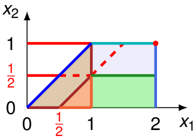
Computing robust values in acyclic WTG

Symbolic computation

A combination of two existing methods

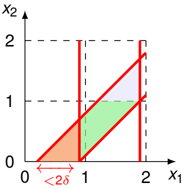
Cells

Affine equations:
$y = \sum_i a_i \, x_i + b$



Shrunk DBM

Matrix: $M - \delta \, P$
where $\delta \to 0$



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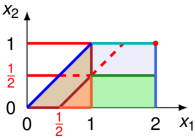

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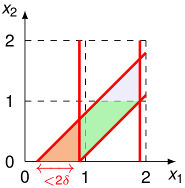

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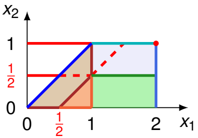

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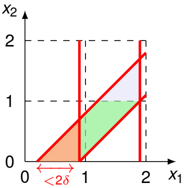

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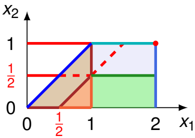

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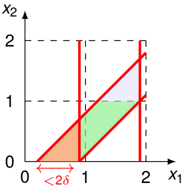

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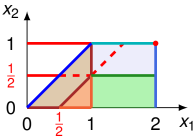

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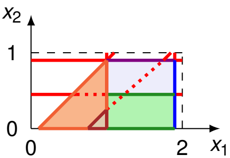

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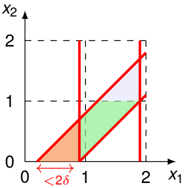

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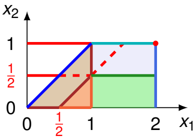

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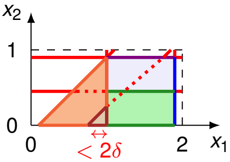

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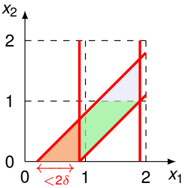

Computing robust values in acyclic WTG

ℓ belongs to I

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Computing robust values in acyclic WTG

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$$V_\ell = \min_{e=(\ell,g,Y,\ell')} \left[\mathrm{wt}(e) + \mathrm{Pre}_\ell(\mathrm{Perturb}_\ell^\delta(\mathrm{Guard}_g(\mathrm{Unreset}_Y(V_{\ell'}))))\right]$$

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Perturb operator

$$\mathrm{Perturb}^\delta_\ell(V_{\ell'})(\nu) = \begin{cases} V_{\ell'}(\nu) & \text{if } \ell \text{ belongs to Max} \\ \sup_{d\in[0,2\delta]}[d\,\mathrm{wt}(\ell) + V_{\ell'}(\nu + d)] & \text{if } \ell \text{ belongs to Min} \end{cases}$$

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Conclusion

	conservative	excessive
Reach TA	PSPACE-complete	EXPTIME-c
Reach TG	EXPTIME-c	EXPTIME-c
WTG	Undecidable	Undecidable
WTA	PSPACE-c	Undecidable
Acyclic WTG	Decidable	
Strictly non-zeno WTG	Decidable	

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- Adaptation of cells and shrunk DBM

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- Others classes of WTG

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Perspective

- Others classes of WTG
- Robustness with probabilistic strategies

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Thank you! Questions?