Stochastic Strategies in Quantitative or Timed Games

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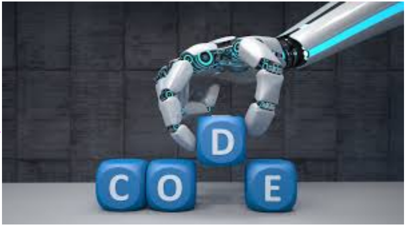
February 4, 2020

Motivation : games theory for synthesis



- Check the correctness of a system
- Interaction between two antagonistic agents : environment and controller
- Correct by construction : synthesis of controller

Different sorts of games

Quantitative games

- Consider quantitative parameters : energy consumption, ...
- Compare distinct strategies

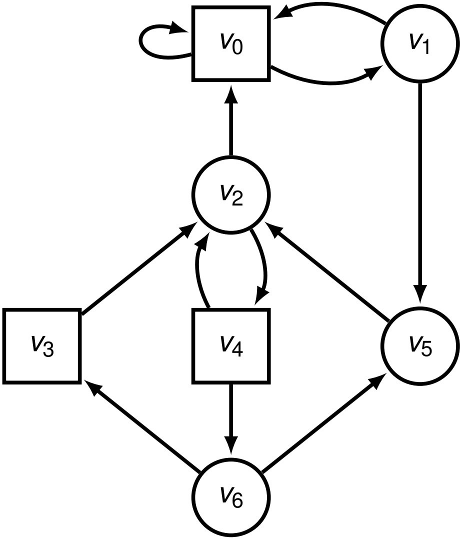
Timed games

- Consider timed issues : receive a message, ...
- Infinite games

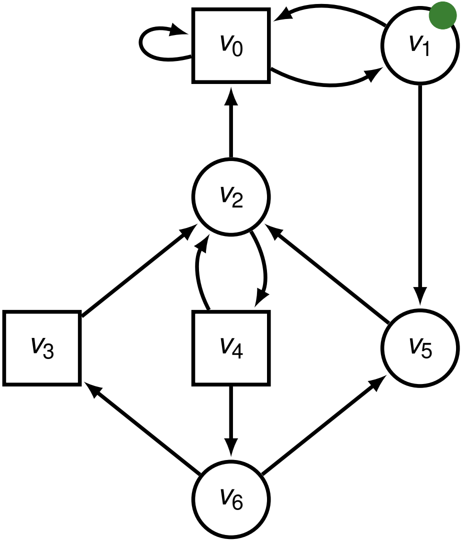
Priced Timed games

Combination of both.

Games on graph



Games on graph

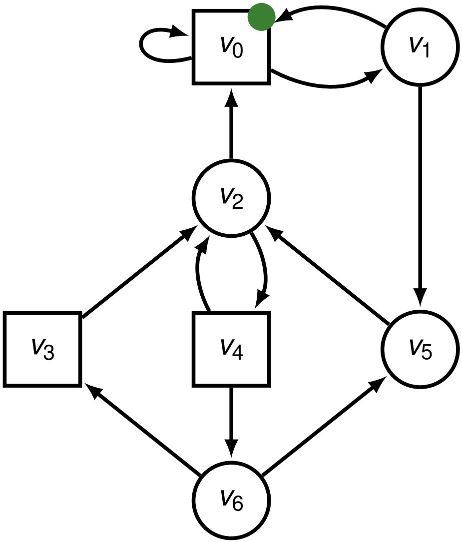


How to play?

Move a token along edge

$$\pi = v_1$$

Games on graph



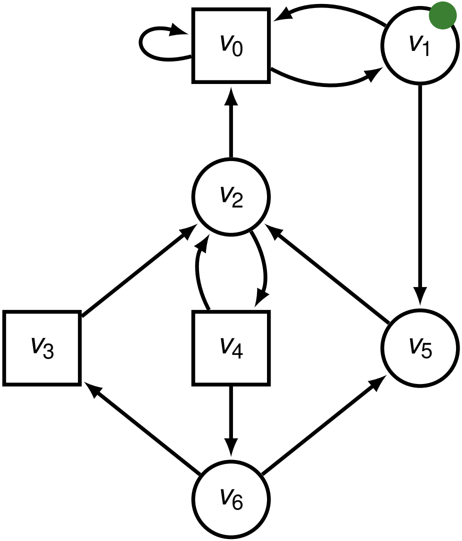
□ Adam ○ Eve

How to play?

Move a token along edge

$$\pi = v_1 v_0$$

Games on graph



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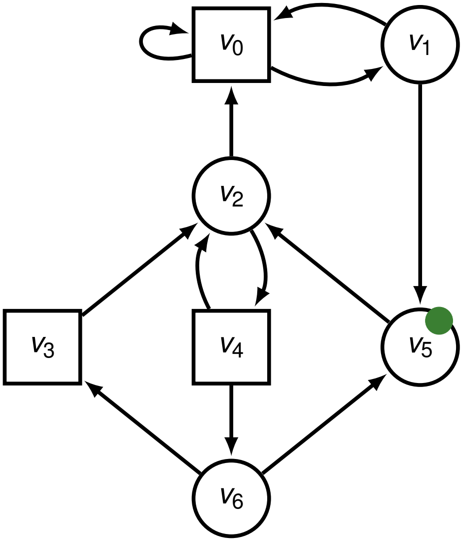
Games on graph



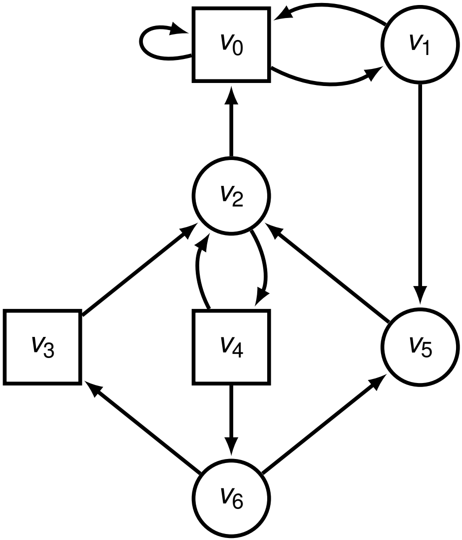
□ Adam ○ Eve

How to play?

Move a token along edge

$$\pi = v_1 v_0 v_1 v_5$$

Games on graph



Adam ☐ Eve ◯

Run

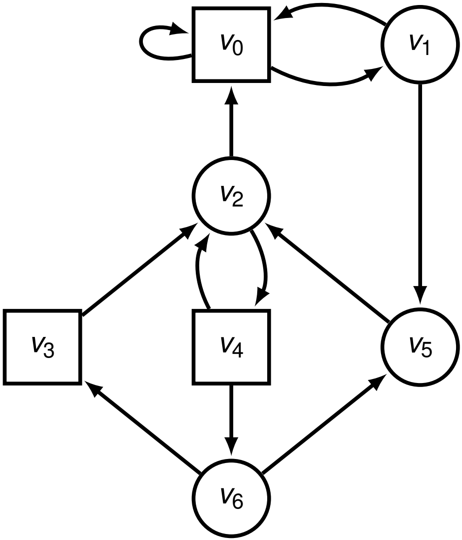
$\pi = (v_i)_i \in V^\omega$: an infinite path in the arena

How to play?

Move a token along edge

$$\pi = v_1 v_0 v_1 v_5 (v_2 v_4)^\omega$$

Games on graph



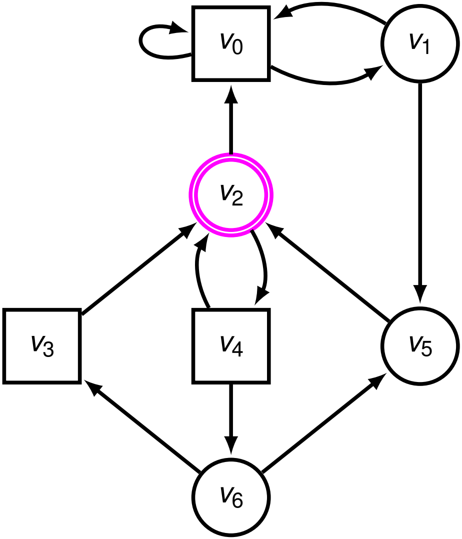
□ Adam ○ Eve

Objective

Eve $W \subset V^\omega$

Adam $V^\omega \setminus W$

Games on graph



□ Adam ○ Eve

Objective

Eve $W \subset V^\omega$

Adam $V^\omega \setminus W$

Reachability objective

Let T be a set of vertices.

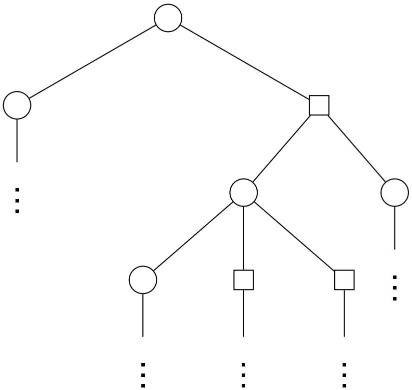
Eve $\exists i,\ v_i \in T$

Adam $\forall i,\ v_i \notin T$

Who wins?

Let $\pi = v_1 v_0 v_1 v_5 (v_2 v_4)^\omega$ be a run : Eve wins.

Strategies for Eve



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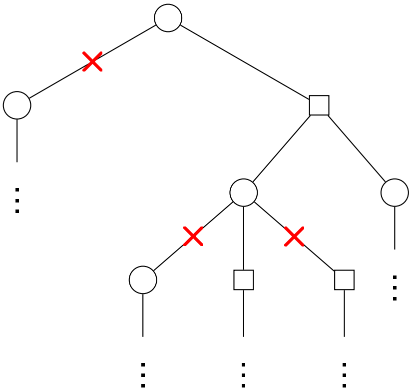
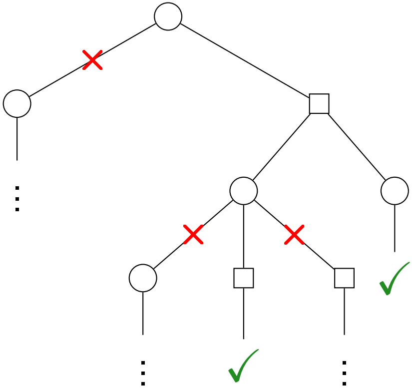


Strategies for Eve

A strategy $\sigma : V^* V_{Eve} \to V$.

Strategies for Eve



Strategies for Eve

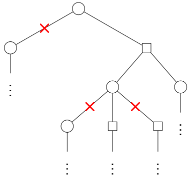
A strategy $\sigma : V^* V_{Eve} \to V$.

A winning strategy all paths are winning

Strategies for Eve

Infinite memory

$\sigma : V^* V_{Eve} \to V$



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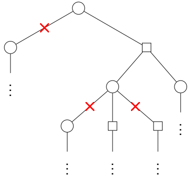
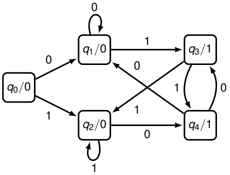


Finite memory

Moore machine



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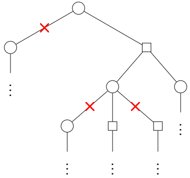
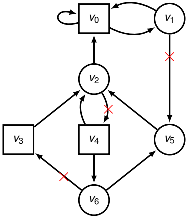


Memoryless

$\sigma : V_{Eve} \to V$



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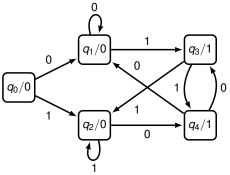

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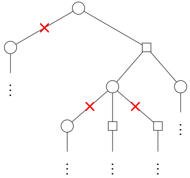

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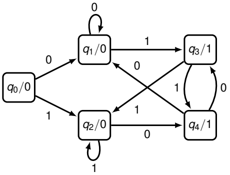

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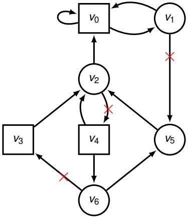
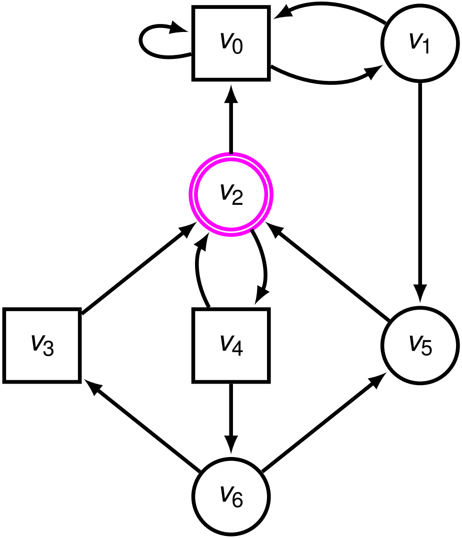


Probability [1]

$\sigma : V^* V_{Eve} \to Dist(E)$

[1] *Trading Memory for Randomness*, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

An attractor strategy [2]



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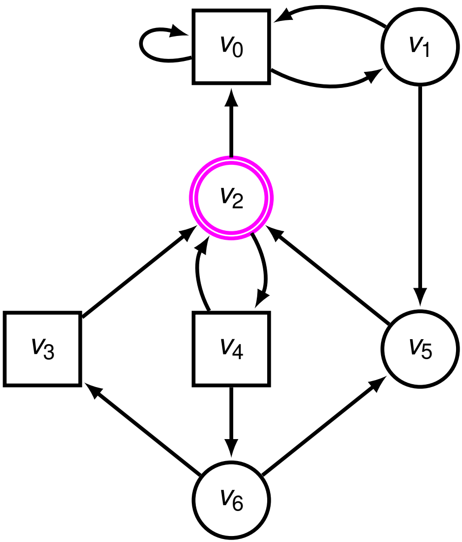

Attractor

$$X \mapsto T \cup Pre(X)$$

where $Pre(X) =$
$\{v \in V_{Eve} | \exists (v, v') \in E, v' \in X\} \cup$
$\{v \in V_{Adam} | \forall (v, v') \in E, v' \in X\}$

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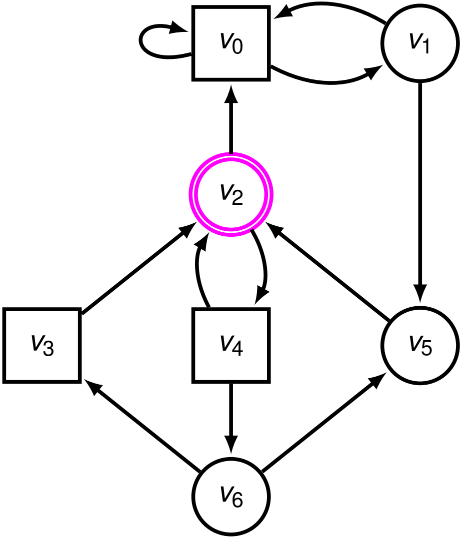

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Winning strategy for Eve

Witness for Eve's vertices

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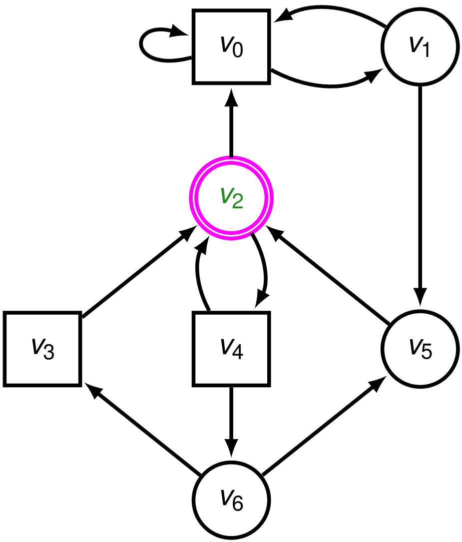

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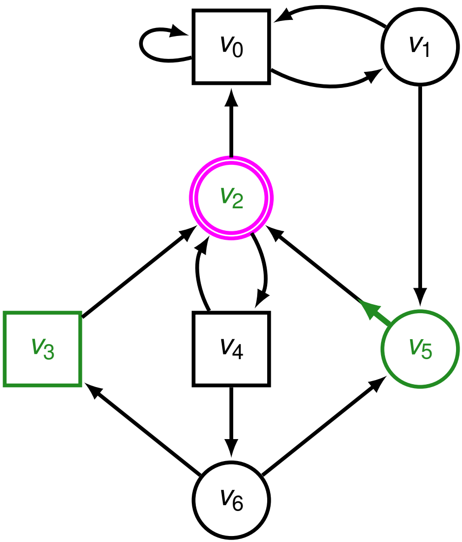

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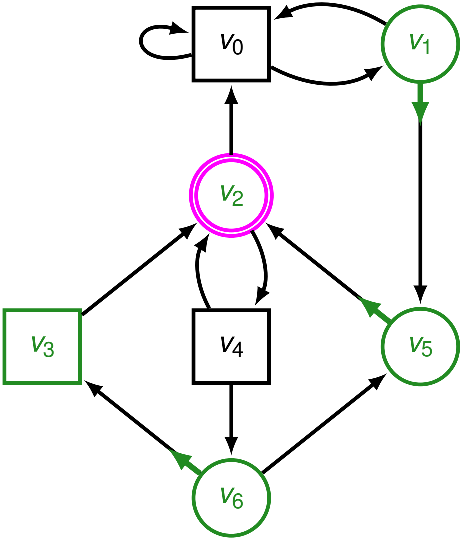

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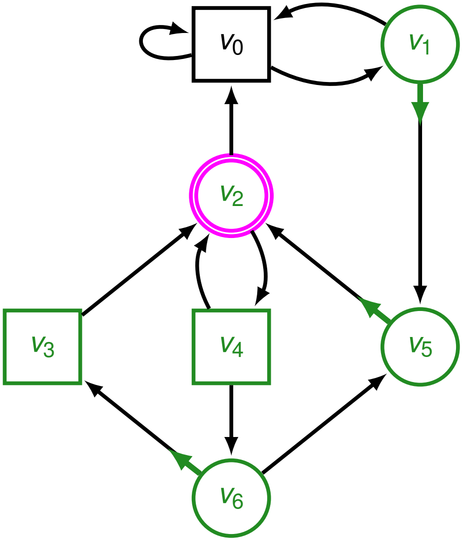

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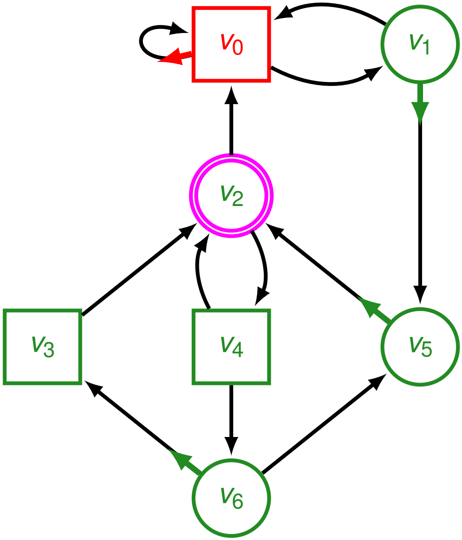

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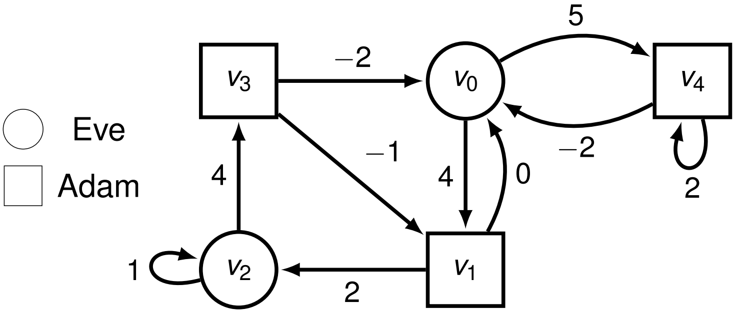
Determinacy

For all vertices, Adam or Eve have a winning strategy

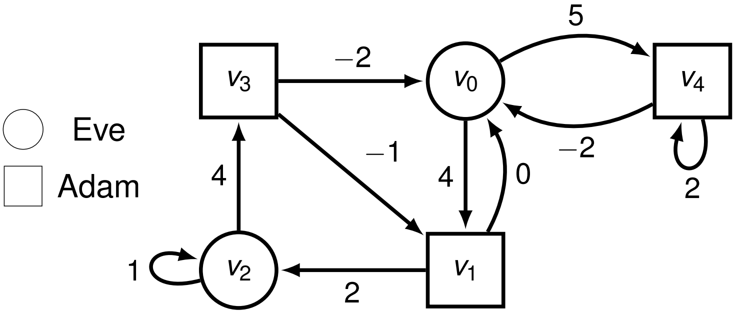
[2] *Automata Logics, and Infinite Games: A Guide to Current Research*, E. Grädel, W. Thomas and T. Wilke, 2002, Springer-Verlag New York, Inc.

Quantitative games



A run : $\pi = v_0(v_1 v_2 v_3)^\omega$

Quantitative games



Mean-Payoff

Let $\pi = (\pi_i)_i$ be a run.

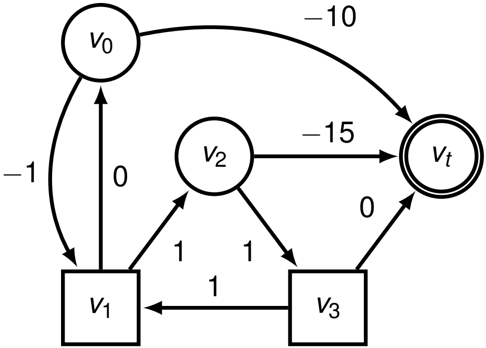
$$\liminf_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} w((\pi_i, \pi_{i+1}))$$

Mean-Payoff on a run

Let $\pi = v_0(v_1 v_2 v_3)^\omega$ be a run.

$$\text{Mean-Payoff}(\pi) = \frac{5}{3}$$

Focus on Shortest Path objective [3]



Shortest Path payoff

Let π be a run.

$$\begin{cases} \infty & \text{if } \pi \text{ does not reach } T \\ \sum_{i=0}^{n-1} w((\pi_i, \pi_{i+1})) & \text{if } n \text{ is the smallest index such that } \pi_n \in T \end{cases}$$

[3] *Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games*, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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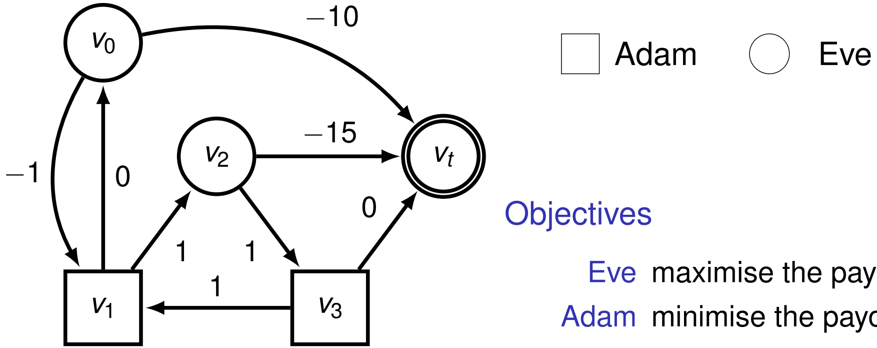

Objectives

Eve maximise the payoff

Adam minimise the payoff

Shortest Path payoff

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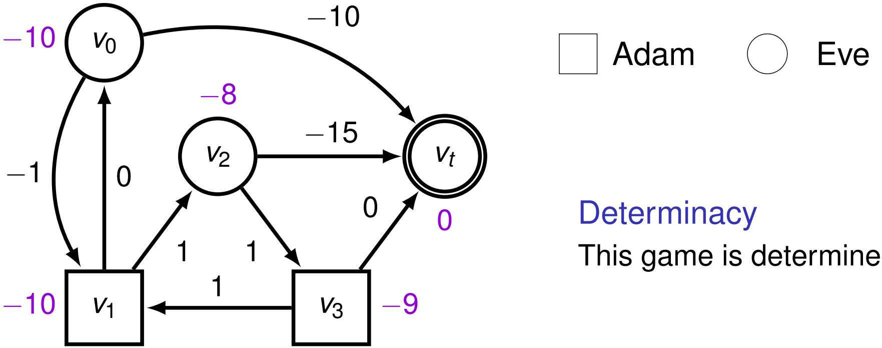

Determinacy

This game is determined.

Shortest Path payoff

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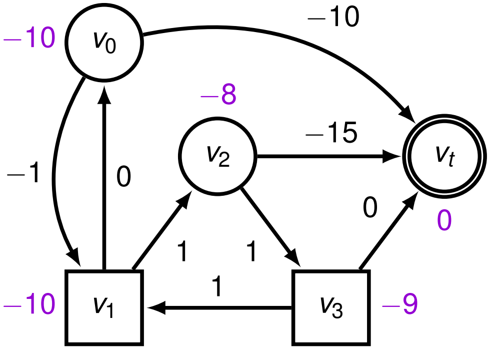

Optimal strategy

An optimal strategy is a strategy that maximise or minimise the payoff.

Shortest Path payoff

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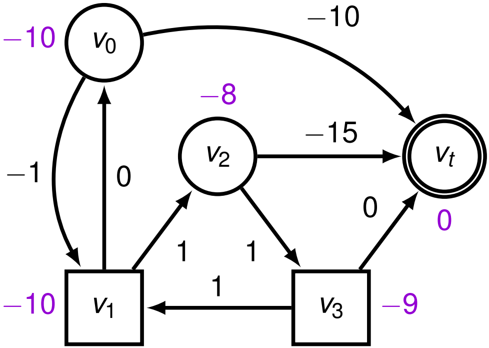

Optimal strategy for Adam

An optimal strategy for Adam may require finite memory.

Shortest Path payoff

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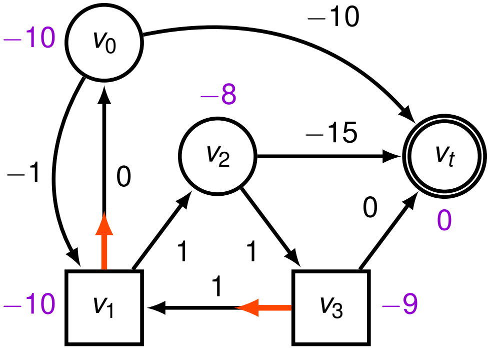

Optimal strategy for Adam

An optimal strategy for Adam may require finite memory.
The switching strategy:

- σ^o : reach the optimal value

Shortest Path payoff

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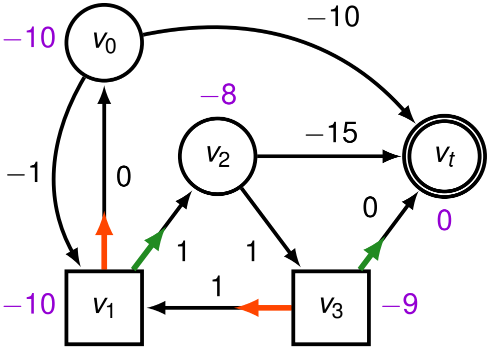

Optimal strategy for Adam

An optimal strategy for Adam may require finite memory.
The switching strategy:

- σ^o : reach the optimal value
- σ^t : reach the target

Shortest Path payoff

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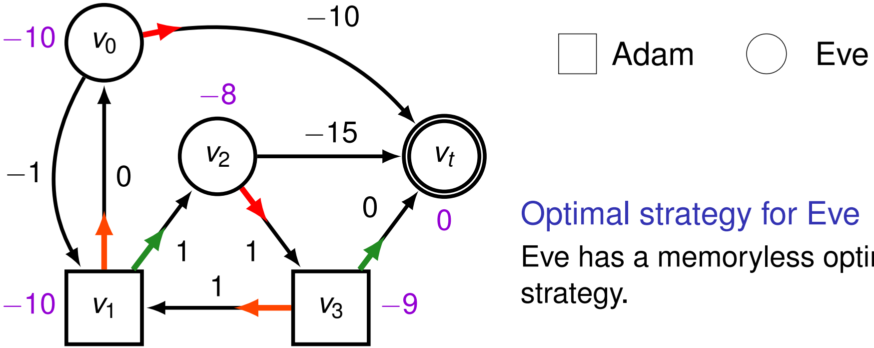

Optimal strategy for Eve

Eve has a memoryless optimal strategy.

Shortest Path payoff

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Internship objective

Make a trade-off between probability and memory.

- For quantitative games : existence of an optimal probabilistic memoryless strategy?
- For Priced Timed games : using probability to reduce memory[4]?

Undecidability result [5]

The problem of existence of an optimal strategy is undecidable for Priced Timed games with at least two clocks.

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[4] *Trading Infinite Memory for Uniform Randomness in Timed Games*, K. Chatterjee, T. Henzinger and V. Prabhu, 2008, HSCC

[5] *Adding Negative Prices to Priced Timed Games*, T. Brihaye, G. Geeraerts, S. N. Krishna, L. Manasa, B. Monmege and A. Trivedi, 2014, CONCUR

First idea for quantitative games

Probabilistic memoryless strategy

Probabilistic choice between the memoryless strategies of the switching strategy.