Reaching Your Goal Optimally by Playing at Random with no Memory

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Benjamin Monmege   Pierre-Alain Reynier

September 24, 2020

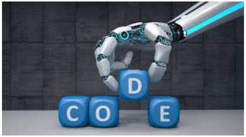
Motivation : game theory for synthesis

Classic approach

Check the correctness of a system

Game theory

Interaction between two antagonistic agents : environment and controller



Code synthesis

Correct by construction : synthesis of controller

Different sorts of games

Qualitative games

Reach or avoid some (sequences of) states

Quantitative games

- Consider quantitative parameters : energy consumption...
- Compare distinct strategies

Different sorts of games

Qualitative games

Reach or avoid some (sequences of) states

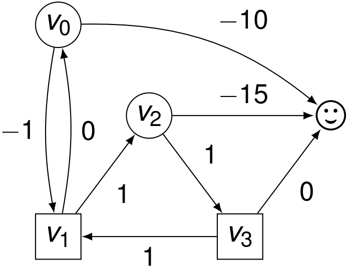
Quantitative games

- Consider quantitative parameters : energy consumption...
- Compare distinct strategies

Shortest-Path games

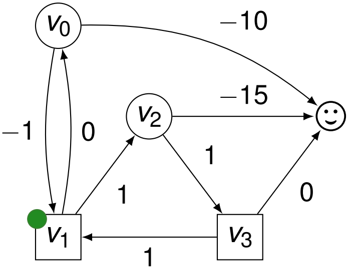
- Combination of a qualitative with a quantitative objective
- Reach a target with a minimum cost

Shortest Path Game



Shortest Path Game

□ Adam ○ Eve



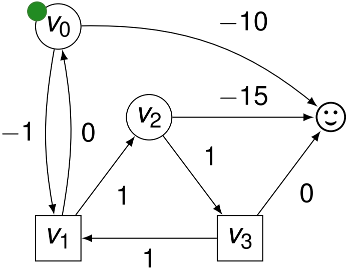
How to play?

Move a token along an edge

$$\pi = v_1$$

Shortest Path Game

Adam Eve



How to play?

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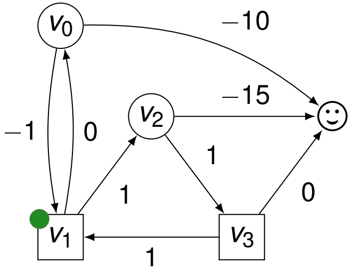

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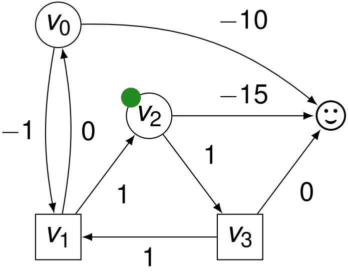

How to play?

Move a token along an edge

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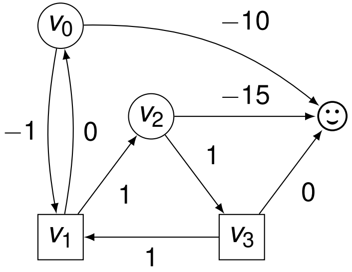

Play

Infinite path or reach the target

$$\pi = (v_i)_i \in V^\omega \qquad \pi = (v_i)_i ☺$$

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$$\pi = v_1 v_0 v_1 v_2 v_3 ☺$$

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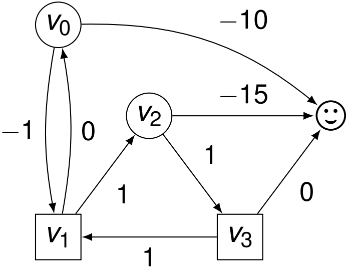

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Shortest Path payoff of a play π

$$\mathbf{SP}(\pi) = \begin{cases} \sum_{i=0}^{n-1} w((\pi_i, \pi_{i+1})) & \text{if } \exists n \text{ (the smallest) s.t. } \pi_n = ☺ \\ +\infty & \text{if } \pi \text{ does not reach } ☺ \end{cases}$$

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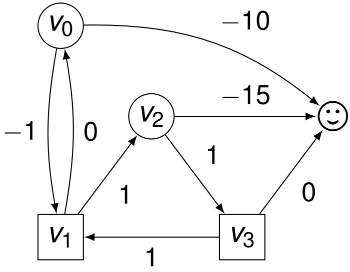

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$$\mathbf{SP}(\pi) = 0 + (-1) + 1 + 1 + 0 = 1$$

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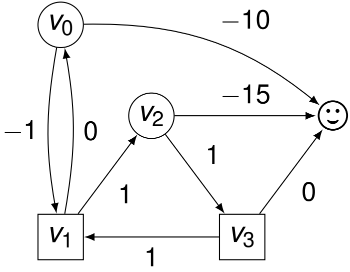

Objectives

Eve maximise the payoff

Adam minimise the payoff

Shortest Path payoff of a play π

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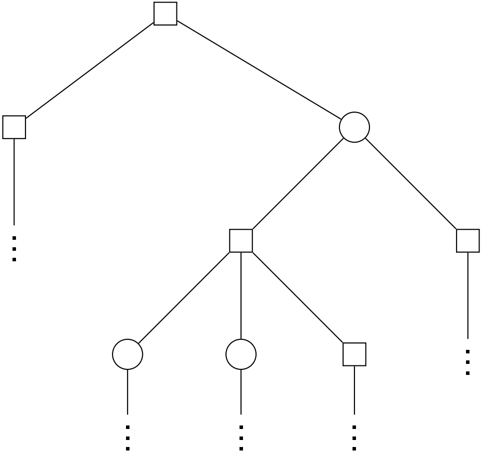
Strategies for Adam



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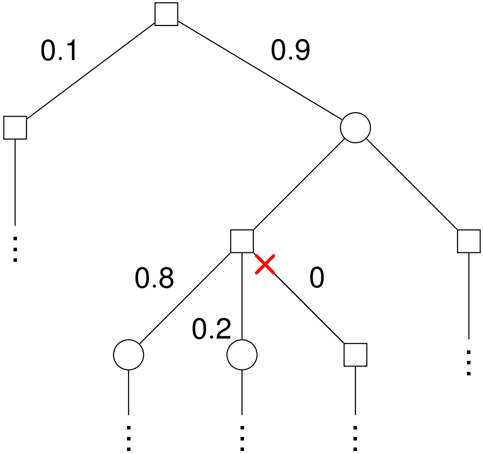


A strategy

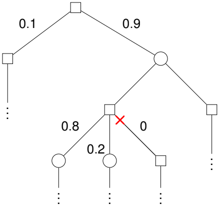
$\sigma : V^* V_{Adam} \to \Delta(V)$

Trading Memory for Randomness, K. Chatterjee, L. Alfaro and T. Henzinger, 2004, QEST

Strategies for Adam

Infinite memory

$\sigma : V^* V_{Adam} \rightarrow \Delta(V)$



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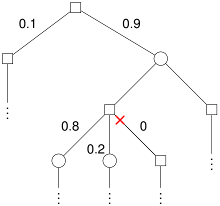
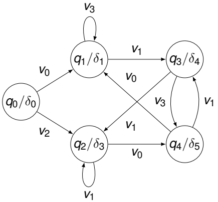


Finite memory

Moore machine



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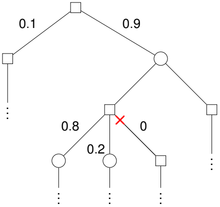

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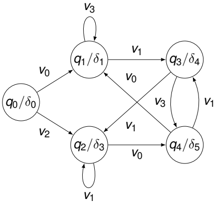
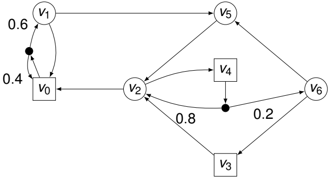


Memoryless

$\sigma : V_{Adam} \to \Delta(V)$



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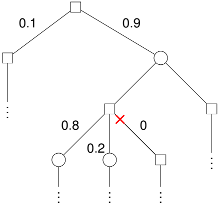

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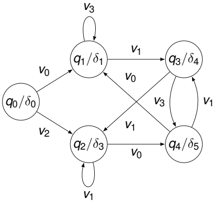

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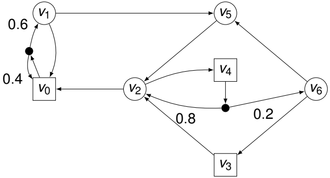
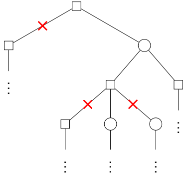


Deterministic

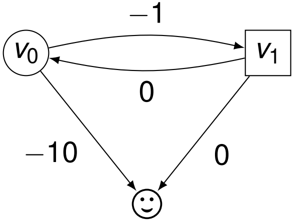
$\sigma : V^* V_{Adam} \to V$



Deterministic Strategies

$\sigma : V^* \times V_{\text{Adam}} \to V$

$\boxed{\sigma}$ Adam $\quad (\tau)$ Eve



Value

$$\overline{\text{dVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \textbf{SP}(\text{Play}(v, \sigma, \tau))}_{\text{dVal}^{\sigma}(v)}$$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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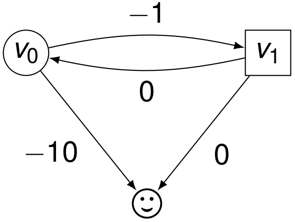

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Determinacy

$$\text{dVal}(v) = \overline{\text{dVal}}(v) = \underline{\text{dVal}}(v)$$

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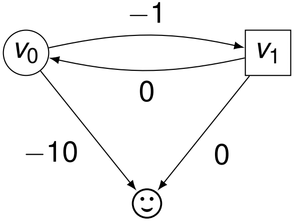

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Bellman equation

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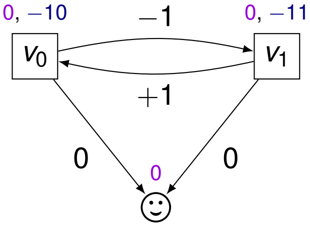

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Unicity

Bellman equation may have many solutions.

Bellman equation

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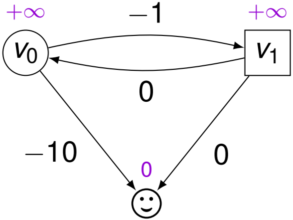

Value

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Value iteration

- Compute dVal as a greatest fixed point

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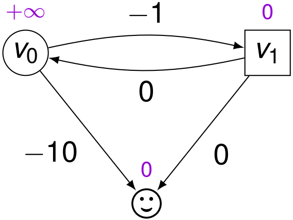

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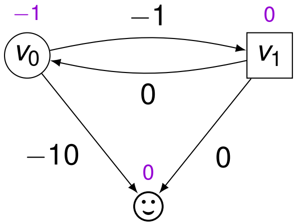

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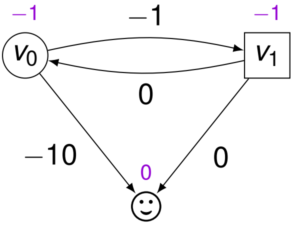

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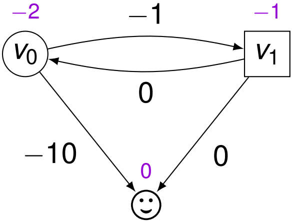

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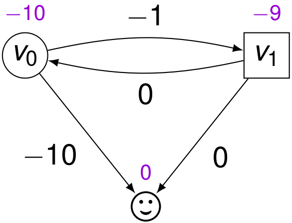

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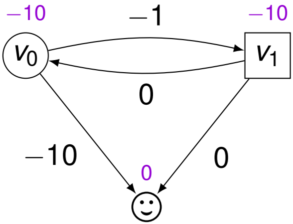

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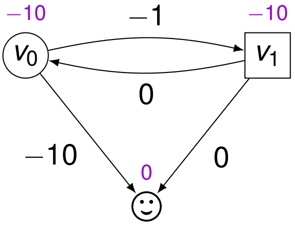

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Value iteration

- Compute dVal as a greatest fixed point
- Complexity: pseudo-polynomial

Bellman equation

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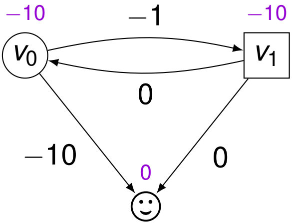

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Optimal strategy

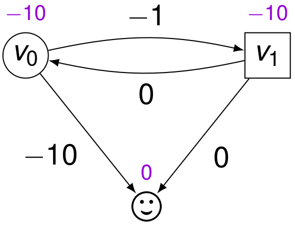
$\text{dVal}^{\sigma^*}(v) \leqslant \text{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\text{Adam}} \to V$

σ Adam $\quad$ τ Eve



Value

$$\text{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\text{Play}(v, \sigma, \tau))}_{\text{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

An optimal strategy for Adam may require finite memory.

Optimal strategy

$\text{dVal}^{\sigma^*}(v) \leqslant \text{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

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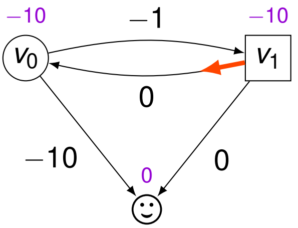
$\boxed{\sigma}$ Adam $\quad \textcircled{\tau}$ Eve



Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle

Optimal strategy

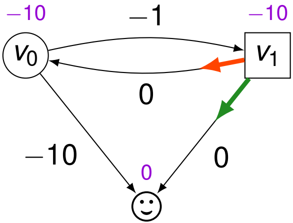
$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\mathsf{Adam}} \to V$

$\boxed{\sigma}$ Adam $\quad \textcircled{\tau}$ Eve



Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle
- σ_2 : reach ☺

Optimal strategy

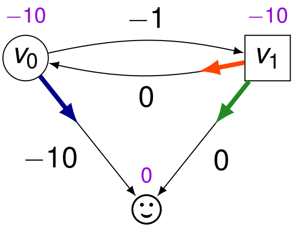
$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Deterministic Strategies

$\sigma : V^* \times V_{\text{Adam}} \to V$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

$$\mathsf{dVal}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbf{SP}(\mathsf{Play}(v, \sigma, \tau))}_{\mathsf{dVal}^{\sigma}(v)}$$

Optimal strategy for Adam

Switching strategy:

- σ_1 : reach negative cycle
- σ_2 : reach ☺

Optimal strategy

$\mathsf{dVal}^{\sigma^*}(v) \leqslant \mathsf{dVal}(v)$

Optimal strategy for Eve

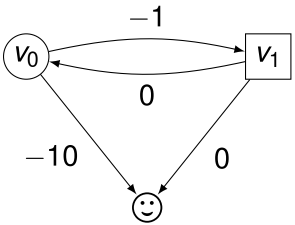
Eve has a memoryless optimal strategy.

Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve

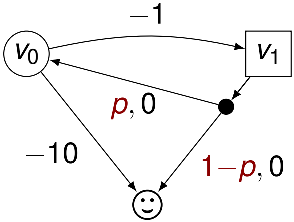


Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



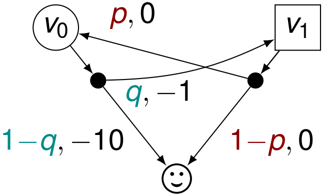
$\boxed{\sigma}$ Adam $\quad$ τ Eve

Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



σ Adam τ Eve

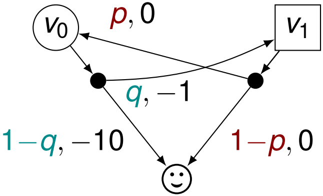
Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Value

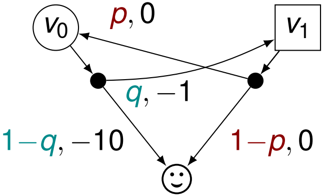
$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Unicity

A unique fix point : $\text{mVal}^{\sigma,\tau}$

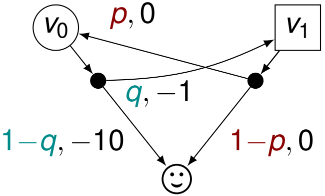
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p \times \text{mVal}^{\sigma,\tau}(v_0)$$
$$\text{mVal}^{\sigma,\tau}(v_0) = q(\text{mVal}^{\sigma,\tau}(v_1) - 1) - 10(1 - q)$$

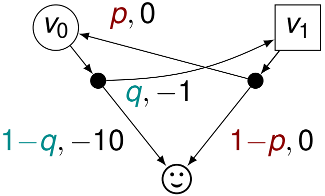
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

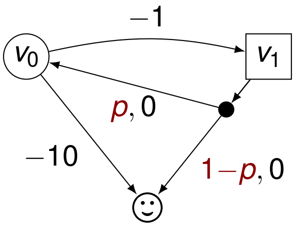
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad (\tau)$ Eve



Value

$$\overline{\text{mVal}}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^\sigma(v)}$$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^σ

- If $p < \frac{9}{10}$
- If $p \geqslant \frac{9}{10}$

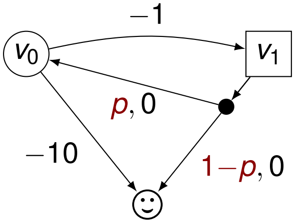
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:
- If $p \geqslant \frac{9}{10}$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

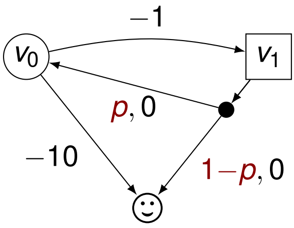
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ $\textcircled{\tau}$ Eve



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

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- If $p < \frac{9}{10}$, then $q = 1$:

  $\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$

  $\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$

- If $p \geqslant \frac{9}{10}$

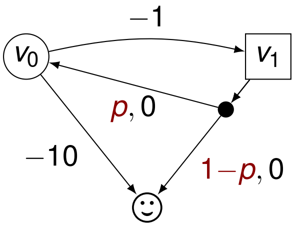
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ $\textcircled{\tau}$ Eve



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\overline{\mathrm{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

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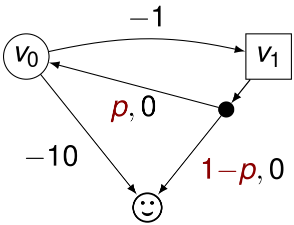
- If $p \geqslant \frac{9}{10}$, then $q = 0$:

Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

$\boxed{\sigma}$ Adam $\quad$ $\textcircled{\tau}$ Eve

Value

$$\overline{\mathrm{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $$\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p}$$

  $$\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $$\mathrm{mVal}^{\sigma}(v_1) = -10p$$

  $$\mathrm{mVal}^{\sigma}(v_0) = -10$$

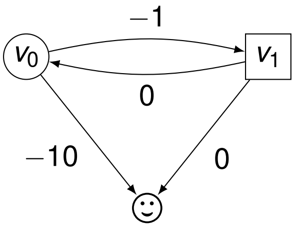
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\overline{\mathrm{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

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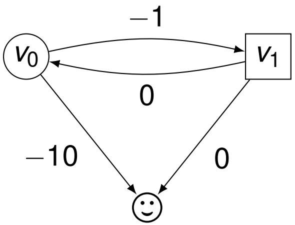
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ (τ) Eve



Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $$\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} < -9$$

  $$\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p}$$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $$\text{mVal}^{\sigma}(v_1) = -10p$$

  $$\text{mVal}^{\sigma}(v_0) = -10$$

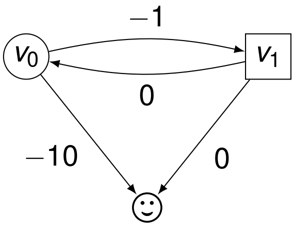
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\textcircled{\tau}$ Eve



Compute $\mathrm{mVal}^{\sigma,\tau}$

$$\mathrm{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

$$\mathrm{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Value

$$\overline{\mathrm{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\mathbf{SP})}_{\mathrm{mVal}^{\sigma}(v)}$$

Compute mVal^{σ}

- If $p < \frac{9}{10}$, then $q = 1$:

  $\mathrm{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} < -9$

  $\mathrm{mVal}^{\sigma}(v_0) = \frac{-1}{1-p} < -10$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $\mathrm{mVal}^{\sigma}(v_1) = -10p$

  $\mathrm{mVal}^{\sigma}(v_0) = -10$

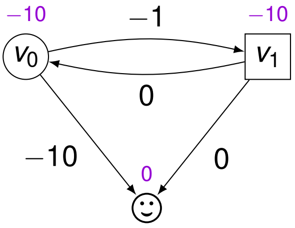
Bellman equation in a Markov Chain

$$\mathrm{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \mathrm{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$

$\boxed{\sigma}$ Adam $\quad$ τ Eve



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- If $p < \frac{9}{10}$, then $q = 1$:

  $\text{mVal}^{\sigma}(v_1) = \frac{-p}{1-p} < -9$

  $\text{mVal}^{\sigma}(v_0) = \frac{-1}{1-p} < -10$

- If $p \geqslant \frac{9}{10}$, then $q = 0$:

  $\text{mVal}^{\sigma}(v_1) = -10p$

  $\text{mVal}^{\sigma}(v_0) = -10$

Compute $\text{mVal}^{\sigma,\tau}$

$$\text{mVal}^{\sigma,\tau}(v_1) = p\frac{-q-10(1-q)}{1-pq}$$

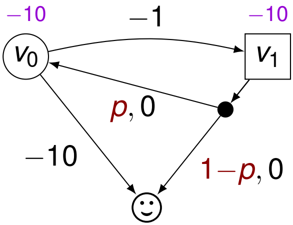
$$\text{mVal}^{\sigma,\tau}(v_0) = \frac{-q-10(1-q)}{1-pq}$$

Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



σ Adam τ Eve

Value

$$\overline{\text{mVal}}(v) = \inf_\sigma \underbrace{\sup_\tau \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^\sigma(v)}$$

Value in a MDP

Computable in polynomial time

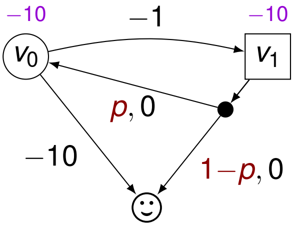
Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v,v')(w(v,v') + \text{mVal}^{\sigma,\tau}(v'))$$

Stochastic Shortest Paths and Weight-Bounded Properties in Markov Decision Processes, C. Baier, N. Bertrand, C. Dubslaff, D. Gburek and O. Sankur, 2018, LICS.

Memoryless strategies

$\sigma : V_{\text{Adam}} \to \Delta(V)$



$\boxed{\sigma}$ Adam $\quad$ τ Eve

Value

$$\overline{\text{mVal}}(v) = \inf_{\sigma} \underbrace{\sup_{\tau} \mathbb{E}^{\sigma,\tau}(\textbf{SP})}_{\text{mVal}^{\sigma}(v)}$$

Value in a MDP

Computable in polynomial time

ε-optimal strategy

$$\text{mVal}^{\sigma^*}(v) \leqslant \overline{\text{mVal}}(v) + \varepsilon$$

Bellman equation in a Markov Chain

$$\text{mVal}^{\sigma,\tau}(v) = \sum_{v'} \mathbb{P}^{\sigma,\tau}(v, v')(w(v, v') + \text{mVal}^{\sigma,\tau}(v'))$$

Stochastic Shortest Paths and Weight-Bounded Properties in Markov Decision Processes, C. Baier, N. Bertrand, C. Dubslaff, D. Gburek and O. Sankur, 2018, LICS.

Contribution

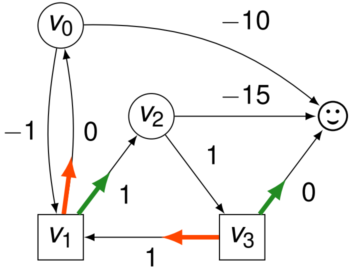
$$\mathsf{dVal} = \overline{\mathsf{mVal}}$$

Memoryless simulate deterministic

Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Memoryless simulate deterministic



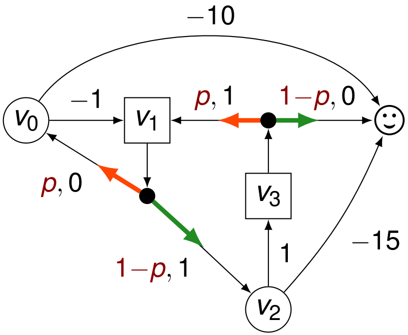
Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy,

Memoryless simulate deterministic



Claim

For all v, there exists p such that $\mathrm{mVal}^{\rho_p}(v) \leqslant \mathrm{dVal}(v)$.

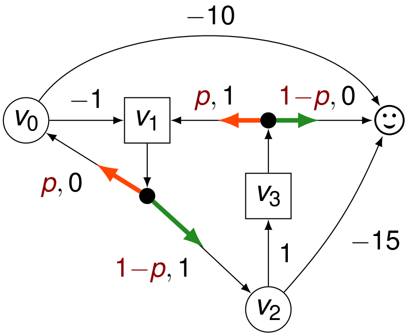
Strategy ρ_p

Let $\langle \sigma_1, \sigma_2 \rangle$ be an optimal switching strategy, for all $p \in (0,1)$,

$$\rho_p = p \times \sigma_1 + (1-p) \times \sigma_2$$

Memoryless simulate deterministic

□ Adam ○ Eve



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Properties of ρ_p

- For all τ, $\mathbb{P}^{\rho_p,\tau}(\diamond ☺) = 1$

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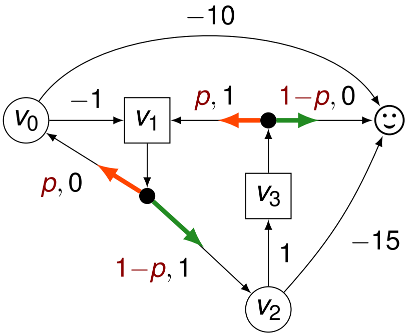

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Properties of ρ_p

- For all τ, $\mathbb{P}^{\rho_p,\tau}(\diamond ☺) = 1$
- Eve has an optimal memoryless deterministic strategy.

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An analysis of stochastic shortest path problems, D. Bertsekas and J. Tsitsiklis, 1991, Mathematics of Operations Research.

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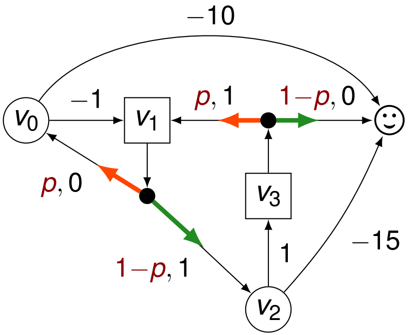

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Presence of non-negative cycles

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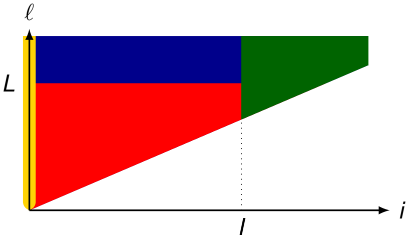

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Control the non-negative cycles with a partition of plays

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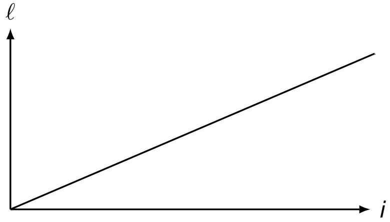
Focus on the partition of plays

ℓ size of play reaching the target
i number of non-negative cycles



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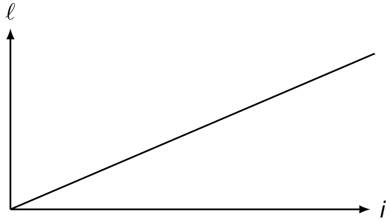

Focus on the partition of plays

ℓ size of play reaching the target
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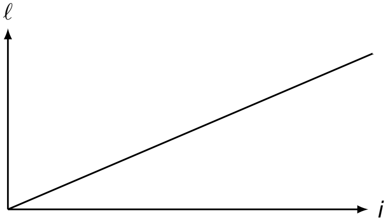
Fix a memoryless strategy for Eve

Good zones

SP $\leqslant$ dVal
$\Rightarrow \mathbb{E}(\textbf{SP}) \leqslant$ dVal



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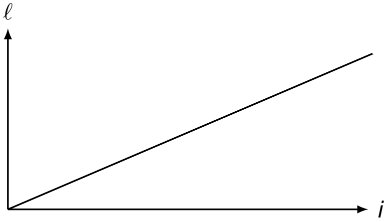

Zones to control

$\mathbb{E}(\textbf{SP}) \leqslant \varepsilon$

Focus on the partition of plays

ℓ size of play reaching the target
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Fix a memoryless strategy for Eve

Yellow zone

All plays conforming to σ_1

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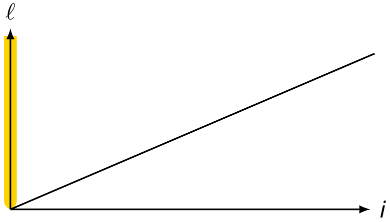

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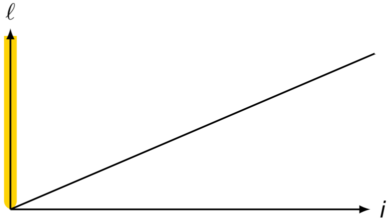

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Pseudopolynomial iterative algorithm to solve total-payoff games and min-cost reachability games, T. Brihaye, G. Geeraerts, A. Haddad and B. Monmege, 2016, Acta Informatica

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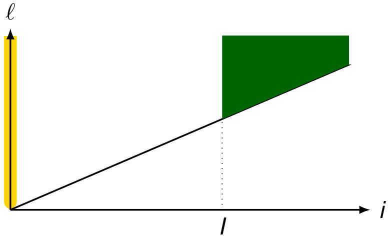
$\Rightarrow \mathbb{E}(\mathbf{SP}) \leqslant \text{dVal}$

Green zone
Plays contain many non-negative cycles

Zones to control
$\mathbb{E}(\mathbf{SP}) \leqslant \varepsilon$



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Weight of each play is $\leqslant$ dVal

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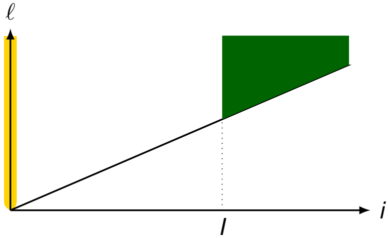
Plays contain many non-negative cycles
$\forall p, \exists I$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$



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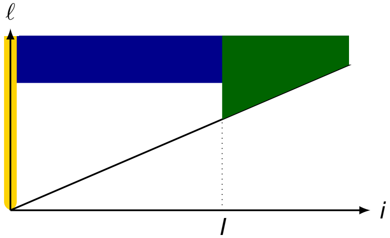

Zones to control

$\mathbb{E}(\textbf{SP}) \leqslant \varepsilon$

Blue zone

Plays with many negative cycles and few non-negative cycles

Focus on the partition of plays

ℓ size of play reaching the target

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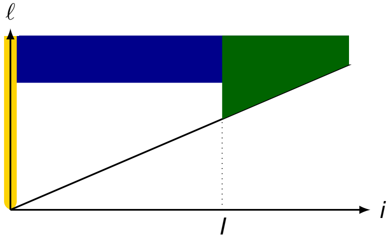

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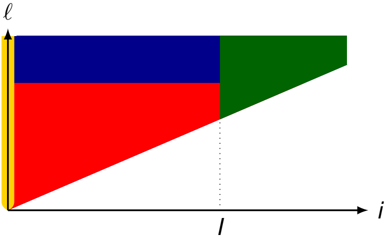

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Plays with many negative cycles and few non-negative cycles

$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Red zone

Rest of plays

Focus on the partition of plays

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Fix a memoryless strategy for Eve

Yellow zone
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Weight of each play is $\leqslant$ dVal

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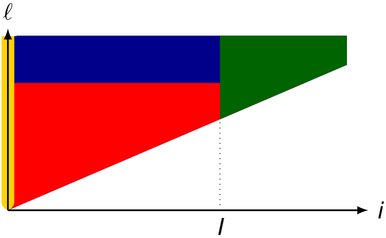

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Plays with many negative cycles and few non-negative cycles
$\forall p, I, \ \exists L$ s.t. weights $\leqslant$ dVal

Red zone
Rest of plays
$\exists p$ s.t. expectation $\leqslant \frac{\varepsilon}{2}$

Deterministic simulate memoryless

□ Adam ○ Eve

Claim

For all v and for all memoryless strategies ρ, there exists a deterministic strategy σ such that

$$\mathrm{dVal}^{\sigma}(v) \leqslant \mathrm{mVal}^{\rho}(v)$$

Deterministic simulate memoryless

Adam Eve

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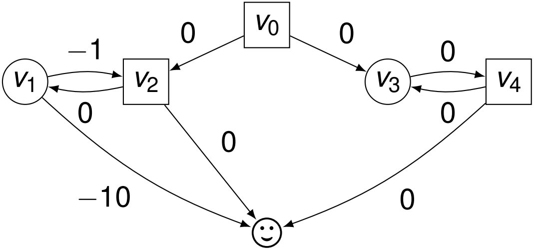

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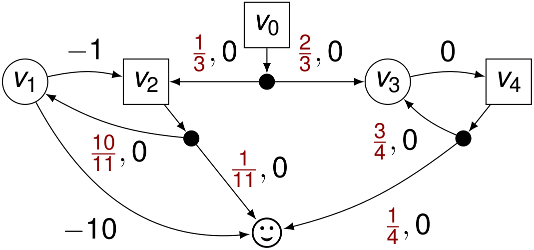

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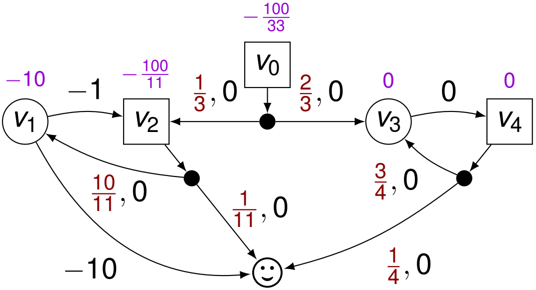

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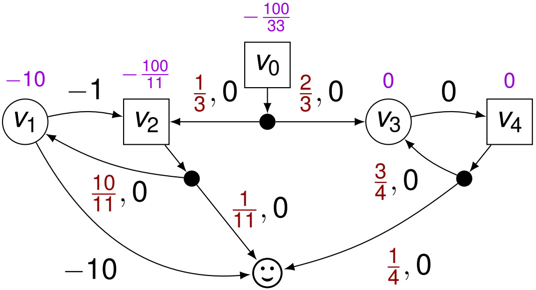

First Idea: Adam's strategy

- v_2: 10 times v_1 and 1 time ☺

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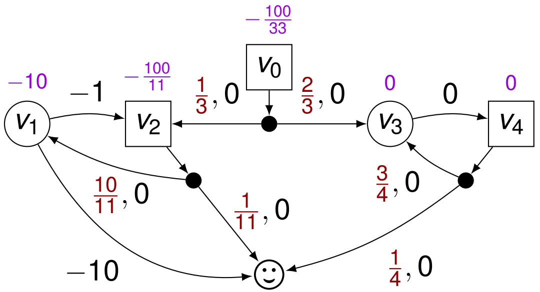

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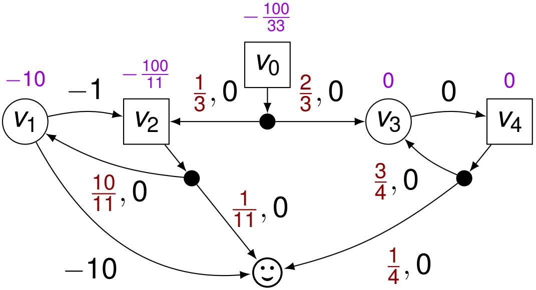

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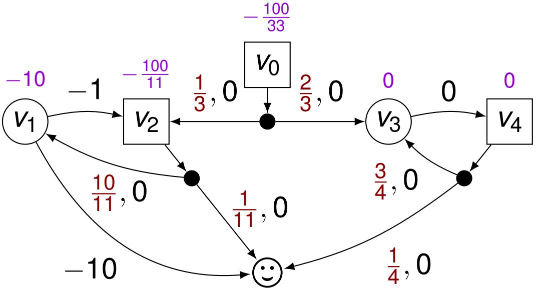

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Counter-example

$$\mathrm{dVal}^{\sigma}(v_1) = 0 > -\frac{100}{33} = \mathrm{mVal}^{\rho}(v_1)$$

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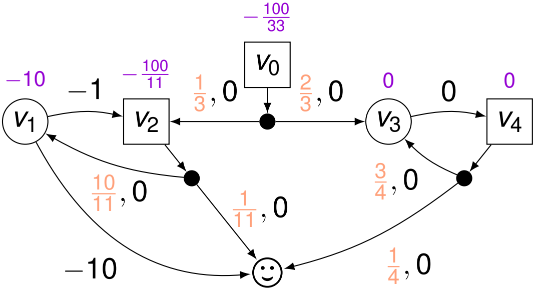

Tools for the proof

- Build a switching strategy $\sigma = \langle \sigma_1, \sigma_2 \rangle$ for ρ
- Value iteration for the fixpoint that gives the value

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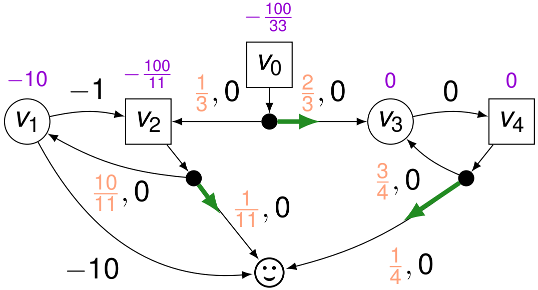

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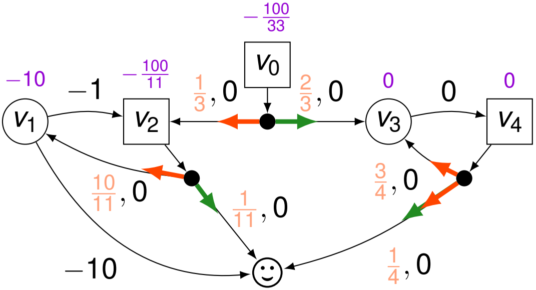

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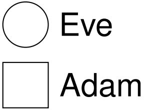
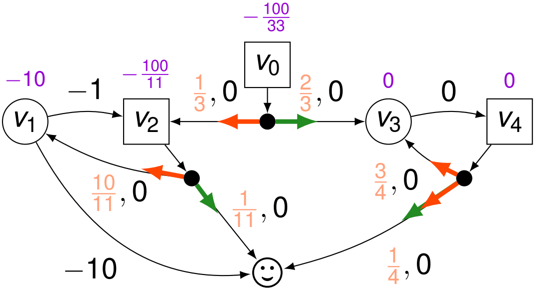
How do we choose the good vertex?





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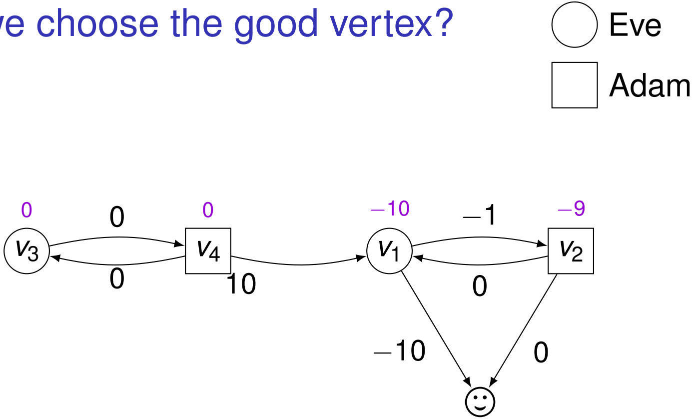

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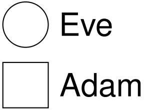
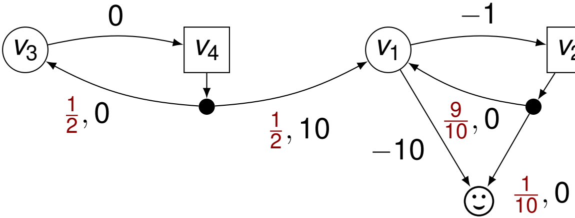
Let ρ be a memoryless strategy





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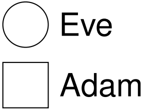
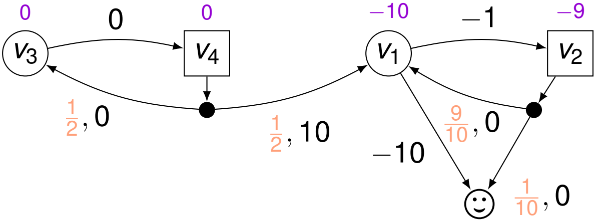


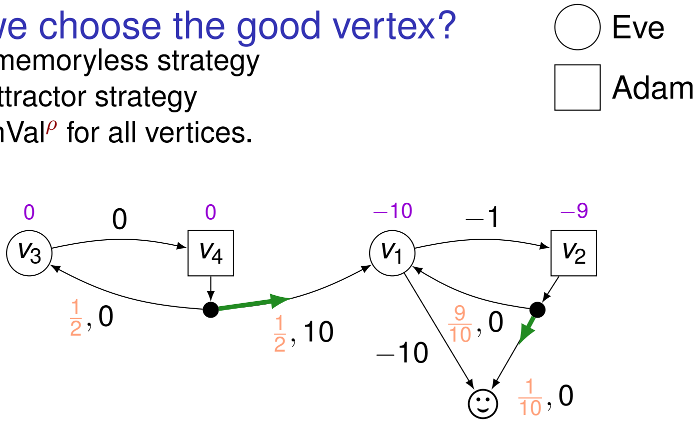
Let ρ be a memoryless strategy

We know mVal^{ρ} for all vertices.



How do we choose the good vertex?

Let ρ be a memoryless strategy
Fix σ_2 an attractor strategy
We know mVal^{ρ} for all vertices.



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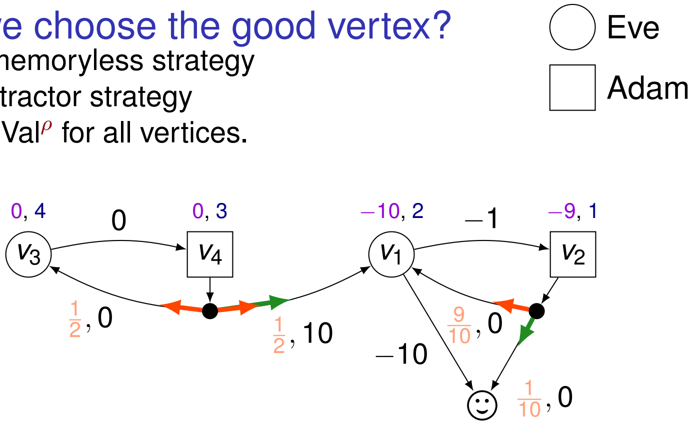

Attractor distance

How do we choose the good vertex?

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Fix σ_2 an attractor strategy
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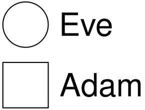
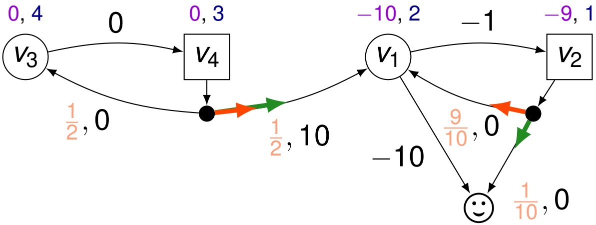
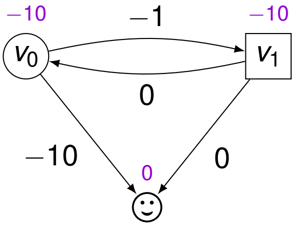




Attractor distance

$\sigma_1(v)$ chooses the minimal distance

Optimality



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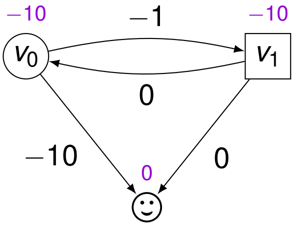

No optimal memoryless strategy

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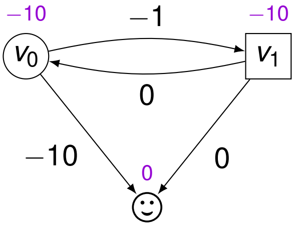

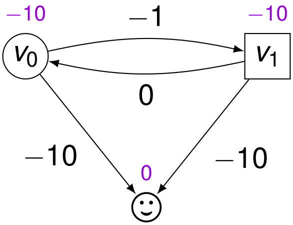

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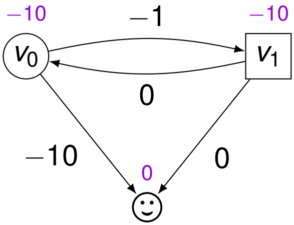

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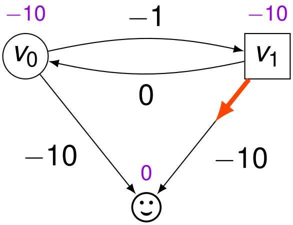
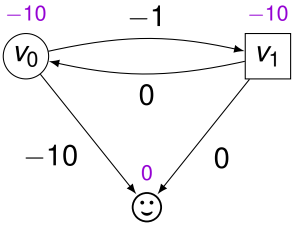
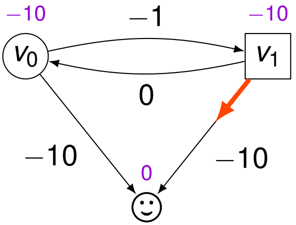


Optimal memoryless strategy

Optimality



No optimal memoryless strategy



Optimal memoryless strategy

Optimality proposition

1. We can characterize and test in polynomial time the existence of an optimal memoryless strategy.
2. A memoryless strategy optimal is deterministic.

Conclusion

Contributions

1. Adam has the same hope using memory or randomness.
2. Existence of an optimal memoryless strategy for Adam is testable in polynomial time.

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Perspectives

- A polynomial-time algorithm to compute the value
- Extension to probabilistic value (memory and randomisation)

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Thank you! Questions?