Evolution of error types in the understanding of fractions

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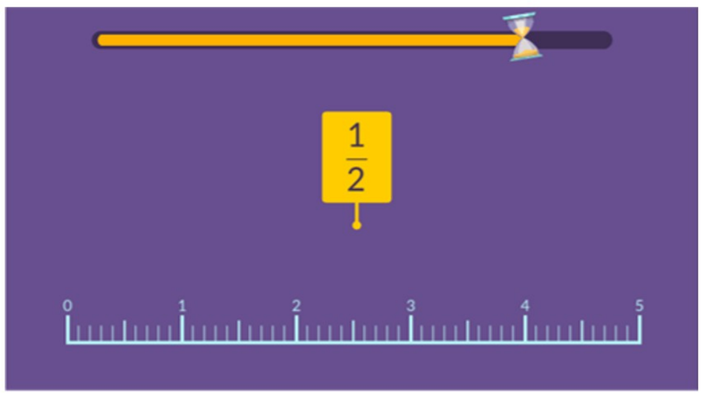
Introduction

Most children fail to manipulate fractions...
→ 42% of Greek 9th graders fail to order 1/7, 5/6, 1 and 4/3 [1]
→ 43% of US 8th graders fail on basic arithmetic operations with fractions [2]
→ 50% of Finnish 7th graders fail to shade 3/4 of an 8-piece bar [3]

...but non-failure does not even mean that children actually understand what they are doing!
→ Children may simply apply procedures memorized and reinforced by exercises [4]

How can we know whether a child truly understands fractions?
→ Number-to-line tasks are a great probe, because they force children to think about linear **magnitude** [5], a central representation for numbers in the brain [6], including for fractions [7].

A. The experiment



→ Computerized number-to-line task

→ 21 fractions, 5 per child

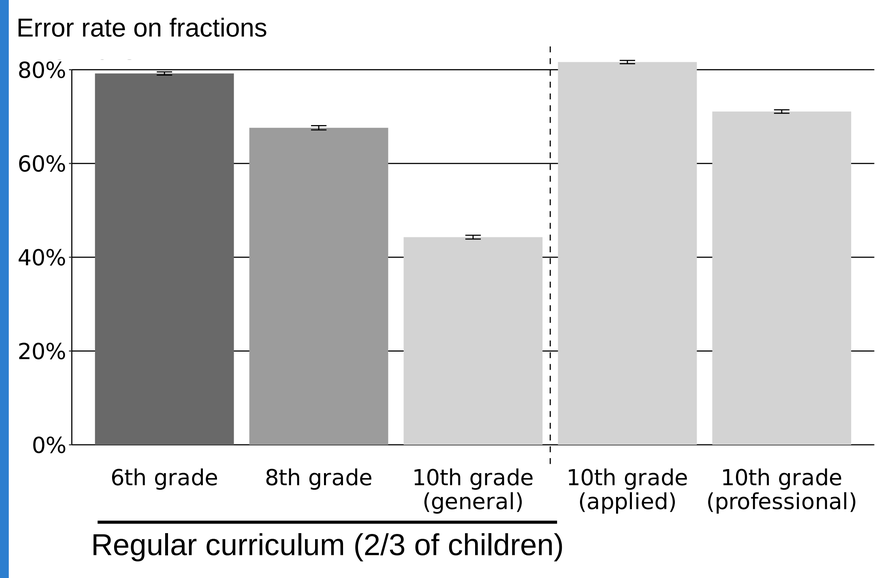
→ Data acquired at the start of school year 2022 (early September), in France

→ Representative sample of ~26,000 6th to 10th graders

Detailed sample :
- 5,707 6th graders
- 4,530 8th graders
- 15,787 10th graders (general : 6,076, professional : 5,517, applied : 4,194)

More maths in curriculum ⟷ Less maths in curriculum

B. Poor performance in all grades

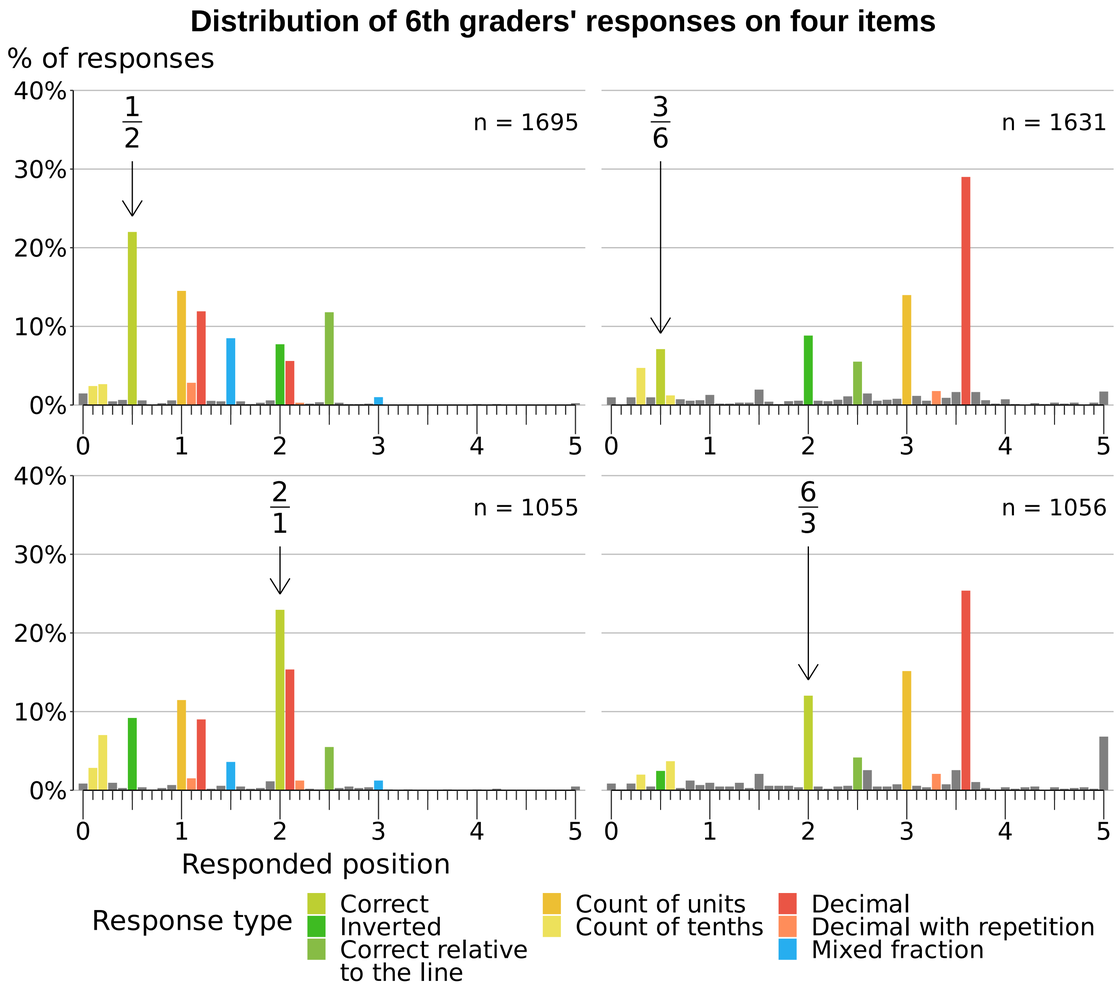


→ In France, children learn fractions from 4th to 6th grade

→ Even 10th graders struggle with them!

→ Improvements in later grades, but huge inequalities

C. Detail of errors in 6th grade



→ Children's reponse distributions are organised as "line spectra"

→ Seven error patterns explain 68% of 6th graders' errors

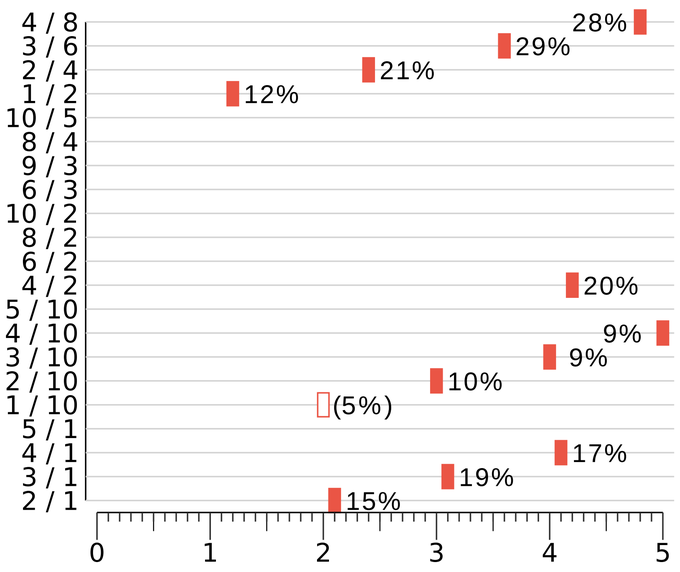
→ Distributions are more similar across inverted fractions than equivalent ones

→ We classify patterns of errors in three levels:

1) confusion with another type of number (**decimals** or **mixed numbers**)

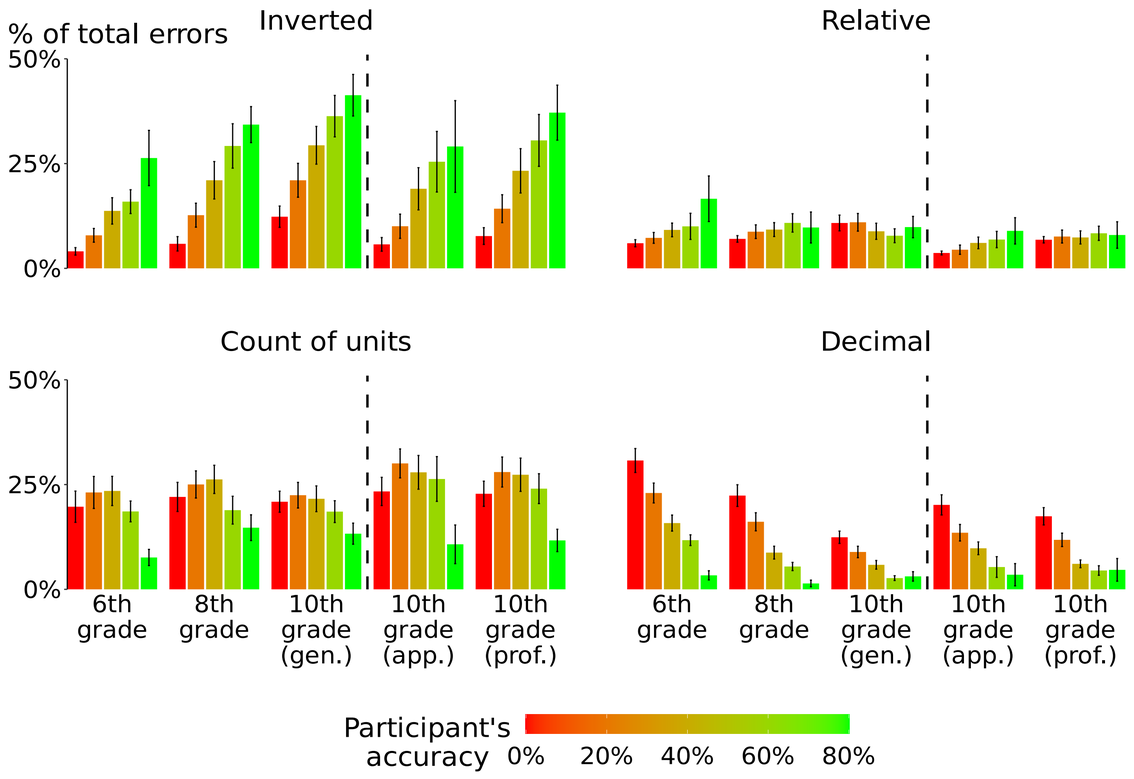
2) count of graduations (**units** or **tenths**)

3) wrong use of the correct fraction (applying it to the **line** itself or giving the **inverse** answer)



→ The main error in 6th grade seems to be a shallow one: confusing fractions and decimals

D. Predicting errors by grade and individual performance



→ Children in later grades make the same errors as 6th graders

→ **Decimal** errors progressively disappear, while **inversion** errors increase

→ Individual performance strongly predicts the type of error, more than grade

Conclusion

→ The number-to-line task is an effective tool to measure children's understanding of fractions, which remain very poor up to 10th grade.

→ Patterns of errors reveal three levels of understanding.

→ These levels define an order in the evolution of errors, within and across grades.

🧠 We propose that children's errors may indicate the presence of "bugs" [8] in their mental algorithms for processing fractions.

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