TA Intro to Logic 2025

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1 Well-formed Formulae

Exercise 1
Are the following formulae well-formed?

(i) $((p \wedge q \vee r) \rightarrow q)$

(ii) $\wedge \vee pq \rightarrow rq$

(iii) $(\neg(p \wedge q) \rightarrow (r \wedge s \wedge t))$

(iv) $(\neg(p \rightarrow q) \vee (r \wedge \neg r))$

2 Logical Entailment

Exercise 2 [from Kleene and Ruyer]
Arthur, Bao, and Clara are accused of painting the neighbor's cat green. They make the following statements:

> **Arthur:** Bao is guilty and Clara is innocent.
> **Bao:** If Arthur is guilty, Clara is too.
> **Clara:** I am innocent, but at least one of the other two is guilty.

Translate each statement into the language of propositional logic, with propositional letters: a = 'Arthur is guilty', b = 'Bao is guilty', c = 'Clara is guilty'. Then, with the help of a truth table, answer the following questions:

(i) If Clara lied, what can be said of Arthur's statement?

(ii) Under the same hypothesis, what can be said of Bao's statement?

(iii) Assuming that all told the truth, who is innocent and who is guilty?

*Exercises amiably provided by Quentin Blomet

(iv) Assuming that all are guilty, who lied and who told the truth?

(v) Assuming that every innocent told the truth and every culprit lied, who is innocent and who is guilty?

(vi) How should be answered the question "Assuming that every innocent lied and every culprit told the truth, who is innocent and who is guilty?"?

Exercise 3
Determine whether the following argument schemata are valid using truth tables.

(i) $p \to q \overset{?}{\models} q \to p$

(ii) $p \to q \overset{?}{\models} \neg p \to \neg q$

(iii) $p \to q \overset{?}{\models} \neg q \to \neg p$

(iv) $p, p \to q \overset{?}{\models} \neg p \to \neg q$

(v) $p \vee q, r \to p, r \overset{?}{\models} q$

(vi) $p \wedge q \overset{?}{\models} q \vee p$

(vii) $p \to (q \to r), (q \to r) \to p, p \overset{?}{\models} r$

(viii) $(p \wedge q) \to r \overset{?}{\models} p \to (q \to r)$

(ix) $p \to q, \neg q \overset{?}{\models} p$

(x) $p \leftrightarrow \neg q \overset{?}{\models} \neg(p \wedge q)$