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The objective of this TA session is to automatize the production of truth tables in order to build your intuitions from them. Even when not explicitly asked, feel free to refer to them.

1 Languages

Exercise 1. What is the difference between $A \wedge B \to C$, and $A, B \models C$?

2 Truth tables

Exercise 2. Write down the truth tables for the following formulaes. What can you say about them ?

1. $p \to q$
2. $p \vee \neg p$
3. $q \wedge \neg q$
4. $(p \vee q) \wedge \neg p$
5. $p \to (q \wedge \neg q)$
6. $q \to (r \vee \neg r)$
7. $(\neg p \to \neg q) \to (q \to p)$
8. $(p \to q) \wedge (r \to p) \wedge (q \to r)$
9. $(p \to \neg p) \to p$
10. $(p \to \neg p) \to \neg p$

Exercise 3. Using a truth table, prove the semantic deduction theorem (seen during previous lecture). Using this very theorem, can you tell whether the following formulaes are tautologies or not ?

1. $((p \vee q) \to r) \to (p \to r)$
2. $((p \wedge q) \to r) \to (p \to r)$
3. $((p \vee q) \to r) \to (p \to r)$
4. $\neg p \to (p \to q)$
5. $(p \to \neg p) \to p$
6. $(p \to \neg p) \to \neg p$

*Template and exercise 1 amiably provided by M. Sablé-Meyer

3 Satisfiability

Exercise 4. Are the following sets satisfiable ? Justify it using a truth table. How would you solve the exercise without it ?

1. $\{p, p \to q\}$
2. $\{p \vee \neg p, q \wedge \neg q\}$
3. $\{p \to q, q \wedge \neg q\}$
4. $\{(p \vee q) \wedge \neg p, \neg q\}$
5. $\{p \to (q \wedge \neg q), p\}$
6. $\{p \to (q \wedge \neg q), \neg q\}$
7. $\{q, q \to (r \vee \neg r)\}$
8. $\{(p \to q), (r \to p), (q \to r), \neg r\}$
9. $\{(p \to q), (r \to p), (q \to r), q\}$
10. $\{(p \to q), (r \to p), (q \to r)\}$

Exercise 5*. [1] Find a set Σ of 5 formulaes so that

— Σ is contradictory

— Every subset of 4 formulaes of Σ is satisfiable.

Exercise 6*. Is the infinite set Σ defined below satisfiable ? If so, what are the valuation(s) that satisfy it ?

$$\Sigma = \{p_1 \to p_2, p_2 \to p_3, \ldots, p_i \to p_{i+1}, \ldots\}$$

1. Star '*' exercises are harder than others. Build up your habits with other exercises beforehand !