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1 Finding equivalent formulae

Exercise 1. For each of the formulae below, write a formula that is equivalent to it and whose only connectives are $\neg$ and $\vee$.

1. $(\neg a \wedge (b \to c)) \to \neg(\neg b \vee c)$
2. $(a \vee \neg(b \wedge c)) \wedge \neg(d \to (\neg a \wedge e))$

Exercise 2. Same thing with $\neg$ and $\wedge$

2 Induction

Exercise 3.* Below are two definitions by inductions on formulaes. How could you describe the corresponding notions in English ?

1. For any atom p, and any binary connector $\bullet$:
$p^\star = 0$
$(\neg\phi)^\star = \phi^\star$
$(\phi \bullet \psi)^\star = \phi^\star + \psi^\star + 1$

2. For any binary connector $\bullet$:
$p^+ = 1$
$(\neg\phi)^+ = \phi^+$
$(\phi \bullet \psi)^+ = \phi^+ + \psi^+$

Can you prove by induction that $\phi^+ = \phi^\star + 1$?

3 Either NAND or NOR

Exercise 4. Reproduce the truth-table defining the Scheffer stroke $\uparrow$. How could you paraphrase $A \uparrow B$ using plain English ? Show that a language whose only connective is $\uparrow$ is as expressive as the one based on $\{neg, \wedge, \vee, \to\}$

Exercise 5. Now considering a connective $\downarrow$ that could be paraphrased by 'Neither A nor B', write down its truth-table. Again, show that a language whose only connective is $\downarrow$ is as expressive as the one based on $\{neg, \wedge, \vee, \to\}$

*Template and exercise 3 amiably provided by M. Sablé-Meyer

4 Naive set theory

Exercise 6. For each statement below, tell whether it is true or false.

1. $\{x : x \text{ is a dog}\} \subseteq \{x : x \text{ is a mammal}\}$
2. $\{x : x \text{ is a dog}\} \subseteq \{y : y \text{ is a dog}\}$
3. $\{x : x \text{ is a mammal}\} \subseteq \{x : x \text{ is a cow}\}$

Exercise 7. Same with the statements below

1. $a \in \{a, b, c\}$
2. $a \subseteq \{a, b, c\}$
3. $\{a\} \in \{a, b, c\}$
4. $\{a\} \subseteq \{a, b, c\}$
5. $\{a\} \in \{\{a\}, b, \{a, c\}\}$
6. $\{a\} \subseteq \{\{a\}, b, \{a, c\}\}$
7. $\{a, b, c\} \in \{\{a\}, \{a, b, c\}, \{\{b, c\}\}\}$
8. $\{a, b, c\} \subseteq \{\{a\}, \{a, b, c\}, \{\{b, c\}\}\}$
9. $\emptyset \in \{\emptyset, \{a\}\}$
10. $\emptyset \subseteq \{\emptyset, \{a\}\}$
11. $\emptyset \in \{\{a\}\}$
12. $\emptyset \subseteq \{\{a\}\}$