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1 Natural Deduction

1.1 Conjunction and Disjunction

Show that

$$1. \quad \frac{A \wedge (B \wedge C)}{C \wedge A}$$

$$4. \quad \frac{A \vee (A \wedge C)}{A}$$

$$2. \quad \frac{A \wedge (B \wedge C)}{(A \wedge B) \wedge C}$$

$$5. \quad \frac{A \wedge (B \vee C)}{(A \wedge B) \vee (A \wedge C)}$$

$$3. \quad \frac{A \quad B}{A \wedge (B \vee C)}$$

$$6. \quad \frac{A \vee B \quad A \vee C}{A \vee (B \wedge C)}$$

1.2 Negation and Contradiction

Show that

$$1. \quad \frac{A}{\neg\neg A}$$

$$3. \quad \frac{A \vee B}{A \vee \neg\neg B}$$

$$2. \quad \frac{\neg\neg A}{A}$$

$$4. \quad \frac{A \vee B \quad \neg B \vee C}{A \vee C}$$

1.3 Implication

Show that

$$1. \quad \frac{A \vee B}{(A \rightarrow B) \rightarrow B}$$

$$2. \quad \frac{A \rightarrow (B \rightarrow C)}{(A \rightarrow B) \rightarrow (A \rightarrow C)}$$

$$3. \quad \frac{A \wedge \neg B}{\neg(A \rightarrow B)}$$

$$4. \quad \frac{}{(A \rightarrow B) \rightarrow (\neg B \rightarrow \neg A)}$$

$$5. \quad \frac{A \rightarrow \neg B}{B \rightarrow \neg A}$$

1.4 Complex Proofs

Show that

$$1. \quad \frac{}{A \vee \neg(A \wedge B)}$$

$$2. \quad \frac{}{(A \wedge B) \vee \neg A \vee \neg B}$$

$$3. \quad \frac{}{\neg A \vee \neg(\neg B \wedge (\neg A \vee B))}$$

$$4. \quad \frac{A \vee (B \wedge C) \quad \neg B \vee \neg C \vee D}{A \vee D}$$

$$5. \quad \frac{}{\neg(A \rightarrow B) \rightarrow (A \wedge \neg B)}$$