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1 Translation into predicate logic

Translate the following sentences into predicate logic

- (i) A door is open or closed.
- (ii) All truths are in the Bible.
- (iii) Frogs are more intelligent than some humans.
- (iv) If frogs are more intelligent than some humans, then there is some frog more intelligent than some humans.
- (v) All that glitter is not gold.
- (vi) There are pains and pleasures, but no pain is a pleasure.
- (vii) The only pains that are a pleasure are love pains.
- (viii) There are good deeds that are not rewarded, but no evil deed is rewarded.
- (ix) Some like it hot.

2 Some semantics

In this exercise, we will consider a standard French-suited card game. We provide the following predicates, with their canonical meanings.

- Color predicates : B(lack) and R(ed).
- Suit predicates : S(pade), C(lubs), H(earts), D(iamonds).
- Binary comparison predicates : $(>^{(2)}, <^{(2)}, \geq^{(2)}, \leq^{(2)})$.

Cards will be designated using similar conventions : $c_{s,1}$ for the ace of spades, $c_{s,q}$ for the queen of spades... For simplicity, we will also use comparison predicates as per the standard mathematical notation, i.e., $c_{h,1} < c_{c,2}$.

*Template and exercise 1 amiably provided by M. Sablé-Meyer

Tell whether the following fomulaes are true or false under the definitions above. Pay attention to the effect of quantifiers on implications! (Note : all variables should be bound – if not, this is an oversight on my side)

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| (i) $R(c_{p2})$ | (x) $\forall x(\forall y(C(x) \wedge C(y)) \wedge ((x \geq y) \vee (x \leq y)))$ |
| (ii) $R(c_{p,2}) \vee R(c_{h,k})$ | (xi) $\forall x(\exists y(C(x) \wedge C(y)) \wedge ((x \geq y) \wedge (x \leq y)))$ |
| (iii) $\exists x(H(x))$ | (xii) $\exists x(\neg(\exists y((C(x) \wedge C(y)) \wedge (x \geq y))))$ |
| (iv) $\forall x(H(x))$ | (xiii) $\forall x(H(x) \rightarrow R(x))$ |
| (v) $\forall x(B(x) \vee R(x))$ | (xiv) $\exists x(H(x) \rightarrow B(x))$ |
| (vi) $\exists x(B(x) \wedge R(x))$ | (xv) $\exists x(B(x) \rightarrow H(x))$ |
| (vii) $\forall x(\forall y(B(x) \vee R(y)))$ | (xvi) $\forall x(\exists y(C(x) \wedge C(y)) \rightarrow (x > y))$ |
| (viii) $\exists x(\exists y(C(x) \wedge C(y) \wedge ((x > y) \vee (x < y))))$ | (xvii) $\forall x(\exists y(C(x) \wedge C(y)) \rightarrow (x \geq y))$ |
| (ix) $\exists x(\exists y(C(x) \wedge C(y)) \rightarrow ((x > y) \vee (x < y)))$ | |

3 Free/bound variables and substitution

For each formula ϕ , replace the free occurences of x by the variable c — pay attention to the scope of quantifiers and whether variables are free or bound!

- (i) $A(x, y)$
- (ii) $A(x, x)$
- (iii) $\forall x(A(x, x))$
- (iv) $A(y)$
- (v) $A(c, x)$
- (vi) $A(x, x) \wedge \exists x(B(x))$
- (vii) $\forall x(B(y))$
- (viii) $\exists x(\exists y(A(x, y) \rightarrow B(x)))$
- (ix) $\forall x(\forall y(A(y, y)) \rightarrow B(x))$