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1 Model

Construct a model in which the following statements are true:

- Every guest arrived with a present.
- Everybody saw the last guest.
- There is a guest that is unhappy.
- No one saw someone unhappy.

2 Satisfaction

2.1 Unary Predicates

Consider the language with only two unary predicate symbols O and E. Let $\mathfrak{M}$ be the model with domain $D = \{2, 4, 7\}$ and where the extension of O consists of those elements n of D such that n is odd, and the extension of E consists of those elements n of D such that n is even. For each of the following well-formed formulae, describe the sequences which satisfy it.

1. $E(x_3)$
2. $O(x_2)$
3. $\neg O(x_8)$
4. $\exists x_3 O(x_3)$
5. $\exists x_1 O(x_2)$
6. $O(x_3) \to \forall x_2 O(x_2)$
7. $\forall x_1 (O(x_1) \vee E(x_1))$
8. $O(x_1) \wedge \forall x_1 E(x_1)$

2.2 Binary Predicates

Consider the language with only one binary predicate symbol P and let $\mathfrak{M}$ be the model with domain $D = \{1, 2, 3\}$ and where the extension of P consists of those pairs $\langle n, m \rangle$ such that $m = n + 1$. For each of the following well-formed formulae, describe the sequences which satisfy it.

1. $P(x_1, x_2)$
2. $\exists x_3 P(x_3, x_2)$
3. $\forall x_2 P(x_1, x_2)$
4. $P(x_2, x_2)$
5. $\exists x_1 \neg P(x_1, x_1)$
6. $\forall x_3 P(x_3, x_3)$
7. $P(x_1, x_1) \vee P(x_2, x_3)$
8. $\exists x_1 (P(x_1, x_1) \vee P(x_2, x_4))$

3 Validity

Are the following inferences valid?

1. $\forall x P(x) \overset{?}{\models} \exists x P(x)$
2. $\exists x P(x) \overset{?}{\models} \forall x P(x)$
3. $\exists x (P(x) \wedge Q(x)) \overset{?}{\models} \exists x P(x) \wedge \exists x Q(x)$
4. $\exists x P(x) \wedge \exists x Q(x) \overset{?}{\models} \exists x (P(x) \wedge Q(x))$
5. $\exists x (P(x) \wedge Q(x)) \overset{?}{\models} \exists x (P(x) \to Q(x))$
6. $\exists x (P(x) \to Q(x)) \overset{?}{\models} \exists x (P(x) \wedge Q(x))$
7. $\forall x (P(x) \to Q(x)) \overset{?}{\models} \forall x (P(x) \wedge Q(x))$
8. $\forall x (P(x) \wedge Q(x)) \overset{?}{\models} \forall x (P(x) \to Q(x))$