Maxime Cauté
maxime.caute@college-de-france.fr*

May 5th, 2025

1 Modal truth values

Consider the following frame and valuation :

— $W_m = \{w_1, w_2\}$

— $R_m = \{(w_1, w_1), (w_1, w_2), (w_2, w_1)\}$

— $V_m(w_1, p) = 0, \quad V_m(w_2, p) = 1$

Draw a diagram of this model, and evaluate the truth of the following formulas in each world :

1. $\Box p$
2. $\Diamond p$
3. $\Box \neg p$
4. $\Diamond \neg p$
5. $\Box \Diamond p$
6. $\Diamond \Box p$
7. $\Box \neg \neg p$

2 Constraints on Kripke Models

For each sentence below, characterize the frames that make them valid.

1. $p \to \Box p$
2. $p \to \Diamond p$

3 The K-axiom

Rewrite the K-axiom and prove it

*Template amiably provided by M. Sablé-Meyer.

4 Negative Introspection

1. Explain why the following constraints are all equivalent.
   (a) For any sentence S, if I don't believe S, I believe that I don't believe S.
   (b) For any sentence S, $\neg\Box S \rightarrow \Box\neg\Box S$
   (c) For any sentence S, $\Diamond S \rightarrow \Box\Diamond S$
2. Show that they for any Kripke Model, they are also equivalent to the following constraint.

   > If a world is accessible, it is accessible from all accessible worlds. In other words : all accessible worlds are accessible from each other.

   $$\forall u \forall v \forall w \left(uRv \wedge uRw \rightarrow vRw \right)$$

3. What can you tell about R's properties ?