Include Course Administrative Data
CEA CAPA Paris Study Center
Course Code & Course Title
Instructor Name: Lecoq Raphaël
Date: 4th of June, 2026

Midterm Examination

Time Allotted: 120 Minutes

Student Name or Student CEA CAPA ID number : ____

EXAMINATION RULES

1. Desks must be cleared of all books, notes and papers. All unauthorized materials must be put away and remain out of sight throughout the examination.
2. Special needs or accommodations must be discussed with and approved by the local Academic Director or Center Director before the examination begins.
3. All telephones and electronic devices must be turned off and stored out of reach during the examination.
4. Talking to or communicating with other students during the exam is forbidden. Should assistance be required, please attract the attention of the instructor.
5. Anyone found cheating will earn a “0” on this examination. Additional sanctions may apply.
6. Anyone arriving after the start of the examination will not be allowed into the classroom to take the examination, subject to the decision of the instructor.
7. At the end of the exam, remain seated until your blue book is collected. No one may leave the room until excused.
8. Violations of these rules will result in a “0” on this examination. Additional sanctions may apply.

The exam is 2 hours long.
No calculator, use of internet or use of AI are allowed.
Solve any exercises you want to achieve 70 points
. Exercises with a ($\star$) are longer and may be done after the shorter ones.

1. (10 points) Pay attention to clarity and readability.

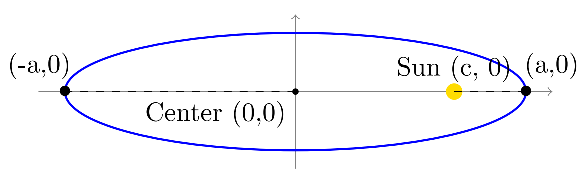
Section 10

2. (5 points) Classify the following conics as *circle*, *ellipse*, *parabola* or *hyperbola* (explain your reasoning).
   A. $y^2 + 6y + 8x + 25 = 0$.
   B. $9x^2 + 4y^2 + 36x - 24y = 36$.
   C. $x^2 - 2x - 4y - 7 = 0$.
   D. $y^2 = 16x^2 - 64x + 208$.
   E. $y^2 - 4y = x + 5$.
   F. $25x^2 - 10x - 200y - 119 = 0$.

3. (10 points) ($\star$)

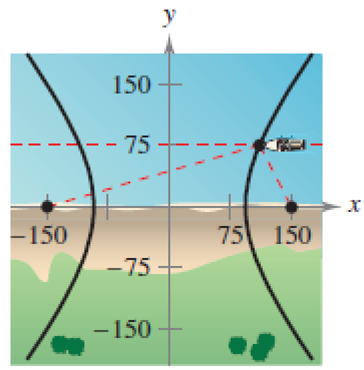
   (a) Find the vertex, focus, and directrix of the following parabola, and sketch its graph :
   $$y^2 + 6y + 8x + 25 = 0.$$
   (b) Find the center, foci, vertices, and eccentricity of the following ellipse, and sketch its graph :
   $$(x + 4)^2 + 4(y + 6)^2 = 1.$$
   (c) Find the center, foci, vertices, and eccentricity of the following hyperbola, and sketch its graph using asymptotes as an aid :
   $$9x^2 - y^2 - 36x - 6y + 18.$$

4. (4 points) ($\star$) Probably the most famous of all comets, Halley's comet, has an elliptical orbit with the sun at one focus. Its maximum distance from the sun is approximately 35.29 AU (1 astronomical unit is approximately 92.956×106 miles), and its minimum distance is approximately 0.59 AU.
Find the eccentricity of the orbit.

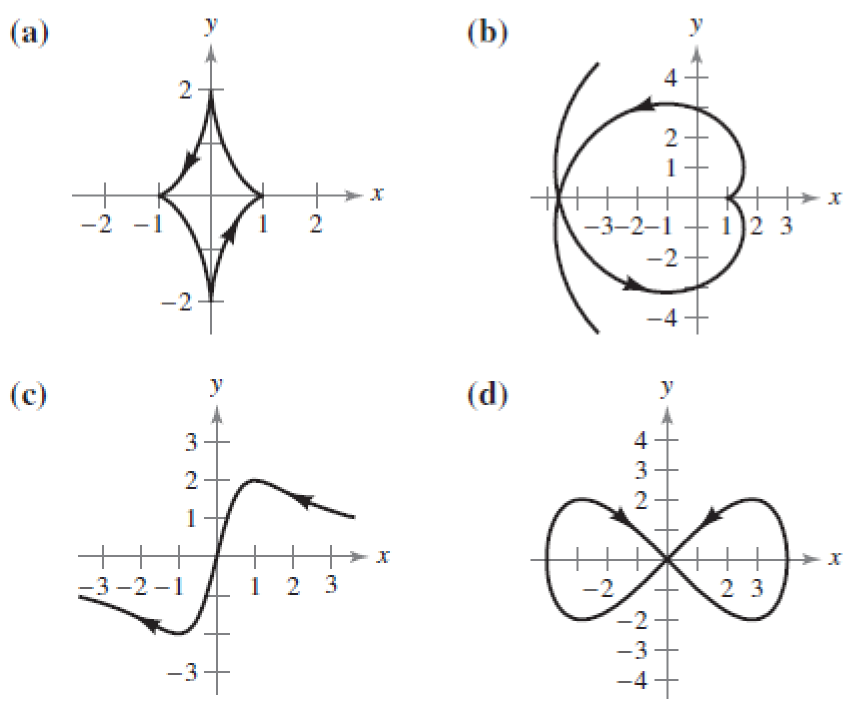


5. (3 points) True or False ? If it is false, explain why or give an example.
   (a) The graph of the parametrics equations $x = t^2$ and $y = t^2$ is the line $y = x$.
   (b) If y is a function of t and x is a function of t then y is a function of x.
   (c) The curve represented by the parametrics equations $x = t$ and $y = \cos t$ can be written as an equation of the form $y = f(x)$.

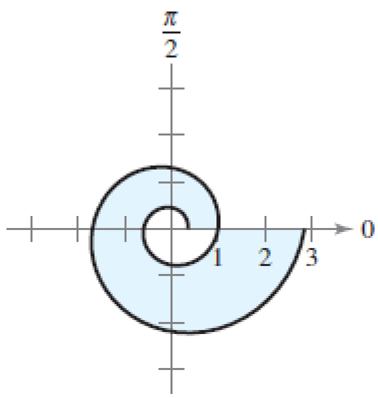
6. (10 points) ($\star$) LORAN (long distance radio navigation) for aircraft and ships uses synchronized pulses transmitted by widely separated transmitting stations. These pulses travel at the speed of light (186,000 miles per second). The difference in the times of arrival of these pulses at an aircraft or ship is constant on a hyperbola having the transmitting stations as foci. Assume that two stations, 300 miles apart, are positioned on a rectangular coordinate system at (-150, 0) and (150, 0) and that a ship is traveling on a path with coordinates (x, 75) (see figure).
   Find the x-coordinate of the position of the ship when the time difference between the pulses from the transmitting stations is 1000 microseconds (0.001 second).



7. (4 points) Match each set of parametrics equations to a graph. Explain your reasoning:
   A. Lissajous curve : $x = 4\cos\theta,\ y = 2\sin 2\theta$.
   B. Evolute of ellipse : $x = \cos^3\theta, y = 2\sin^3\theta$.
   C. Involute of circle: $x = \cos\theta + \theta\sin\theta,\ y = \sin\theta - \theta\cos\theta$.
   D. Serpentine curve: $x = \cot\ \theta,\ y = 4\sin\theta\cos\theta$.



8. (7 points) ($\star$) The curve represented by the equation $r = ae^{b\theta}$, where a and b are constants, is called a logarithmic spiral. The figure shows the graph of $r = e^{\theta/6}$, $-2\pi \leq \theta \leq 2\pi$. Find the area of the shaded region.



9. (15 points) ($\star$) Use the parametric equations $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$, $a > 0$, to answer the following.

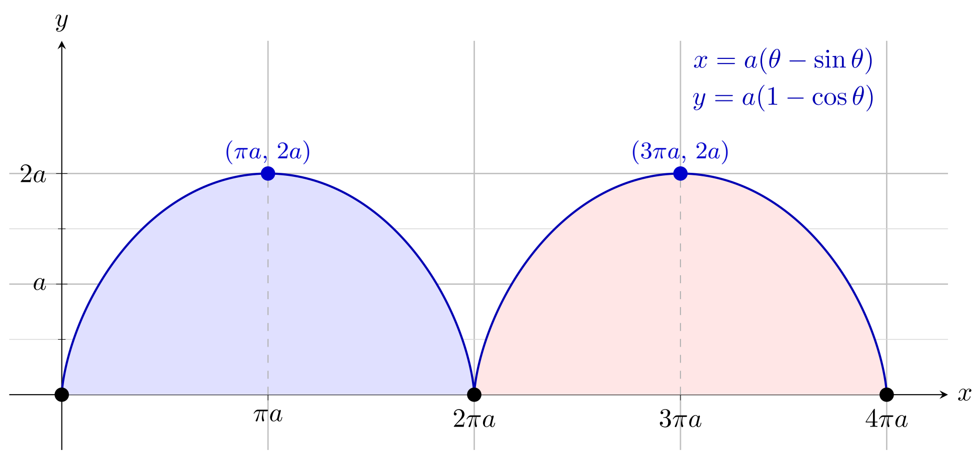
   (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

   (b) Find the equation of the tangent line at the point where $\theta = \pi/6$.

   (c) Find all points of horizontal tangency.

   (d) Determine where the curve is concave upward or concave downward.

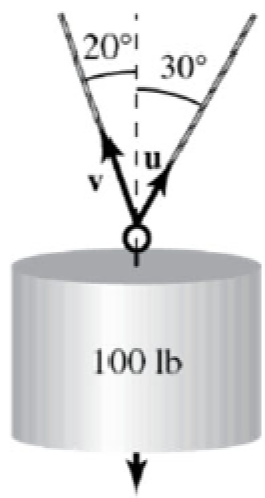
   (e) Find the length of one arc of the curve.



Section 11

10. (3 points) The initial and terminal points of a vector $\boldsymbol{v}$ are given: P(2,1,3) and Q(-1,2,1).
    (a) Sketch the given directed line segment.
    (b) Write the vector in a component form.
    (c) Write the vector as the linear combination of standard unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$.
    (d) Sketch the vector with its initial point at the origin.
    (e) Calculate the magnitude (norm) of the vector.
    (f) Indicate the unit vector associated with $\boldsymbol{v}$.

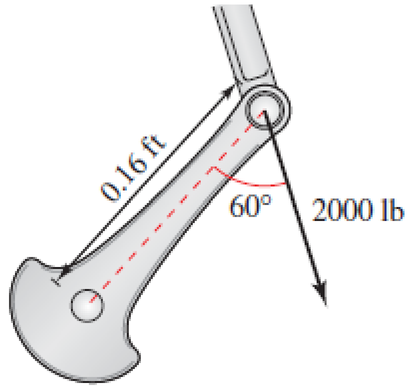
11. (6 points) ($\star$) To carry a 100-pound cylindrical weight, two workers lift on the ends of short ropes tied to an eyelet on the top center of the cylinder. One rope makes a 20° angle away from the vertical and the other makes a 30° angle (see figure).
    (a) Find each rope's tension when the resultant force is vertical.
    Hint : To lift the weight vertically, the sum of the vertical components of $\boldsymbol{u}$ *and* $\boldsymbol{v}$ *must be 100 and the sum of the horizontal components must be 0.*
    (b) Find the vertical component of each worker's force.
    $\sin 20 = 0.34$ *and* $\cos 20 = 0.94$



12. (3 points) Let $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j}$, $\boldsymbol{v} = \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{w} = a\boldsymbol{u} + b\boldsymbol{v}$.
    (a) Sketch $\boldsymbol{u}$ and $\boldsymbol{v}$.
    (b) If $\boldsymbol{w} = 0$, show that a and b must be both 0.
    (c) Find a and b such that $\boldsymbol{w} = \boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$.

13. (5 points) (a) ($\star$) Prove the triangular inequality : $\|\boldsymbol{u} + \boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$.
    Hint : use $\|\boldsymbol{u}\|^2 = \boldsymbol{u} \cdot \boldsymbol{u}$.
    (b) Make a drawing highlighting this property.

14. (7 points) ($\star$) Both the magnitude and the direction of the force on a crankshaft change as the crankshaft rotates. Find the torque on the crankshaft using the position and data shown in the figure.

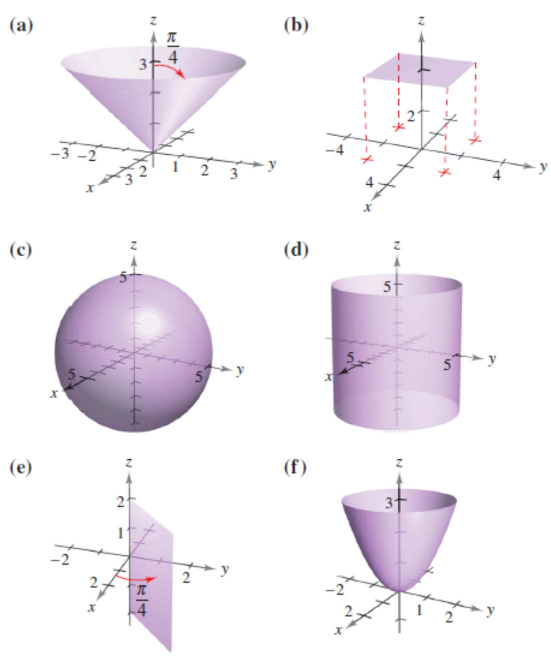


15. (3 points) (a) Find the set of parametric equations of the line that passes through the point (-2,0,3) and is parallel to the vector $\boldsymbol{v} = 2\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$.

    (b) Find the equation of the plane that passes through the point (3,2,2) and is perpendicular to the vector $\boldsymbol{n} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$.

16. (2 points) Convert the point (2,-2,-4) from rectangular coordinates to cylindrical coordinates, and to spherical coordinates.

17. (4 points) Match the correspond equation (written in cylindrical coordinates or spherical coordinates) to its graph. A. $r = 5$. B. $\theta = \pi/4$. C. $\rho = 5$. D. $\Phi = \pi/4$. E. $r^2 = z$. F. $\rho = 4/\cos\Phi$.



Section 12

18. (2 points) (a) Evaluate the vector-valued function $\boldsymbol{r}(t) = \begin{pmatrix} \cos t \\ 2\sin t \end{pmatrix}$ at each given value of t.

    A. $t = 1$. B. $t = 0$. C. $\boldsymbol{r}(1+s)$. D. $\boldsymbol{r}(t+\Delta t) - \boldsymbol{r}(t)$.

    (b) Compute $\boldsymbol{r}'(t)$.

19. (4 points) True or False ? If false, explain why or give an example that shows it is false.

    (a) If a particle moves along a sphere centered at the origin, then its derivative vector is always tangent to the sphere.

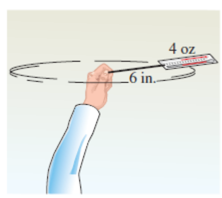
    (b) The definite integral of a vector-valued function is a real number.

    (c) $\dfrac{\mathrm{d}}{\mathrm{d}t}[\|\boldsymbol{r}(t)\|] = \|\boldsymbol{r}'(t)\|$.

    (d) If $\boldsymbol{r}$ and $\boldsymbol{u}$ are differentiable vector-valued functions of t, then

$$\frac{\mathrm{d}}{\mathrm{d}t}[\boldsymbol{r}(t) \cdot \boldsymbol{u}(t)] = \boldsymbol{r}'(t) \cdot \boldsymbol{u}'(t).$$

20. (7 points) ($\star$) A psychrometer (an instrument used to measure humidity) weighing 4 ounces is whirled horizontally using a six-inch string (see figure). The string will break under a force of 2 pounds. Find the maximum speed in feet/s the instrument can attain without breaking the string. (Use $\boldsymbol{F} = m\boldsymbol{a}$, where $m = 1/128$.).
    Hint : for a rotating motion, $\|\boldsymbol{a}(t)\| = b\omega^2$ where b is the radius of the circle described by $\boldsymbol{r}(t)$.
    Hint : 12 inches is 1 foot and 1 pound is 16 ounces.



21. (4 points) An object moves along the path given by $\boldsymbol{r}(t) = 3t\boldsymbol{i} + 4t\boldsymbol{j}$.

    (a) Find $\boldsymbol{v}(t)$, $\boldsymbol{a}(t)$, $\boldsymbol{T}(t)$, and $\boldsymbol{N}(t)$ (if it exists).

    (b) What is the form of the path?

    (c) Is the speed of the object constant or changing?

22. (3 points) Find $\boldsymbol{r}(t)$ that satisfies the initial condition

$$\boldsymbol{r}'(t) = 2t\boldsymbol{i} + e^t\boldsymbol{j} + e^{-t}\boldsymbol{k}, \qquad \boldsymbol{r}(0) = \boldsymbol{i} + 3\boldsymbol{j} - 5\boldsymbol{k}.$$

23. (13 points) ($\star$) The quarterback of a football team releases a pass at a height of 7 feet above the playing field, and the football is caught by a receiver 30 yards directly downfield at a height of 4 feet. The pass is released at an angle of 35° with the horizontal.

    (a) Find the speed of the football when it is released.
    Hint : 30 yards = 90 feet. Don't compute $\cos 35$ *or* $\sin 35$.

    (b) Find the maximum height of the football.

    (c) Find the time the receiver has to reach the proper position after the quarterback releases the football.

Question	Points	Score
1	10	
2	5	
3	10	
4	4	
5	3	
6	10	
7	4	
8	7	
9	15	
10	3	
11	6	
12	3	
13	5	
14	7	
15	3	
16	2	
17	4	
18	2	
19	4	
20	7	
21	4	
22	3	
23	13	
Total:	134	