Quantum Pseudorandomness Cannot be Shrunk in a Black-Box Way

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Classical Pseudorandomness

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Randomness in cryptography

Randomness is essential to build cryptographic primitives.
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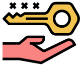

The relevance of One-Way Functions

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- For example, a hash function has to be a one-way function.
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Theorem

One-way functions and Pseudorandom Number Generators are equivalent, in a black-box way.

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Black-box impossibility results

A black-box impossibility result of A from B consist of exhibiting an oracle O such that, relative to O, B exists but not A.

Some results about classical cryptography

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$\exists$  $\Rightarrow P \neq NP$

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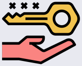
$\exists$  $\Rightarrow \exists$ 

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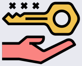
$\exists$  $\Rightarrow \exists$ 

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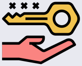
$\exists$  $\not\Rightarrow \exists$ 

Worlds of cryptography

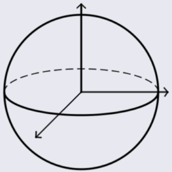
Different worlds where we might live in (Imp'95):

- 😰 **Algorithmica** P=NP
- 😵‍💫 **Heuristica** NP problems are easy on average but hard on the worst case
- 😟 **Pessiland** P$\neq$NP but $\nexists$ one-way function.
- 😌 **Minicrypt** One-Way Functions exist!
- 🤩 **Cryptomania** Public Key Encryption exists!

Quantum Randomness

We can also consider quantum randomness.

The equivalent to the uniform distribution is the *Haar measure* μ_{2^n}.



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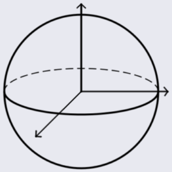

Quantum Pseudorandom States Generators

A function $F : \{0,1\}^\lambda \to \left(\mathbb{C}^2\right)^{\otimes n}$ is a Pseudorandom Quantum State generator (PRS) if:

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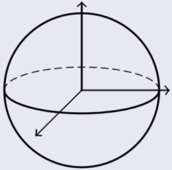

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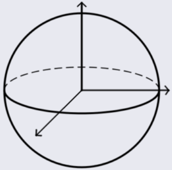

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Formal definition of PRSs

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A keyed family of n-qubit quantum states $\{|\varphi_k\rangle\}_{k\in\{0,1\}^\lambda}$ is *pseudorandom* if the following two conditions hold:

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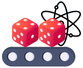
If $n \approx \lambda$, it is a **long-PRS**, or just PRS. 

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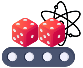
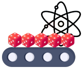
If $n \approx \lambda$, it is a **long-PRS**, or just PRS. 

If $n \approx \log\lambda$, it is a **short-PRS**. 

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Relation between quantum primitives

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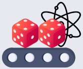
 $\Longrightarrow$ 

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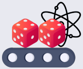
 $\Rightarrow$ 

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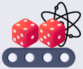 $\not\Rightarrow$ 

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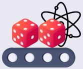
 $\Longrightarrow$ 

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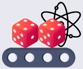 $\not\Longrightarrow$ 

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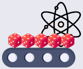 $\Longrightarrow$ 

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Idea: Use Tomography, with cost $O(2^d) = \mathrm{poly}(n)$

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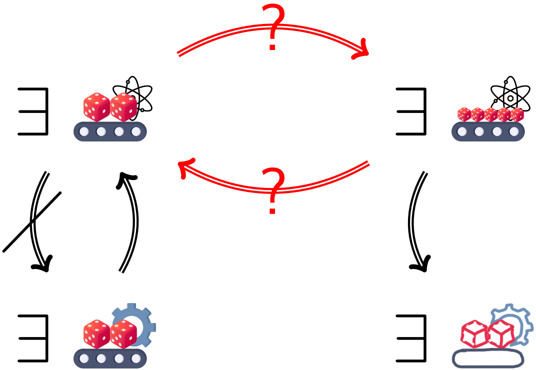
This work: long-PRSs do not imply short-PRSs.

Relations between primitives



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It suffices to show that PD-OWF imply
PromiseBQP $\neq$ PromiseQMA (in a black-box way)!

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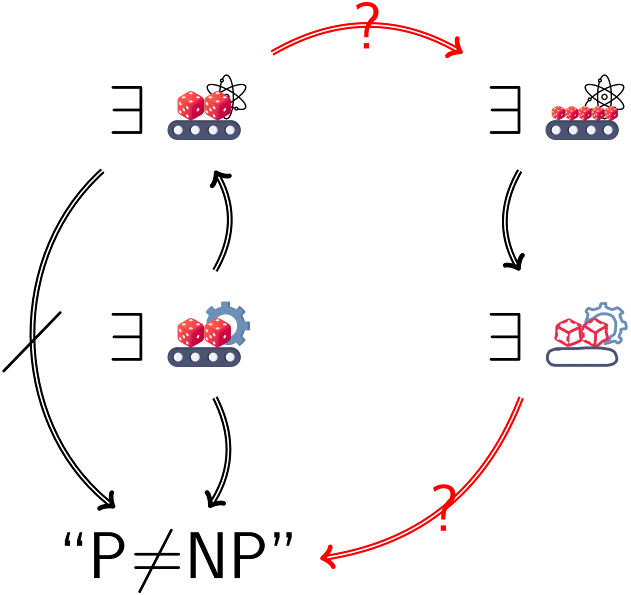

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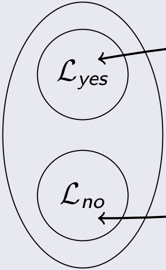
- **Security**. For every QPT inverter $\mathcal{A}$:

$$\Pr_{x \leftarrow \{0,1\}^{m(\lambda)}}\left[F\left(\mathcal{A}(F(x))\right) = F(x)\right] \leq \mathrm{negl}(\lambda). \tag{2}$$

Quantum complexity classes

Definition (PromiseBQP)

$\mathcal{L} = (\mathcal{L}_{yes}, \mathcal{L}_{no}) \in$ PromiseBQP if $\exists$ a QTP $\mathcal{A}$ such that:



$x \in \mathcal{L}_{yes}$, $\Pr[\mathcal{A}(x) = 1] \geq 2/3$

$x \in \mathcal{L}_{no}$, $\Pr[\mathcal{A}(x) = 0] \geq 2/3$

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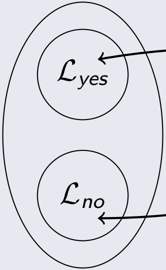

$x \in \mathcal{L}_{yes}$, $\exists \left|\phi\right\rangle, \Pr\left[\mathcal{A}(x, \left|\phi\right\rangle) = 1\right] \geq 2/3$

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Sketch proof

Assume we have a PD-OWF F. The goal is to exhibit a language $\mathcal{L}$ such that $\mathcal{L} \in$ PromiseQMA, and $\mathcal{L} \notin$ PromiseBQP.

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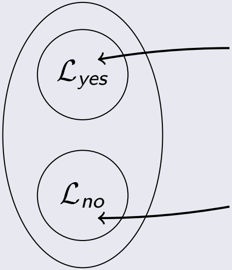

$$\mathcal{L}_{yes} = \left\{ \begin{array}{c} y \text{ that have a pre-image} \\ \text{relative to } F \end{array} \right\}$$

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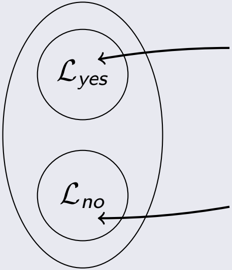

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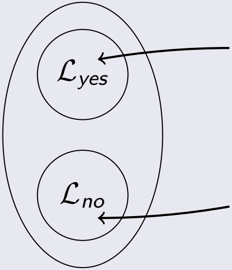

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$Ver(x, y)$ runs $F(x)$ many times and checks that $F(x) = y$ every time.

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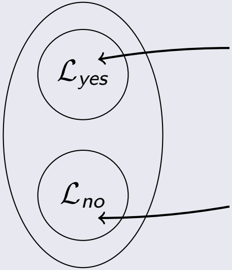

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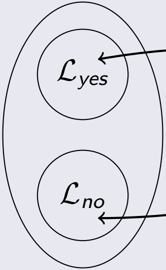

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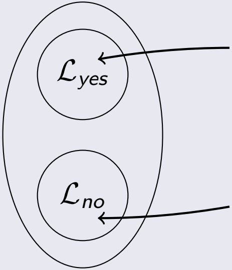

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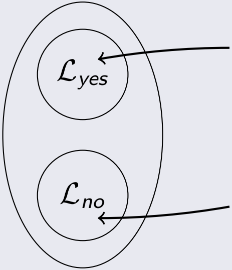

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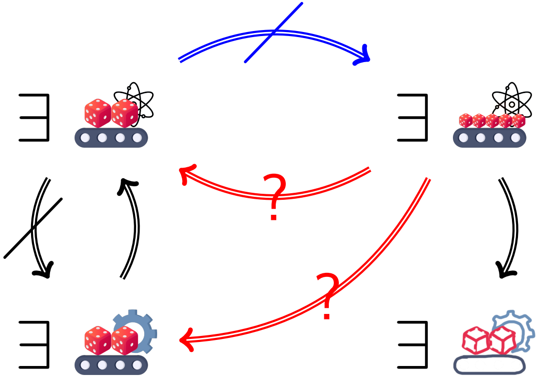
Otherwise, it would break the security definition of PD-OWF: for some y, we can learn a pre-image bit by bit.

Conclusion



Pictures of the presentation are adapted from icons from flaticon.com

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Thank you for your attention!

Bibliography