Discussion of “Diversifying conformal selections”

Yash Nair, Ying Jin, James Yang, and Emmanuel Candès

Ulysse Gazin (LPSM, Université Paris Cité)

ISSI: 26 February 2026

Context and Aim

Prediction:

$$\text{Covariates: } \begin{cases} \text{Predictive Covariate} & X \in \mathcal{X} \\ \text{Diversity Covariate} & Z \in \mathcal{Z} \end{cases}$$

$$\text{Quality Outcome: } Y \in \mathbb{R}$$

Context and Aim

Prediction:

Covariates: $\begin{cases} \text{Predictive Covariate} & X \in \mathcal{X} \\ \text{Diversity Covariate} & Z \in \mathcal{Z} \end{cases}$

Quality Outcome: $Y \in \mathbb{R}$

Aim: Select X with good quality ($Y \geq 0$)
Having diversity for the selected (various Z)

Context and Aim

Prediction: Covariates: $\begin{cases} \text{Predictive Covariate} & X \in \mathcal{X} \\ \text{Diversity Covariate} & Z \in \mathcal{Z} \end{cases}$

Quality Outcome: $Y \in \mathbb{R}$

Aim: Select X with good quality ($Y \geq 0$)
Having diversity for the selected (various Z)

$\mathcal{D}_{\text{test}} = ((X_{n+1}, Z_{n+1}, Y_{n+1}), \cdots, (X_{n+m}, Z_{n+m}, Y_{n+m}))$ observe only X and Z.

$\rightarrow \varphi(\mathcal{S}, \mathbf{Z}) \in \mathbb{R}$: measures diversity of set $\mathcal{S} \subset [\![m]\!]$

Context and Aim

Prediction: Covariates: $\begin{cases} \text{Predictive Covariate} & X \in \mathcal{X} \\ \text{Diversity Covariate} & Z \in \mathcal{Z} \end{cases}$

Quality Outcome: $Y \in \mathbb{R}$

Aim: Select X with good quality ($Y \geq 0$)

Having diversity for the selected (various Z)

$\mathcal{D}_{\text{test}} = ((X_{n+1}, Z_{n+1}, Y_{n+1}), \cdots, (X_{n+m}, Z_{n+m}, Y_{n+m}))$ observe only X and Z.

$\rightarrow \varphi(\mathcal{S}, \mathbf{Z}) \in \mathbb{R}$: measures diversity of set $\mathcal{S} \subset [\![m]\!]$

Diverse FDR control

Construct $\mathcal{R} \subset [\![m]\!]$ such that:

$$\text{FDR}(\mathcal{R}) = \mathbb{E}\left[\frac{\sum_{i \in \mathcal{R}} \mathbb{1}\{Y_{n+i} \leq 0\}}{|\mathcal{R}| \wedge 1}\right] \leq \alpha,$$

$\mathcal{R}$ maximising $\varphi(\mathcal{S}, Z_{n+1}, \cdots, Z_{n+m})$.

- Guarantee over the selected.
- The selection is diverse.

The e-values lens: FDR control

- Quality scores $V(x, y) = -\widehat{\mu}_{\text{train}}(x)\mathbb{1}\{y \leq 0\} + \infty\mathbb{1}\{y > 0\}$
- $\mathcal{D}_{\text{cal}} = (X_1, Z_1, Y_1), \cdots, (X_n, Z_n, Y_n)$
- $\mathcal{D}_{\text{test}} = (X_{n+1}, Z_{n+1}, Y_{n+1}), \cdots, (X_{n+m}, Z_{n+m}, Y_{n+m})$ only X and Z are observed.

The e-values lens: FDR control

- Quality scores $V(x, y) = -\widehat{\mu}_{\text{train}}(x)\mathbb{1}\{y \leq 0\} + \infty\mathbb{1}\{y > 0\}$
- $\mathcal{D}_{\text{cal}} = (X_1, Z_1, Y_1), \cdots, (X_n, Z_n, Y_n)$
- $\mathcal{D}_{\text{test}} = (X_{n+1}, Z_{n+1}, Y_{n+1}), \cdots, (X_{n+m}, Z_{n+m}, Y_{n+m})$ only X and Z are observed.

$\left\{W_{(1)} \geq W_{(2)} \geq \cdots \geq W_{(n+m)}\right\} =$
$\{V(X_1, Y_1), \cdots, V(X_n, Y_n), V(X_{n+1}, 0), \cdots, V(X_{n+m}, 0)\}$

Conformal e-values

$$e_i^{(t)} = \frac{(n+1)\mathbb{1}\left\{V(X_{n+i}, 0) \leq W_{(t)}\right\}}{1 + n - \sum_{k \in [\![n]\!]} \mathbb{1}\left\{V(X_k, Y_k) > W_{(t)}\right\}}, \; i \in [\![m]\!], \; t \in [\![n+m]\!]$$

e-values control [Nair et al., 2025]

Backwards filtration $\mathcal{F}_t = \sigma\left(\mathbb{1}\left\{W_{(t+1)} \text{ from } \mathcal{D}_{\text{cal}}\right\}, \cdots, \mathbb{1}\left\{W_{(n+m)} \text{ from } \mathcal{D}_{\text{cal}}\right\}, \mathbf{Z}^{()}\right)$
If τ stopping time and $\mathcal{R} \in [\![m]\!]$ self-consistent w.r.t. $(e_i^{(\tau)})_{i \in [\![m]\!]}$:

$$\text{FDR}(\mathcal{R}) \leq \alpha$$

Optimising diversity: Step-Up procedure

$O_t = \varphi(\mathcal{R}_t^\star, \mathbf{Z})$ with $\mathcal{R}_t^\star$ self consistent w.r.t. $(e_i^{(t)})_{i \in [\![m]\!]}$ optimising $\varphi(\cdot, \mathbf{Z})$

Aim: Choose the good stopping time τ

Answer: Supermartingale properties of *e*-values $\rightarrow$ Snell envelope for martingale

Optimising diversity: Step-Up procedure

$O_t = \varphi(\mathcal{R}_t^\star, \mathbf{Z})$ with $\mathcal{R}_t^\star$ self consistent w.r.t. $(e_i^{(t)})_{i \in [\![m]\!]}$ optimising $\varphi(\cdot, \mathbf{Z})$

Aim: Choose the good stopping time τ

Answer: Supermartingale properties of e-values $\rightarrow$ Snell envelope for martingale

$\triangleright$ Sequentially for $t \in [\![1, \tau_{\text{BH}}]\!]$:

1. Reward: $R_t = \mathbb{E}[O_t | \mathcal{F}_t]$
2. Envelope: $E_t = R_t \vee \mathbb{E}[E_{t-1} | \mathcal{F}_{t-1}] \geq R_t$

$\triangleright$ $\tau^\star = \max\{t \in [\![1, \tau_{\text{BH}}]\!] \ : \ R_t = E_t\}$ and select $\mathcal{R}_{\tau^\star}^\star$

Optimising diversity: Step-Up procedure

$O_t = \varphi(\mathcal{R}_t^\star, \mathbf{Z})$ with $\mathcal{R}_t^\star$ self consistent w.r.t. $(e_i^{(t)})_{i\in[\![m]\!]}$ optimising $\varphi(\cdot, \mathbf{Z})$

Aim: Choose the good stopping time τ

Answer: Supermartingale properties of e-values $\rightarrow$ Snell envelope for martingale

▷ Sequentially for $t \in [\![1, \tau_{\text{BH}}]\!]$:

1. Reward: $R_t = \mathbb{E}[O_t|\mathcal{F}_t]$
2. Envelope: $E_t = R_t \vee \mathbb{E}[E_{t-1}|\mathcal{F}_{t-1}] \geq R_t$

▷ $\tau^\star = \max\{t \in [\![1, \tau_{\text{BH}}]\!] \;:\; R_t = E_t\}$ and select $\mathcal{R}_{\tau^\star}^\star$

Snell Envelope optimisation [Nair et al., 2025]

If $\tau \leq \tau_{\text{BH}}$ stopping time and $\mathcal{R} \in [\![m]\!]$ self-consistent w.r.t. $(e_i^{(\tau)})_{i\in[\![m]\!]}$:

$$\mathbb{E}[\varphi(\mathcal{R}_{\tau^\star}^\star, \mathbf{Z})|\mathcal{F}_{\tau_{\text{BH}}}] \geq \mathbb{E}[\varphi(\mathcal{R}, \mathbf{Z})|\mathcal{F}_{\tau_{\text{BH}}}]$$

Optimising diversity: Step-Up procedure

$O_t = \varphi(\mathcal{R}_t^\star, \mathbf{Z})$ with $\mathcal{R}_t^\star$ self consistent w.r.t. $(e_i^{(t)})_{i\in[\![m]\!]}$ optimising $\varphi(\cdot, \mathbf{Z})$

Aim: Choose the good stopping time τ
Answer: Supermartingale properties of e-values $\to$ Snell envelope for martingale

$\triangleright$ Sequentially for $t \in [\![1, \tau_{\text{BH}}]\!]$:

1. Reward: $R_t = \mathbb{E}[O_t | \mathcal{F}_t]$
2. Envelope: $E_t = R_t \vee \mathbb{E}[E_{t-1} | \mathcal{F}_{t-1}] \geq R_t$

$\triangleright$ $\tau^\star = \max\{t \in [\![1, \tau_{\text{BH}}]\!] \ : \ R_t = E_t\}$ and select $\mathcal{R}_{\tau^\star}^\star$

Snell Envelope optimisation [Nair et al., 2025]

If $\tau \leq \tau_{\text{BH}}$ stopping time and $\mathcal{R} \in [\![m]\!]$ self-consistent w.r.t. $(e_i^{(\tau)})_{i\in[\![m]\!]}$:

$$\mathbb{E}[\varphi(\mathcal{R}_{\tau^\star}^\star, \mathbf{Z}) | \mathcal{F}_{\tau_{\text{BH}}}] \geq \mathbb{E}[\varphi(\mathcal{R}, \mathbf{Z}) | \mathcal{F}_{\tau_{\text{BH}}}]$$

Problem: $\mathbb{E}$ is not known (distribution free)

Optimising diversity: Step-Up procedure

$O_t = \varphi(\mathcal{R}_t^\star, \mathbf{Z})$ with $\mathcal{R}_t^\star$ self consistent w.r.t. $(e_i^{(t)})_{i\in[\![m]\!]}$ optimising $\varphi(\cdot, \mathbf{Z})$

Aim: Choose the good stopping time τ
Answer: Supermartingale properties of e-values $\rightarrow$ Snell envelope for martingale

▷ Sequentially for $t \in [\![1, \tau_{\text{BH}}]\!]$:

1. Reward: $R_t = \mathbb{E}_{\text{exch}}[O_t | \mathcal{F}_t]$
2. Envelope: $E_t = R_t \vee \mathbb{E}_{\text{exch}}[E_{t-1} | \mathcal{F}_{t-1}] \geq R_t$

▷ $\tau^\star = \max\{t \in [\![1, \tau_{\text{BH}}]\!] \ : \ R_t = E_t\}$ and select $\mathcal{R}_{\tau^\star}^\star$

Snell Envelope optimisation [Nair et al., 2025]

If $\tau \leq \tau_{\text{BH}}$ stopping time and $\mathcal{R} \in [\![m]\!]$ self-consistent w.r.t. $(e_i^{(\tau)})_{i\in[\![m]\!]}$:

$$\mathbb{E}_{\text{exch}}[\varphi(\mathcal{R}_{\tau^\star}^\star, \mathbf{Z}) | \mathcal{F}_{\tau_{\text{BH}}}] \geq \mathbb{E}_{\text{exch}}[\varphi(\mathcal{R}, \mathbf{Z}) | \mathcal{F}_{\tau_{\text{BH}}}]$$

Problem: $\mathbb{E}$ is not known (distribution free)$\Rightarrow$ Use computable $\mathbb{E}_{\text{exch}}$

Summary

Congratulations on the great work & talk!
Strong application in “fair” ML!
Paves the way to general selection while optimising!

Summary

Congratulations on the great work & talk!
Strong application in "fair" ML!
Paves the way to general selection while optimising!

What I like

- Use of $\mathbb{E}_{\text{exch}}$ and exchangeability properties
- General theoretical methods that can applied for various problems
- With various shortcuts

Summary

Congratulations on the great work & talk!
Strong application in "fair" ML!
Paves the way to general selection while optimising!

What I like

- Use of $\mathbb{E}_{\text{exch}}$ and exchangeability properties
- General theoretical methods that can applied for various problems
- With various shortcuts

Questions

1. Can we apply this in a diverse testing problem? Replace Y by $\mathbb{1}\{Y > 0\}$?
2. How far is $\mathbb{E}_{\text{exch}}[\varphi(\mathcal{R}, \mathbf{Z})]$ of $\mathbb{E}[\varphi(\mathcal{R}, \mathbf{Z})]$? Coupling properties? Wasserstein distance between real distribution of (X, Z, Y) and ones such that $\mathbb{E}_{\text{exch}}$ is defined?
3. Can we create prediction region with FCR control? Informative diverse selection with e-values? And V not the clipped score
4. Why the 1.3α control in the relaxed problem? Is it possible to remove it?

Reference I

Nair, Y., Jin, Y., Yang, J., and Candes, E. (2025).
Diversifying conformal selections.
arXiv preprint arXiv:2506.16229.