Conference Decorated Tree-like structures for singular dynamics

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Abstract

Conference organised in Nancy from 27 to 29 of May 2024, funded by the ERC Starting Grant LoRDeT (Low Regularity Dynamics via Decorated Trees) `https://cordis.europa.eu/project/id/101075208`. The PI of the project is Yvain Bruned.

Monday 27 May 2024

Talks take place at IECL, Salle Döblin.

- 14h00-14h50: Dominique Manchon.

  Free Novikov algebras and Hopf algebras of multi-indices

  The free pre-Lie algebra generated by a set A projects itself on the free Novikov algebra generated by the same set. The first one was described by F. Chapoton and M. Livernet, and by A. Dzhumadildaev and C. Löfwall independently, as the linear span of A-decorated rooted trees. The second one was explicitly described by the second group of authors as the linear span of decorated multi-indices subject to a population condition. By duality, one therefore gets a commutative Hopf algebra of polynomials in multi-indices, together with an embedding into the Hopf algebra of A-decorated forests. We shall give in this talk a proposal for choosing a pairing such that the coproduct is explicitly given in terms of admissible cuts, similarly to A. Connes' and D. Kreimer's formula for rooted forests. Joint work with X. Gao and Z. Zhu (Lanzhou University).

- 14H50-15h40: Paul Laubie.

  Operadic twisting of rooted trees and application to hypertrees

  A celebrated result in operad theory is the theorem of Chapoton-Livernet describing the operadic structure of PreLie as a combinatorial construction on rooted trees. The operadic twisting of PreLie, a construction arising from deformation theory consideration, also admits a combinatorial interpretation using rooted trees which was explicitly described by Dotsenko and Khoroshkin.

An important observation is that the operadic twisting of PreLie allows one to delicately extract the Lie operad embedded inside PreLie. From this observation, one may wonder if there could be some analogous story when considering combinatorial object related to rooted trees. We will show that the case of forest of rooted hypertrees allows us to show a non-trivial embedding of operad, giving a positive answer to a conjecture of Dotsenko.

- 15h40-16h10: Coffee break.
- 17h00-18h00: Yvain Bruned. (European Non-Associative Algebra Seminar, online)

Novikov algebras and multi-indices in regularity structures

In this talk, we will present multi-Novikov algebras, a generalisation of Novikov algebras with several binary operations indexed by a given set, and show that the multi-indices recently introduced in the context of singular stochastic partial differential equations can be interpreted as free multi-Novikov algebras. This is parallel to the fact that decorated rooted trees arising in the context of regularity structures are related to free multi-pre-Lie algebras. This is a joint work with Vladimir Dotsenko.

- 19h30-22h30: Conference dinner, Grand Café Foy.

Tuesday 28 May 2024

Talks take place at IECL, Salle Döblin.

- 9h30-10h20: Nils Berglund.

Perturbative renormalisation of the Φ_d^4 model viewed as generalized Wick renormalisation

The Φ_d^4 model is a well-known model from Euclidean quantum field theory, which is known to be super-renormalisable for all dimensions $d < 4$. Most existing proofs of this result rely on quite heavy analytical tools. I will present an approach to the perturbative renormalisation of this model that takes advantage of algebraic techniques to take care of the combinatorics. While the approach still relies on an analytical bound on so-called BPHZ renormalisation of Feynman diagrams, the proof that the perturbative series is renormalisable, as well as explicit computations of the terms of the series, can be viewed in terms of a generalised Wick map acting on polynomials of only two variables. The method has been proved to work for $d = 3$, an may be extendable to all $d < 4$. Joint work with Tom Klose (Warwick) and Nikolas Tapia (Berlin).

- 10h20-10h50: Coffee break.
- 10h50-11h40: Alberto Bonicelli.

Convergence results in the stochastic sine-Gordon model from a QFT

perspective

The importance of the sine-Gordon model in $1+1$ spacetime dimensions resides in the integrability of the field theory that it describes. A recent result showed how, within the setting of algebraic quantum field theory, this property translates into a convergence result for both the formal series associated to the S-matrix and to the interacting field. After introducing an algebraic approach to the perturbative study of singular stochastic PDEs, I will show how an adaptation of the aforementioned results yields convergence of the solution momenta for a stochastic version of the sine-Gordon equation. Interestingly enough, our two-step procedure passes through the quantum theory and recollects the stochastic information via a classical limit. We shall present and implement in simple situations Ecalle's alien calculus when several critical times are involved, within the mechanism of acceleration. The talk will be essentially self-contained, all relevant notions will be introduced, with elementary examples.

- 11h40-12h30: Yingtong Hou.

  Multi-indice B-series

- 12h30-14h30: Lunch.

  Talks take place at IECL, Salle Conférence.

- 14h30-15h20: Damien Calaque.

  Discrete models for E_n-algebras in QFT

  I will explain a circle of ideas that aim at combining discretization methods with homotopical algebra (mainly E_n-algebras, but also factorisation algebras in the sense of Costello-Gwilliam) in order to construct observables for quatum fieeld theories. Depending on the time and audience, I will present (a subset of) three ongoing projects:

  - one connects E_n-algebras with vertex models in statistical physics, solving a conjecture of Kontsevich (in progress with Damien Lejay).
  - another one aims at explaining the claim that renormalization is trivial for topologial field theories , using E_n-algebras and factorisation algebras (joint with Victor Carmona).
  - yet another one reinterprets some results of Migdal and Witten on 2d Yang-Mills theory in terms of factorisation algebras (in progress with Giovanni Felder).

- 15h20-15H50: Coffee break.

- 15H50-16h40: Camille Laurent-Gengoux.

  On Poisson integrators

  I will in fact present the work of my PhD student Oscar Cosserat (in codirection

with Vladimir Salnikov at La Rochelle). For symplectic Hamiltonian ODEs, there is the well-known notion of symplectic (= integrator integrators preserving the symplectic structure) that have their merits. But several ODE of interest are only Hamiltonian for a Poisson structure, not a symplectic structure. It is therefore logical to look for Poisson integrator (= integrators that preserve the Poisson structure). In his PhD, Cosserat used an old tool, Weinstein's symplectic groupoid of a Poisson structure, to find extremely explicit Poisson integrators at any order. Theory and numerical examples will be developped.

- 16h40-17h30: Harprit Singh

Renormalisation of singular SPDEs on Riemannian Manifolds

I shall discuss a generalisation of the theory of regularity structures which is able to treat singular SPDEs on manifolds with values in vector bundles. After presenting the analytic modifications required in comparison to the flat setting, I shall discuss how positive and negative renormalisation can be implemented in an intrinsic manner. Based on joint work with M. Hairer

Wednesday 29 May 2024

Talks take place at IECL, Salle Conférence.

- 9h00-9h50: Pierre Clavier.

Shuffle of rooted forests

I will present a shuffle product of rooted forests which has applications in number theory and rough paths theory. After introducing this product, I wil state some of its properties, and in particular its related coalgebraic structures. This provide an example of non-associative Hopf algebra. Join work with Douglas Modesto.

- 9h50-10h20: Coffee break.

- 10h20-11h10: Abdenacer Makhlouf.

An overview of Hom-type twisted algebra structures

In this talk we deal with Hom-type algebras and BiHom-type, bialgebras and Hopf algebras. The first instances comes from quantum deformations of algebras of vector fields. The relevant examples are obtained for Witt and Virasoro algebra, where usual derivation is replaced by a Jackson derivation, the Leibniz identity being twisted by an algebra map. The description of the new structure gave rise to a structure generalizing Lie algebras, called Hom-Lie algebras or quasi-Lie algebras introduced by Larsson and Silvestrov. A pending Hom-associative algebras and some other Hom-type generalizations of non-associative algebras and their dualization were introduced by Makhlouf and Silvestrov. The main feature is that the classical identities are twisted by homomorphisms. A study from the point of view of monoidal categories were considered by Caenepeel and Goyvaerts for Hom-algebras, then generalized by Graziani, Makhlouf, Mennini

and Panaite to introduce BiHom-type algebras. The purpose of my talk is to review the main results, provide some key constructions and discuss their universal algebra (free algebra, trees, operads. . .)

- 11h10-12h00: Oscar Cosserat.

  Hamilton-Jacobi equation for Poisson Hamiltonian dynamics and tree-like algebraic structures

  We recall how Hamiltonian dynamics on a Poisson manifold is related to its symplectic groupoid by a Hamilton-Jacobi equation. The induced correspondence is of prime interest in the numerical analysis of geometrical integration (see Symplectic groupoids for Poisson integrators, O.C., 2023). We then introduce the so-called Farmer sequence to explain some underlying tree-like algebraic structure solving this Hamilton-Jacobi equation. This work is an ongoing discussion with various people including A. Laurent and C. Zhu.