Conference Singular SPDEs, invariant measures and discrete models

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Abstract

Conference organised in Nancy from 4 to 6 of December 2024, funded by the ERC Starting Grant LoRDeT (Low Regularity Dynamics via Decorated Trees) https://cordis.europa.eu/project/id/101075208. The PI of the project is Yvain Bruned.

Wednesday 4 December 2024

Talks take place at FST, Bâtiment Victor Grignard, Amphi 5.

- 14h00-14h50: Lorenzo Zambotti.

  Convergence of dynamical stationary fluctuations

  We present a general result that ensures convergence of a sequence of stationary Markov processes, provided a few assumptions are satisfied. Our approach builds on the characterization of the resolvents of the limit Markov process through an identity involving the associated Dirichlet form. We first illustrate our result by presenting a simple proof of the convergence of the fluctuations of the one-dimensional zero-range process towards the stochastic heat equation, in which case our approach allows to circumvent the use of the classical Boltzmann-Gibbs principle. Then we apply our result to an evolving pair of reflected interfaces. The result is new and it is unclear whether existing methods could prove such a convergence. Joint work with Cyril Labbé, Benoît Laslier and Fabio Toninelli.

- 14h50-15h40: Pawel Duch.

  Construction of measure of fractional Φ^4_3 model in full subcritical regime

  We present a construction of the Gibbs measure of the fractional Φ^4 model of Euclidean quantum field theory in three-dimensions. The measure is obtained as a perturbation of the Gaussian measure with covariance given by the inverse of a fractional Laplacian. Since the Gaussian measure is supported in the space of Schwartz distributions and the quartic interaction potential of the model involves pointwise products, to construct the measure it is necessary to solve the so-called

renormalization problem. To this end, we study the stochastic quantization equation, which is a nonlinear parabolic PDE driven by the white noise. We prove a certain a priori estimate for solutions of this equation using the flow equation approach to singular stochastic PDEs and the maximum principle. We consider the entire range of powers of the fractional Laplacian for which the model is subcritical (i.e. super-renormalizable). Based on joint work with M. Gubinelli and P. Rinaldi.

- 15h40-16h10: Coffee break.
- 16h10-17h00: Xue-Mei Li.

- 19h30-22h30: Conference dinner at Grand Café Foy.

Thursday 5 December 2024

Talks take place at FST, Bâtiment Victor Grignard, Amphi 5.

- 9h30-10h20: Sarah-Jean Meyer.

  A martingale problem and a FBSDE for sine-Gordon

  I will present a new characterization of the sine-Gordon quantum field theory (QFT) based on a martingale problem and a forward backward SDE (FBSDE). I will first explain the general idea of the FBSDE approach and our construction of the sine-Gordon QFT with this approach. From there, we can formulate a martingale problem for the sine-Gordon QFT for $\beta^2 < 6\pi$. Under certain conditions, the martingale problem gives a unique characterization of the measure. This is based on joint work in progress with Massimiliano Gubinelli.

- 10h20-10h50: Coffee break.
- 10h50-11h40: Martin Hairer.

- 11h40-12h30: Máté Gerencsér,.

  Discretisations of stochastic Allen-Cahn equations

  We study discrete approximations of the Allen-Cahn equations driven by space-time white noise. Focusing on strong convergence rates, we address the questions of the effect of the error topology on the error rate, revisit an old question on accelerating schemes by sampling from the underlying Ornstein-Uhlenbeck process, and take a look at higher dimensions, where renormalisation also has to be taken into accoount. Based on joint works with Ana Djurdjevac, Helena Kremp, and Harprit Singh.

- 12h30-14h30: Lunch.

  Talks take place at FST, Bâtiment Victor Grignard, Amphi 5.

- 14h30-15h20: Antoine Gloria.

  Homogenization of the 2d Euler system in porous media

  In this talk I will discuss some homogenization results for the 2D Euler system with impermeable inclusions. The main difficulty is the homogenization of the transport equation for the vorticity. Localization for the latter could indeed rule out separation of scales. To prevent this and establish homogenization towards a variant of the Euler system, we combine classical results from different areas : homogenization of elliptic systems with stiff inclusions, unique ergodicity for dynamical systems, complex analysis à la Rado-Kneser-Choquet, and large-scale regularity. I will conclude with some open problems on invariant measures in the random case. This is joint work with Mitia Duerinckx (ULB).

- 15h20-15h50: Coffee break.

- 15h50-16h40: Guillaume Barraquand.

  Stationary measures for the open KPZ equation

  It is well-known that a Brownian motion is left invariant by the KPZ equation on R, modulo a global height shift. For the KPZ equation on domains with boundaries, such as $\mathbb{R}_+$ or $[0, L]$, stationary measures are more complicated and have been determined recently, taking the scaling limit of discrete integrable models. My talk will review this, presenting works by Corwin-Knizel, Bryc-Kuznetsov-Wang-Wesolowski, as well as my joint works with Pierre Le Doussal and with Ivan Corwin and Zongrui Yang

- 16h40-17h30: Giuseppe Cannizzaro.

  Superdiffusive Central Limit Theorem for the critical Stochastic Burgers Equation

  The Stochastic Burgers Equation (SBE) was introduced in the eighties by van Beijren, Kutner and Spohn as a way to encode the fluctuations of driven diffusive systems with one conserved quantity. In the subcritical dimension $d = 1$, it coincides with the derivative of the KPZ equation whose large-scale behaviour is polynomially superdiffusive and given by the KPZ Fixed Point, and in the super-critical dimensions $d > 2$, it was recently shown to be diffusive and rescale to a biased Stochastic Heat equation. At the critical dimension $d = 2$, the SBE was conjectured to be logarithmically superdiffusive with a precise exponent but this has only been shown up to lower order corrections. In the present talk, we pin down the logarithmic superdiffusivity exactly by identifying the limit of the so-called diffusion coefficient and show that, once the logarithmic corrections to the scaling are taken into account, the solution of the SBE satisfies a central limit theorem. Joint work with Q. Moulard and F. Toninelli.

Friday 6 December 2024

Talks take place at FST, Bâtiment Victor Grignard, Amphi 5.

- 9h00-9h50: Ajay Chandra.

  Non-commutative singular SPDE

  In this talk, I will describe some recent progress on singular stochastic partial differential equations in the setting of non-commutative probability theory - examples will include the stochastic quantization of Fermionic quantum field theories and also the setting of free probability. This is based on joint work with Martin Hairer and Martin Peev.

- 9h50-10h20: Coffee break.

- 10h20-11h10: Ludovic Goudenège.

  Order of convergence of a time-space discretization of the stochastic heat equation with distributional reaction term

  We present the order of convergence of a time-space discretization scheme applied to the stochastic heat equation driven by space-time white noise, with a reaction term b modeled as a distribution. In the numerical scheme, we account for the singularity of the reaction term by employing a regularization approach that is compatible with its potential negative Besov regularity. Our numerical procedure combines a finite difference method in space with an explicit Euler scheme, along with the taming of the reaction term. The main result establishes the rate of convergence of the scheme in terms of the discretization parameters, which depends on the Besov regularity of b. In particular, one can consider two interesting cases: first, even when b is only a (finite) measure, as in a skew-type stochastic heat equation, a rate of convergence is obtained. On the other hand, when b is a bounded measurable function, we recover the (almost) optimal rate of convergence $(\frac{1}{2} - \varepsilon)$-in space and $(\frac{1}{4} - \varepsilon)$-in time. This is joint work with El Mehdi Haress and Alexandre Richard.

- 11h10-12h00: Cyril Labbé.

  Edge of the spectrum of the Anderson hamiltonian with correlated gaussian noise

  I will consider the discrete Laplacian on $\mathbb{Z}^d$ perturbed by a correlated gaussian noise. I will present some results on the largest eigenvalues / eigenfunctions of this random operator, and their relationship with the maximas of the noise. I will also discuss the i.i.d. case, which was already covered in the literature. Based on a joint work with Giuseppe Cannizzaro and Willem van Zuijlen.