Operads, Symmetries for QFT and Singular SPDEs

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Abstract

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Wednesday 3 December 2025

Talks take place at IECL, Salle de Conférences.

- 14h00-14h50: Vladimir Dotsenko.

  Yamaguti algebras and noncrossing partitions

  Among various types of algebras originating in differential geometry, those known as Lie-Yamaguti algebras still remain somewhat mysterious algebraically. Some months ago, A.Das introduced an "associative" version of Yamaguti algebras which are supposed to serve as envelopes of Lie-Yamaguti algebras. Together with Frédéric Chapoton, we found a combinatorial description of free Yamaguti algebras and the Yamaguti operad via noncrossing partitions. I will report on that work.

- 14h50-15h40: Benoit Fresse.

  Partition Lie algebras of graphs and Maurer-Cartan elements associated to E_n-operads

  I will revisit the results of previous research, carried out in collaborations with Victor Turchin and Thomas Willwacher, which give an interpretation of certain Lie algebras of graphs in terms of the deformation theory of E_n-operads. These Lie algebras of graphs are variations of the graph complexes introduced by Kontsevich, with ideas coming from the theory of Feynman diagrams, in subjects at the interface of geometric topology and mathematical physics.
  In the first part of the talk, I will review the definition of the Lie algebras of graphs considered in our work, of the E_n-operad interpretation of certain graph

complex models, and I will give a survey of known results in characteristic zero, including the connection of graph complex models with the theory of associators and the Grothendieck-Teichmüller theory in the E_2-operad case.
Then I will tackle a work in progress about the definition of complexes of graphs equipped with a partition Lie algebra structure in the odd characteristic setting (partition Lie algebras are derived versions of usual restricted Lie algebra structures). I will explain that classical formality constructions can be adapted in order to associate, to certain models of E_n-operads, a solution of the Maurer-Cartan equation in these partition Lie algebras of graphs.

- 15h40-16h10: Coffee break.
- 16h10-17h00: Muriel Livernet.

  Algebraic structures on m-Dyck paths.

  In this talk, I will present ongoing joint work with Maria Ronco on algebraic structures defined on m-Dyck paths, originally introduced by Lopez, Préville-Ratelle, and Ronco. These structures decompose an associative operation into m distinct operations. I will describe this structure as algebras over a (co)operad m-Dyck. I will also describe a related structure on prime m-Dyck paths. This talk is an opportunity to explore algebraic techniques—such as species, operads, and cooperads—applied to a fundamental combinatorial object.
- 19h30-22h30: Conference dinner at Grand Café Foy.

Thursday 4 December 2025

Talks take place at IECL, Salle de Conférences.

- 9h30-10h20: Adrien Busnot Laurent.

  The algebraic structure of exotic (Lie)-Butcher series for the foundations of stochastic geometric numerical integration

  The exotic Butcher series are an algebraic object used in numerical analysis for representing Taylor expansions of the law of stochastic flows. Their use became unavoidable as the calculations of order conditions for high order become increasingly tedious, especially in the recent creation of intrinsic numerical methods on manifolds. In this talk, we will motivate the use of exotic B-series from concrete numerical questions and we will draw deep links between Hopf algebra structures and crucial numerical theories, with an emphasis on backward error analysis. We will further justify the use of such algebraic object with numerical experiments, universal geometric characterisations, and the characterisation of measure-preserving methods. This is joint work with Eugen Bronasco (Chalmers University), Baptiste Huguet (Univ Rennes), and Hans Munthe-Kaas (University of Bergen and University of Tromsø).

- 10h20-10h50: Coffee break.

- 10h50-11h40: Harprit Singh.

  Homogenisation of singular SPDEs

  First I shall motivate the study of SPDEs in heterogeneous environments using the examples of the Φ^4 and the parabolic Anderson model. Then, after recalling the solution theory of the dynamical Φ^4 and g-PAM equation, I shall discuss the periodic homogenisation problem for these singular SPDEs. This is, in part, joint work with M. Hairer.

- 11h40-12h30: Nguyen Viet Dang.

  The Yang–Mills Measure on Surfaces

  We report on various works in progress with Bonthonneau–Chhaibi–Rivière–To and Nohra. On a Riemannian surface, we construct a Gibbs measure on the space of distributional connections, the so called "2-dimensional Yang–Mills measure". We rely on some dynamical system to fix the gauge on the space of connections and study transport equations with random driving force. We show how to recover the formulas for 2D Yang Mills theory as can be found in the works of Witten, Driver, Sengupta and Lévy. In the spirit of the work of Chevyrev, we also discuss how to recover our measure as scaling limit of discrete measures from lattice gauge theories on the surface which is joint work with Nohra.

- 12h30-14h30: Lunch.

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- 14h30-15h20: Alberto Cattaneo.

  Yang–Mills theory from a topological theory

  Four-dimensional Yang–Mills theory can be obtained from a topological field theory of Schwarz type via a procedure known as BV pushforward, which amounts in integrating out some gauge-fixed fields. This turns out to be an equivalence in the sense that it establishes an isomorphism between the observables of the two theories, which in turns implies that their expectation values can be computed in either theory with the same outcome. I will give an introduction to the basic concept, including the BV pushforward, and discuss in details an easier case.

- 15h20-15h50: Coffee break.

- 15h50-16h40: Nils Berglund.

  Perturbative renormalisation of the $\Phi^4_{d-\varepsilon}$ model via generalized Wick maps

We consider the perturbative renormalisation of the Φ^4_d model from Euclidean Quantum Field Theory for any, possibly non-integer dimension $d < 4$. The so-called BPHZ renormalisation, named after Bogoliubov, Parasiuk, Hepp and Zimmermann, is usually encoded into extraction-contraction operations on Feynman diagrams, which have a complicated combinatorics. We show that the same procedure can be encoded in the much simpler algebra of polynomials in two unknowns X and Y, which represent the fourth and second Wick power of the field. In this setting, renormalisation takes the form of a "Wick map" which maps monomials into Bell polynomials. The construction makes use of recent results by Bruned and Hou on multiindices, which are algebraic objects of intermediate complexity between Feynman diagrams and polynomials.
Joint work with Tom Klose and Nikolas Tapia.

- 16h40-17h30: Alberto Bonicelli.

  Representation of the semigroup for Itô diffusions via (multi-indices) exotic B-series and Feynman diagrams

  In this talk, after justifying the expansion of the semigroup of a one-dimensional Itô diffusion as a power series in time, I will build on previous results on expansions labelled by exotic rooted trees to derive an explicit expression for the combinatorial factors involved. A key step is the extension of the notion of tree factorial and Connes-Moscovici weights to this richer family of rooted trees. As a reult, we obtain an exotic Butcher series representation of the semigroup, suitable for a comparison with the perturbative path integral construction of the statistics of the diffusion, known in the literature as Martin-Siggia-Rose formalism. Computations in the latter framework are based on the erroneous assumption that the measure of the path integral can be seen as a perturbation of a Gaussian measure. Resorting to multi-indices to represent pre-Feynman diagrams, I will shed some light on why, even if starting from such an assumption, the results happen to be correct.

Friday 5 December 2025

Talks take place at IECL, Salle de Conférences.

- 9h00-9h50: Nurlan Ismailov.

  Symmetric elements in a free Novikov algebra

  Let $\mathbb{F}$ be a field of characteristic zero. An algebra is called *right-symmetric* if it satisfies the identity

  $$(a, b, c) = (a, c, b)$$

  where $(a, b, c) = (ab)c - a(bc)$ is the associator of a, b, c.

A right-symmetric algebra is said to be *Novikov* if it satisfies the additional identity

$$a(bc) = b(ac).$$

Let $P_n = \mathbb{F}[x_1, \ldots, x_n]$ be the polynomial algebra over $\mathbb{F}$ in the variables $x_1, \ldots, x_n$. Denote by $W_n = \{u\partial_i \mid u \in P_n, 1 \leq i \leq n\}$ the Witt algebra of index n (the Lie algebra of all derivations of P_n). Define

$$u\partial_i \cdot v\partial_j = (v\partial_j(v))\partial_i$$

Then $L_n = (W_n, \cdot)$ is right-symmetric, called the right-symmetric Witt algebra. If $n = 1$, then L_1 becomes a Novikov algebra, but if $n > 1$, it is not Novikov. The right-symmetric Witt algebra L_1, more explicitly Novikov-Witt algebra, generates the variety of all Novikov algebras over $\mathbb{F}$ [1], [2]. The commutator of L_n corresponds to the Witt algebra W_n of index n. Finding a criterion for determining Lie elements and solving the identity problem for W_1 are well-known open questions [3].
In our talk, we consider the symmetrization product $a \circ b = a \cdot b + b \cdot a$ of the multiplication of L_1. We focus on a subalgebra of $L_1^{(+)}$ generated by a set X and refer to its elements as symmetric elements of L_1. We find a basis for the space of symmetric elements in terms of differential polynomials. Furthermore, we provide a criterion for identifying elements of the space of symmetric elements in L_1. Finally, we give a complete description of the module structure of this space over the symmetric group.
The talk is based on the results of joint work with A Dzhumadil'daev.

References

[1] A. Dzhumadildaev, C. Löfwall, *Trees, free right-symmetric algebras, free Novikov algebras and identities*, Homology Homotopy Appl., Vol. 4(2), (2002), 165–190.

[2] L. Makar-Limanov L., U. Umirbaev *The Freiheitssatz for Novikov algebras*,TWMS J. Pure Appl. Math., Vol. 2(2), (2011), 228–235.

[3] Yu. Razmyslov *Identities of Algebras and Their Representations.* Translations of Mathematical Monographs. Vol. 138., Amer. Math. Soc., Providence, RI, 1994.

- 9h50-10h20: Coffee break.
- 10h20-11h10: Carlo Bellingeri.

Wick products and probabilistic Feynmann diagrams

In this talk, I'll give a gentle introduction to one idea that shows up both in Quantum Field Theory and in the study of singular SPDEs: Wick products. They're a way of making sense of polynomial functions of random fields and of keeping track of the many combinatorial interactions that appear when randomness becomes complicated. Differently to the usual way using set partitions or Hopf algebras, we use probabilistic Feynman diagrams to express

the main ideas. This perspective leads to very general identities and suggests how the usual theory of Gaussian chaos can be extended to much broader settings. Joint work with Emilio Ferrucci (University of Oxford).

- 11h10-12h00: Dominique Manchon.

  Rough paths and controlled rough paths: a general Hopf-algebraic setting

  We present rough paths and controlled rough paths associated to a broad class of combinatorial Hopf algebras, encompassing T. Lyons' original formulation, M. Gubinelli's branched presentation, and the planarly branched presentation associated to the Munthe-Kaas–Wright Hopf algebra of planar forests. Joint work with Xing Gao, Nannan Li and Zhicheng Zhu.